

Stereographic Projection

Applications in Rock slope stability

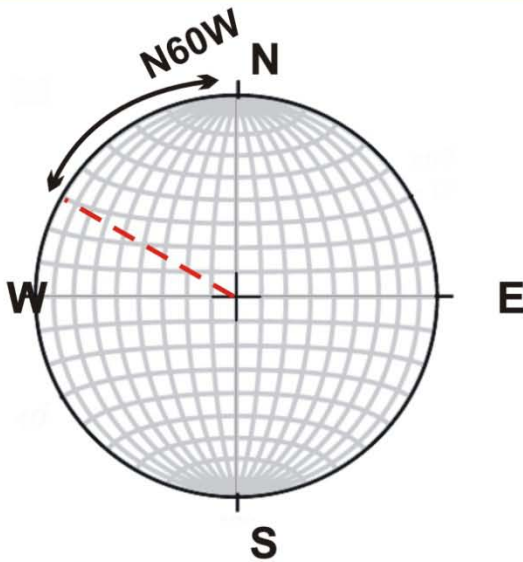
Dr. Mohamed Abdel Wahed

I- Plotting the projection of a line

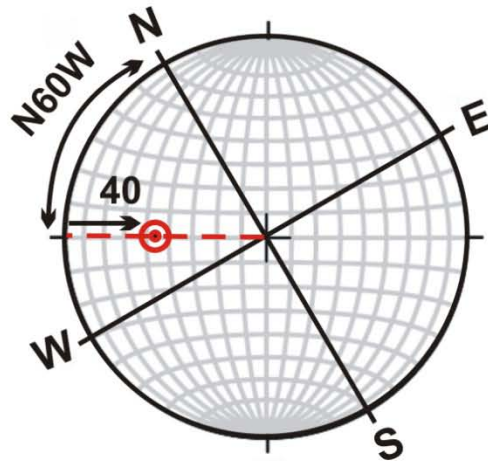
Draw the projection of a line that has the following orientation :

The direction (= trend = bearing) of the line = N60W .

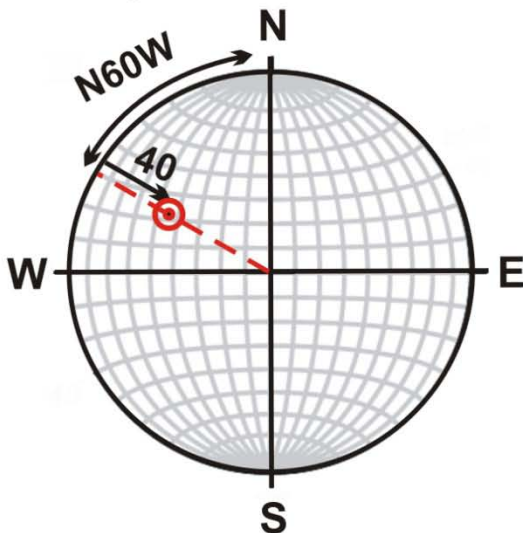
Angle of plunge = 40 .



1- Draw the direction of the line (N60W) to the center of the circle only.



2- Rotate the direction (trend) of the line to the horizontal axis of the net. Measure the angle of plunge of the line (40) and draw a point.



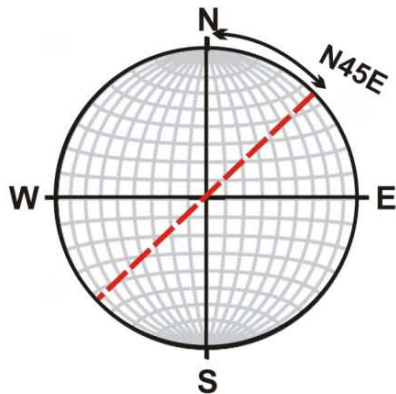
3- Rotate to the north position.

II- Plotting the projection of a plane

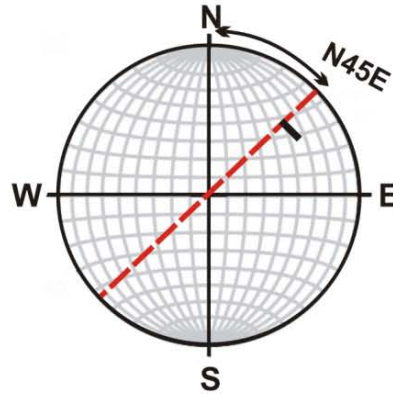
Given : strike- true dip - direction of dip

Draw the projection of a plane of the following orientation :

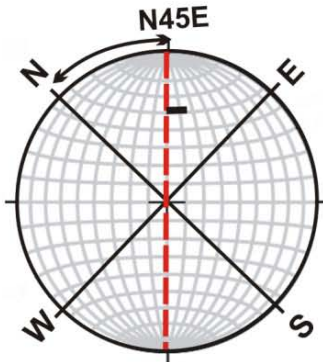
Strike = N45E . Dip = 60 SE .



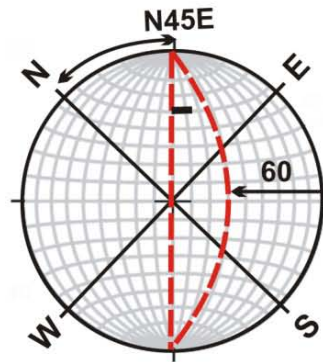
1- Draw the direction of the strike (N45E).



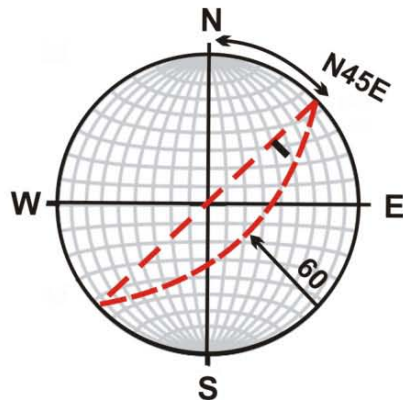
2- Put a mark on the strike line towards the direction of dip.



3- Rotate the strike line to the vertical axis of the net.



4- Measure the angle of dip of the plane (60) on the horizontal axis of the net, and draw the arc. The angle of dip must be measured from the side of the direction of dip.



5- Rotate to the north position.

III- Plotting the projection of a plane

Given : true dip - direction of dip

Draw the projection of a plane of the following orientation :

Dip = 60 , direction of dip = S30W

1- Draw the trend of the true dip (S30W). Draw the line to the center of the circle only.

2- Rotate the line to the horizontal axis of the net. Measure the amount of dip (60) on that line.

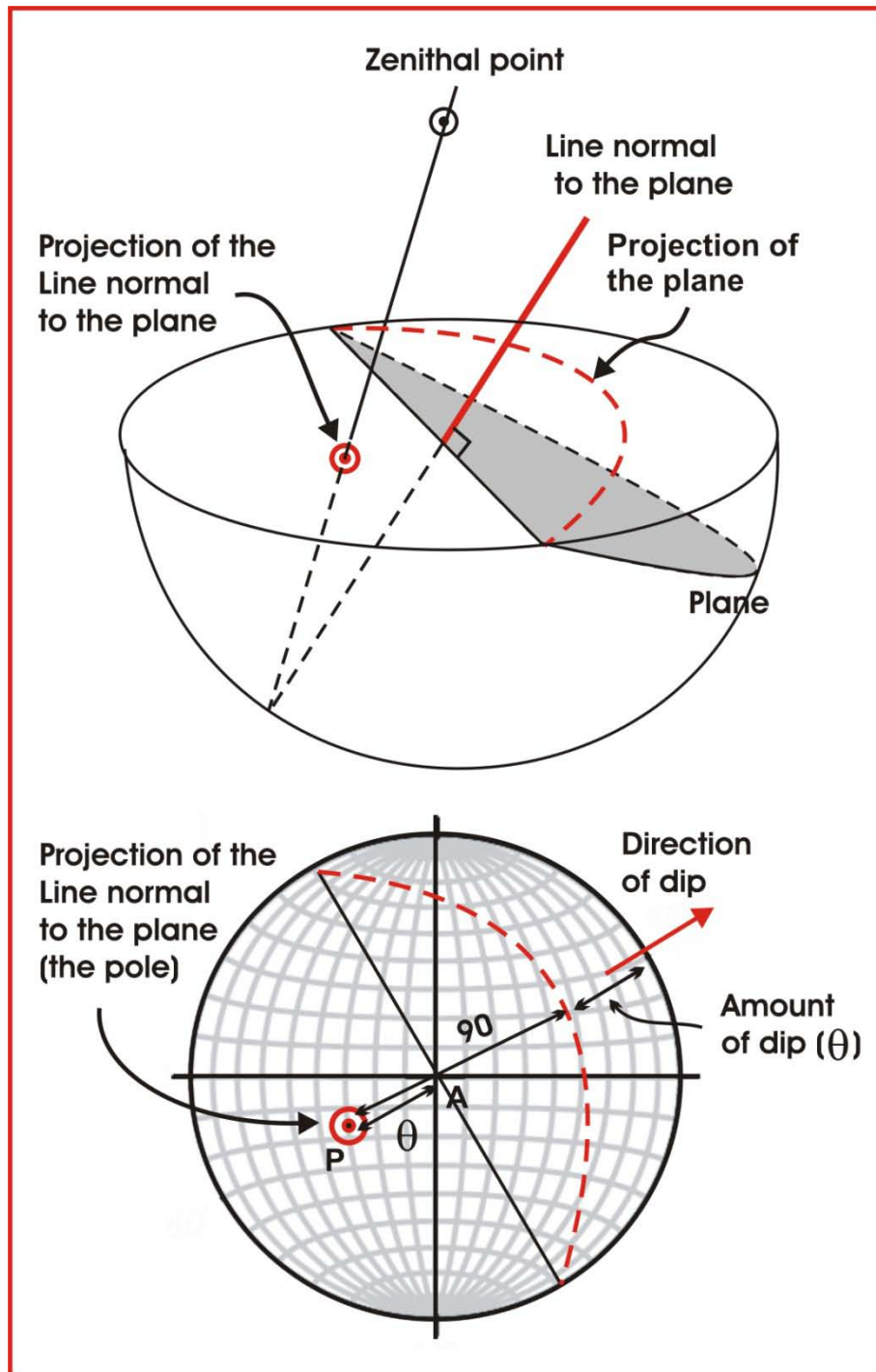
3- Draw the arc passing by this point.

4- Rotate to the north position.

III- Plotting poles to planes

Given : true dip - direction of dip

- **The pole** is the projection of the line normal to the plane, so its projection on the stereographic net is a point as follows.



- The projection of the line normal to the plane (pole) is a point that is characterized by:

- 1- The pole lies on a line normal to the strike of the plane.
- 2- The pole lies in opposite direction to the dip of the plane.
- 3- The pole lies at a distance = angle of dip of the plane measured from the center of the net.

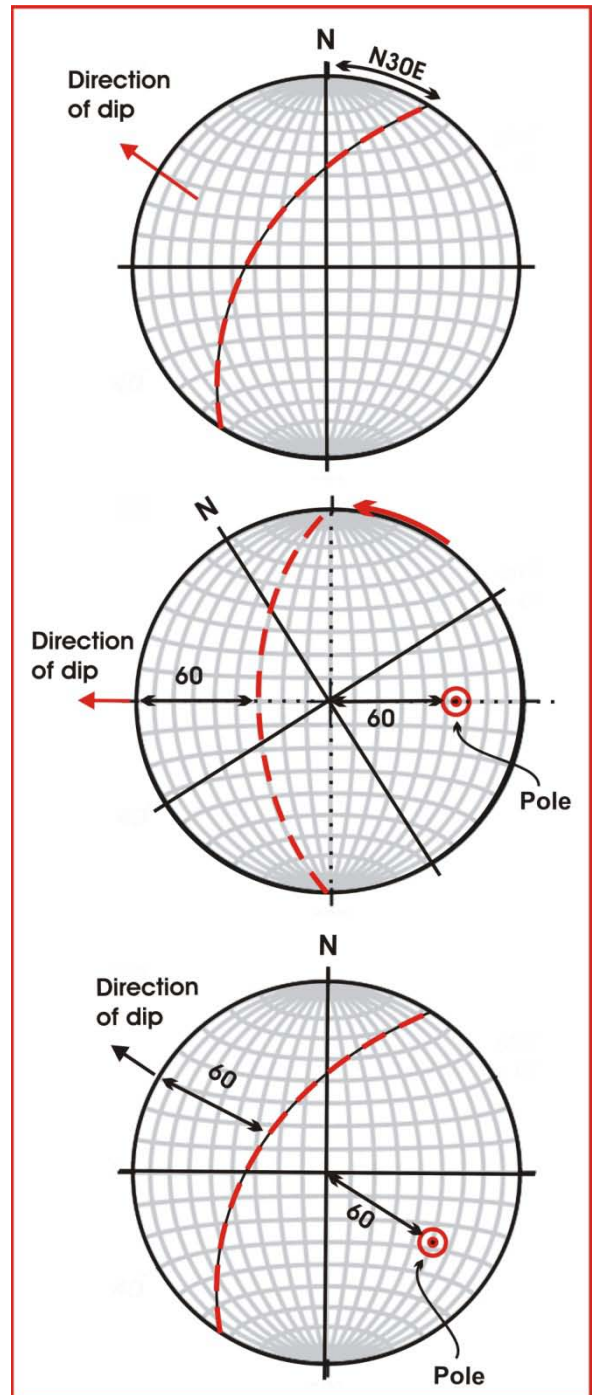
❖ Draw the projection of the pole to a plane as follows:

Draw the pole to the bedding plane that strikes **N30E** and dips **60 NW**.

1- Draw the projection of the plane as an arc.

2- Rotate the arc to the vertical axis of the net. Plot the pole to the arc at an angle = 60 measured from the center of the net and in opposite direction to the dip.

3- Rotate to the North position.

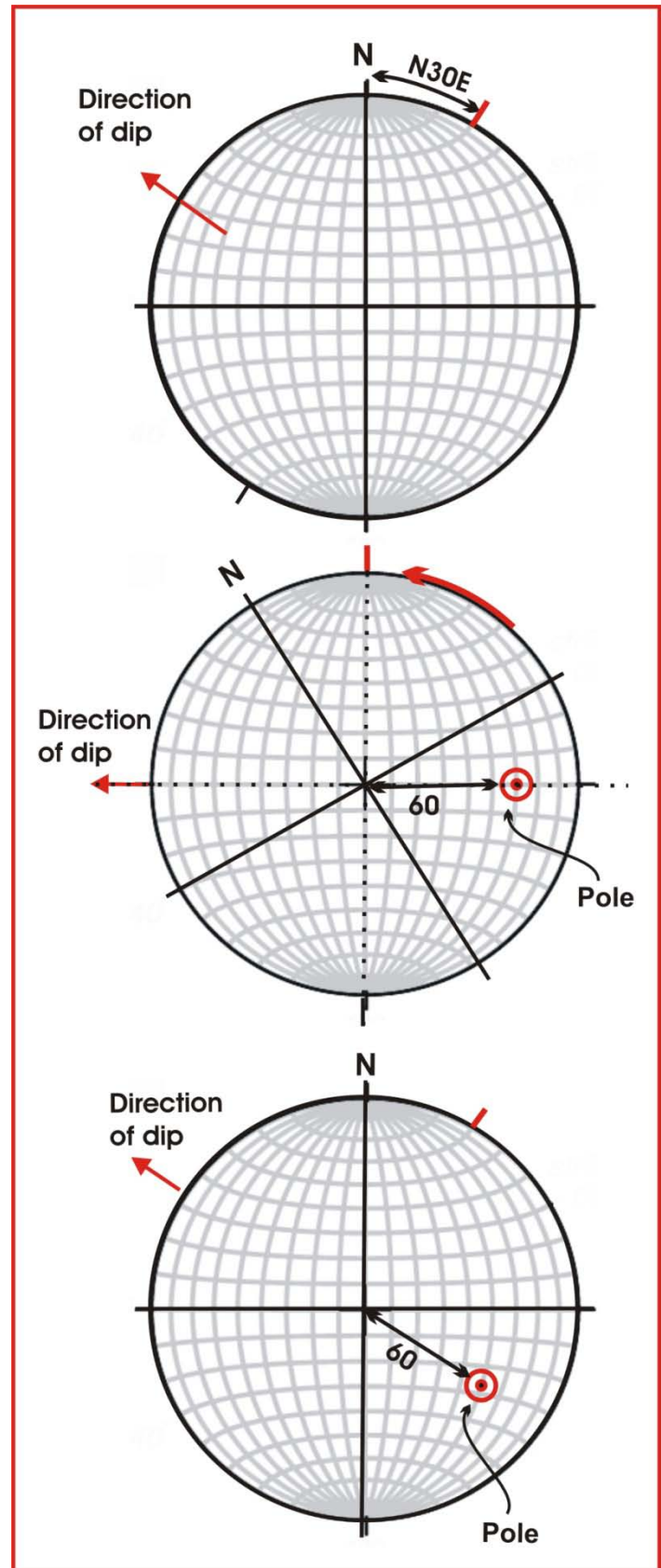


You can plot the pole to the plane
without drawing the arc as follows :

1- Just mark the position of the strike of the plane (at N30E).

2- Draw the pole to the plane at 60 measured from the center of the net in opposite direction to the dip.

3- Rotate to the North position.



Stereographic Projection applications in Rock slope stability

Stereographic projection is an important technique in analyzing and prediction of stability of slopes; it provides:

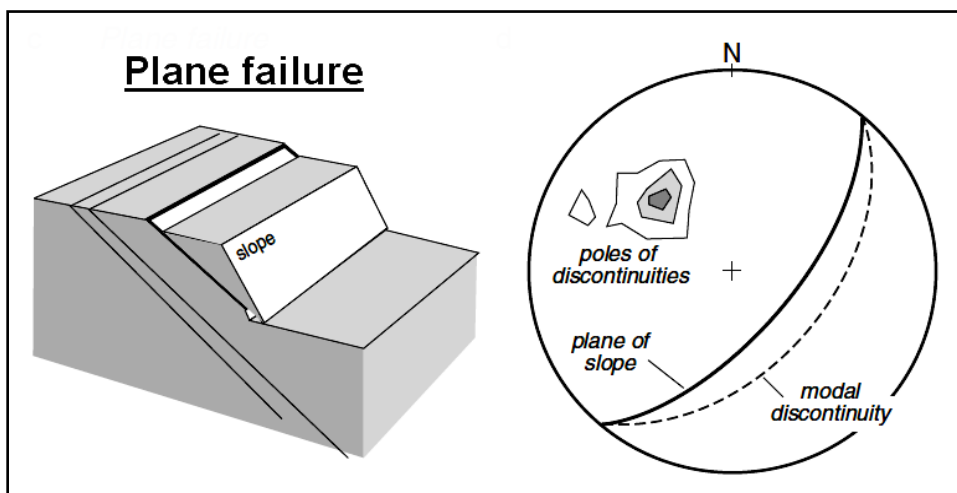
1. Display of the collected data.
2. Identifying the number of discontinuity sets as density contours, and their modal orientations.
3. The angular relationships between the main orientations of discontinuities and its use in solving engineering problems.

Geometrical constructions

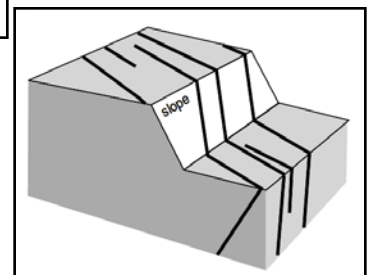
The stereographic projection provides a useful form of displaying the orientation of rock slopes in relation to the sets of discontinuities present. Based on this relationship, it is possible to assess the type of failure most likely to occur:

- 1- **Plane failure**, would be favored in situations where:
 - a) The strike of a set of discontinuities runs parallel to the slope.
 - b) The discontinuity dips towards the slope at a lower angle than the slope itself.
 - c) The angle of dip must be steep enough to produce sliding (**> friction angle of the rock material**),
 - d) i.e. **dip < angle of slope** and **Dip > friction angle**

The stereogram shows, for a given rock slope, the orientations of discontinuities likely to lead to plane failure.

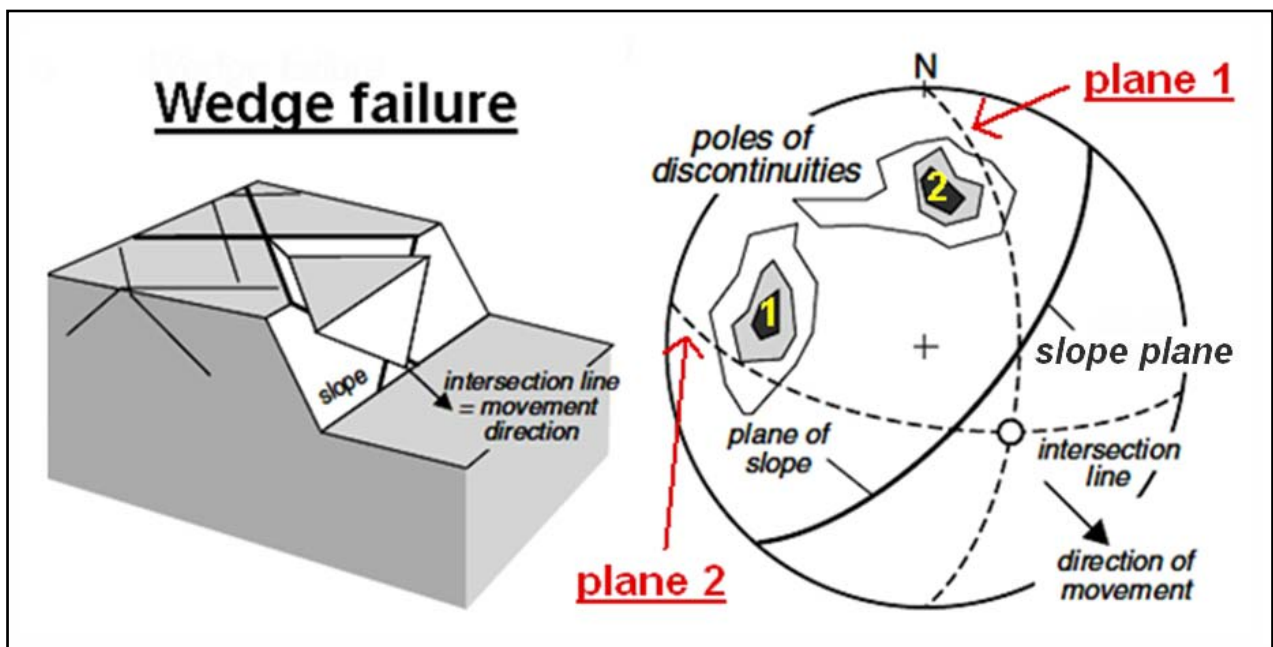


Plane failure is unlikely where joint sets have a strike which is oblique to the rock slope.



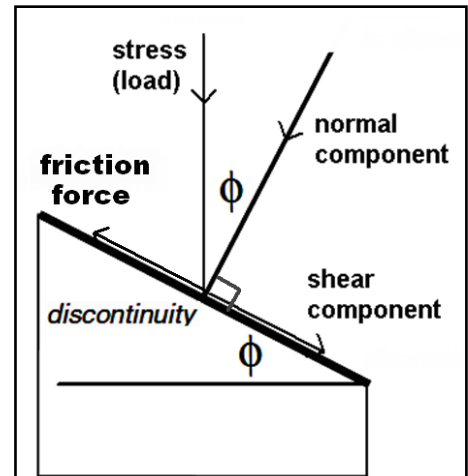
2- **Wedge failure:** occurs when:

- a) Two intersecting sets of joints strike oblique to the slope.
- b) The line of intersection of the two planes plunges in the direction of slope.
- c) The angle of plunge is lower than the angle of slope.
- d) the line of intersection needs to crop out twice; once on the slope and again on the surface above the slope.
- e) The angle of plunge must be steep enough to produce sliding ($>$ friction angle of the rock material),
- f) i.e. plunge $<$ angle of slope and plunge $>$ friction angle



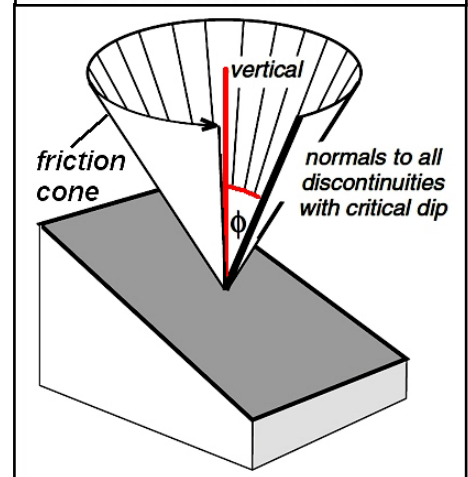
Frictional resistance and Frictional cone of a plane

- The discontinuity surface is under the effect of three stress components:
 - 1- The normal component of the load of the rock mass above the discontinuity acting normal to the discontinuity plane.
 - 2- The shear component of the load of the rock mass above the discontinuity acting down dip parallel to the discontinuity plane.
 - 3- The friction force acting up dip parallel to the discontinuity plane.



at failure: ϕ = friction angle

- **Failure occurs along the discontinuity plane when:**
The shear stress component is > friction force
- **At failure:** the angle of dip of the discontinuity is termed the critical dip angle or angle of friction (ϕ).

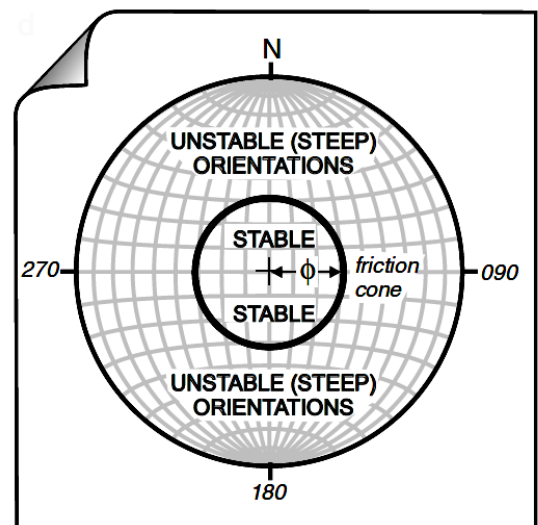


- Any discontinuity **plane** (of any dip direction) but with the same critical dip angle, ϕ , must have a **normal** that deviates from the vertical by the same angle (ϕ). Considering the **normals (poles)** of all such discontinuities with a complete range of dip directions, we find they collectively define a cone with a vertical axis. **This is termed the friction cone**.
- This cone encloses the **normals (poles)** of discontinuities that possess an angle of dip **less** than the critical angle.

- For the analysis of rock slope stability it is convenient to plot the friction cone on a stereogram. This is a small circle made up of lines inclined at the angle (ϕ) away from the vertical.

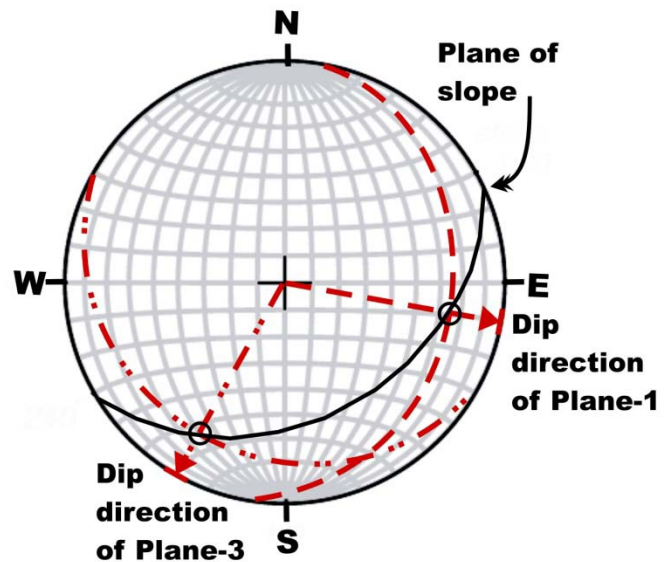
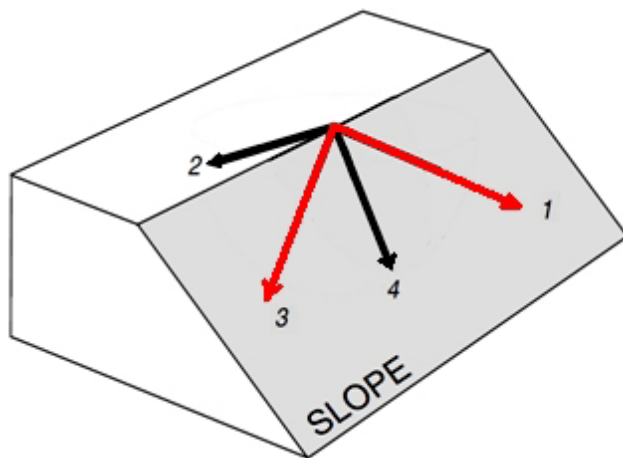
- **To plot the friction cone:** if angle of friction = 30° . Measure 30° radially from the **center of the net**, locate two points to the right and left on the horizontal axis of the net; rotate the tracing paper and locate another two points; repeat the process to add successive points then connect them. .

- **Poles of stable planes will lie within the friction cone; the poles of steep, unstable planes lie outside it.**



Assessing plane failure Daylighting

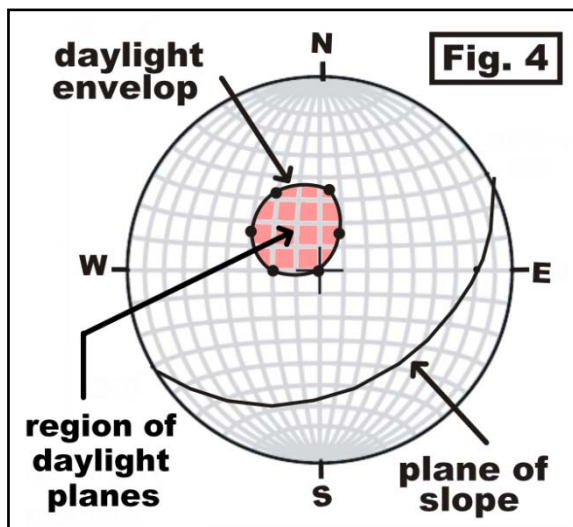
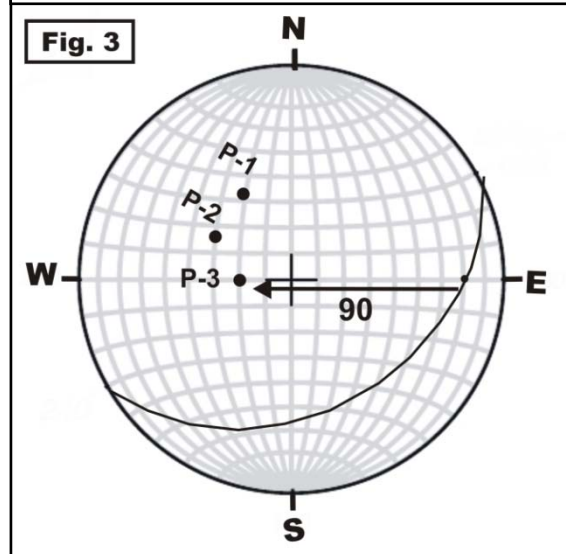
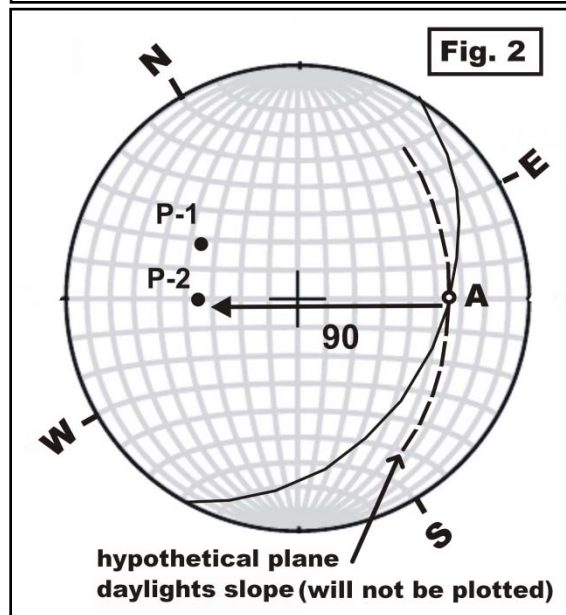
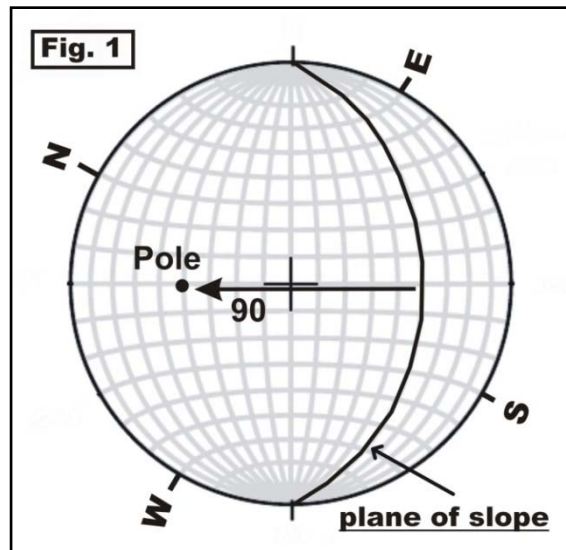
- **Daylighting** means that the discontinuity plane dips in the same direction of slope at an equal or lower angle..
- There are two main factors necessary for plane failure to occur:
 1. The discontinuity must dip in the same direction of the slope at a lower angle (**daylights**).
 2. The discontinuity must dip at a **sufficiently steep** angle to overcome friction.
- The idea of dealing with plane failure through stereographic projection is:
 - 1- To plot the projection of all **possible** planes that daylight the slope surface (**daylight envelop**).
 - 2- To choose which of which **exceeds** the angle of friction.
- In the following figure, the arrows represent the direction of true dip of joint planes. Planes 1, 3, and 4 daylight the plane of slope and may cause plane failure if their angle of dip exceeds the angle of friction.
- plane 2 does not daylight slope. Accordingly there is no possibility for plane failure to occur along this plane.



Planes 1 and 3 daylight the plane of slope.
Dipping towards the slope at a lower angle

• **Construction of daylight envelop:**

- 1- Plot the plane of slope as an arc (Fig.1).
- 2- Plot the pole of this plane (Fig.1).
- 3- Any point on this arc represents the true dip of a plane that daylight the slope.
- 4- Rotate the tracing paper (Fig.2) by an appropriate angle (suppose 30°). Define the point on the slope arc that lies on the horizontal axis of the net (**A**). This point represents the true dip of hypothetical plane daylight the slope. There is no need to plot the arc of this plane.
- 5- Count 90° from point (**A**) across the center of the net and plot point (**P-2**). This point represents the **pole** to the hypothetical plane that daylight the slope.
- 6- Rotate the tracing paper another step (30°) and plot the pole (**P-3**), that represents the **pole** to another hypothetical plane daylight the slope.
- 7- Repeat the process several times and plot a number of poles enough to create the daylight envelop. Connect these point to obtain the **daylight envelop** (Fig. 4).



- **Determination if a discontinuity is favorable for plane failure**

To determine if a discontinuity is favorable for plane failure, the following data is needed:

- 1) The strike and dip of the slope plane to draw the daylight envelop.
- 2) The angle of friction to draw the friction cone,
- 3) The strike and dip of the discontinuity to be tested.

Procedure:

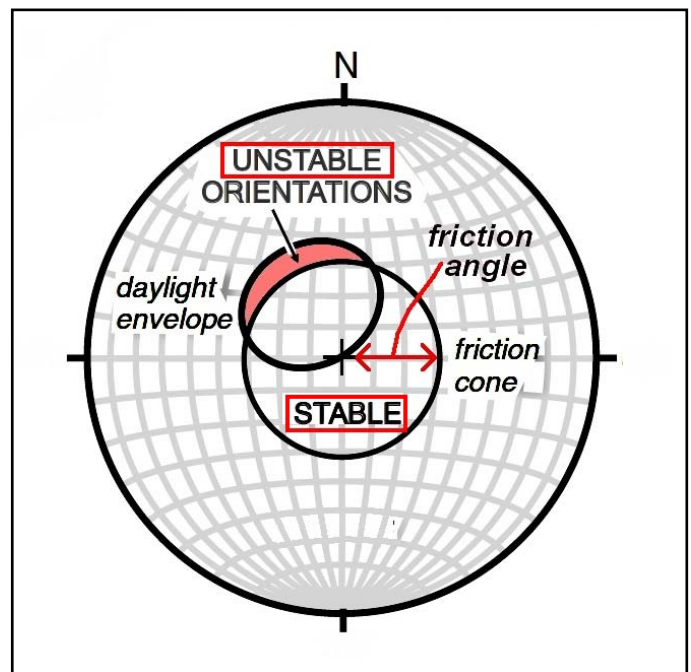
- 1) Plot the **daylight envelop**.
- 2) Plot the **friction cone**.

The intersections between the daylight envelop and the friction cone define a zone of unstable orientations of discontinuities.

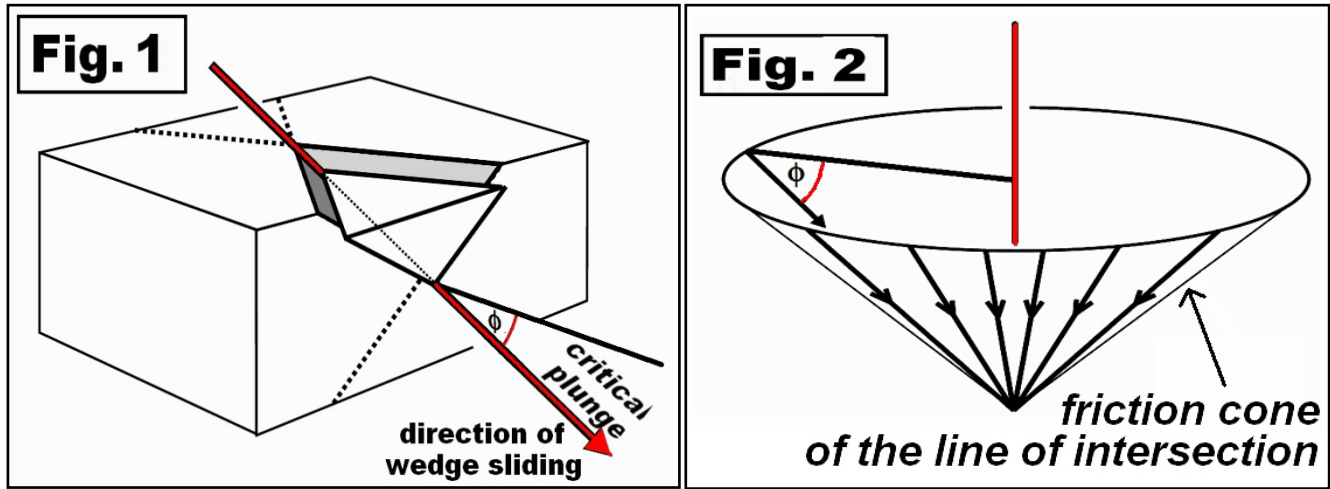
- 3) Plot the **pole** of the discontinuity plane.

If the **pole** of the discontinuity plane lies **within** the unstable region, then the discontinuity is favorable for plane failure,

If the **pole** of the discontinuity plane lies **outside** the unstable region, then the discontinuity is safe and unfavorable for plane failure,



Assessing wedge failure

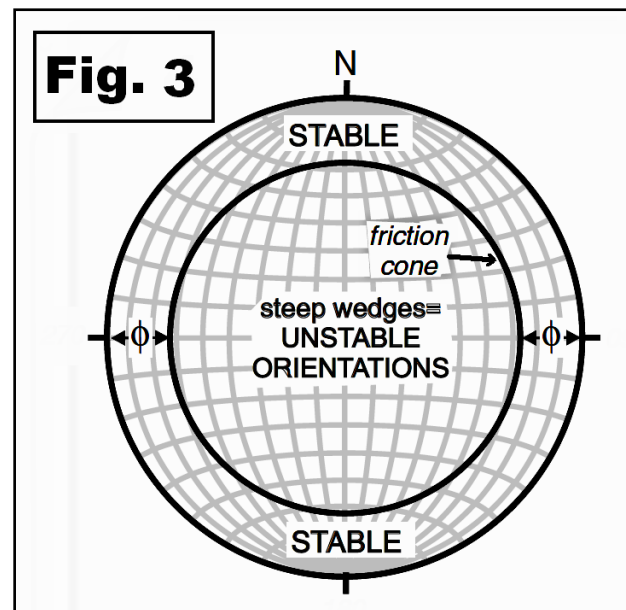


- In wedge failure we consider the orientation of the direction of wedge sliding, parallel to the intersection line of the two sets of discontinuities (Fig. 1).

- **Friction cone**

To overcome frictional resistance, the plunge of the intersection line of the two discontinuities must exceed the sliding friction angle.

All possible orientations of the line of intersection that have plunges equal to the friction angle (critical angle ϕ) must form a cone termed the **friction cone** with an angle = ϕ from **horizontal**, (Fig. 2).



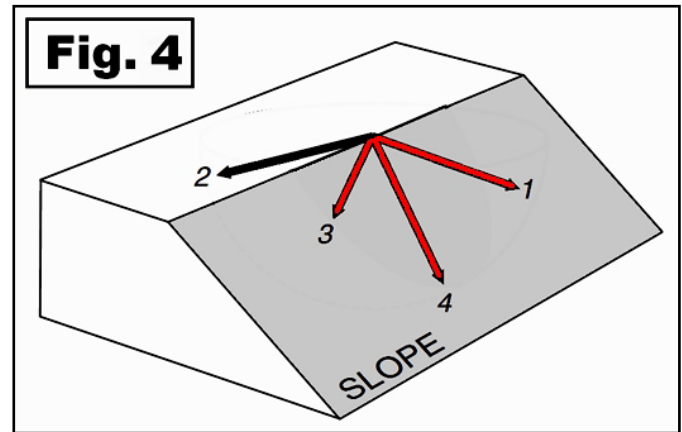
All intersection lines in dangerous (steep) attitudes lie inside the cone.

To plot the friction cone: If the angle of friction = 30. Measure 30 from the **circumference of the net (the outer circle)**, locate successive points then connect them (Fig. 3).

Daylighting

Wedge failure may occur when:

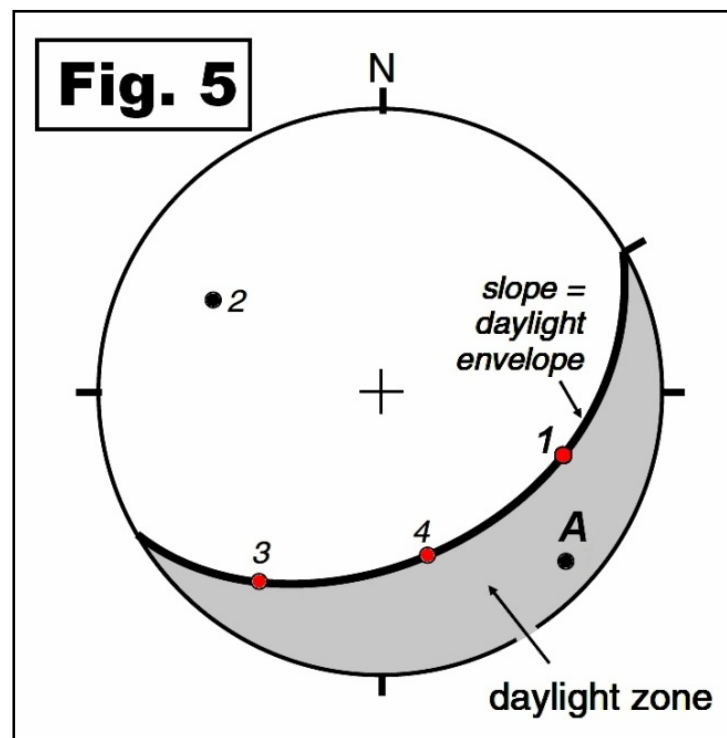
1. The intersection line is plunging towards the slope of the ground.
2. The angle of plunge is less than the apparent dip of the slope in the plunge direction.



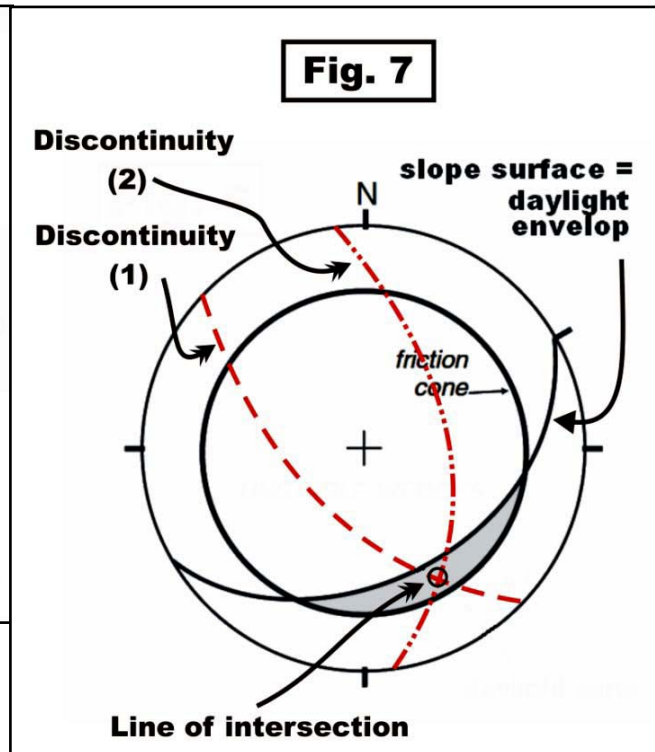
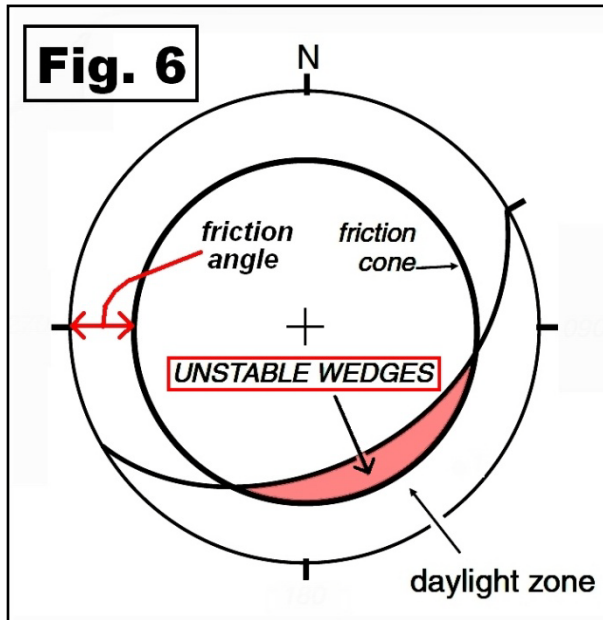
In Fig. 4, Intersection lines 1, 3, and 4 allows the possibility of wedge failure. Intersection line 2 would not permit wedge failure because it has a plunge in the other side of the slope.

Therefore, on the stereogram (Fig. 5), the great circle representing the plane of slope corresponds to the daylight envelope.

Lines of intersection plotted in the crescent area (shaded) are within the daylight zone and may cause wedge failure (e.g. the line of intersection **A**).



Procedure:



1) Plot the slope plane as an arc. This represents the **daylight envelop** (Fig. 6).

2) Plot the **friction cone** (Fig. 6).

The intersections between the daylight envelop and the friction cone define a zone of unstable orientations of discontinuities. The shaded zone in Fig. 6 represents unstable wedges.

3) Plot the two discontinuity planes (Fig. 7). Define the **line of intersection** of the wedge.

If the **line of intersection of the wedge** lies **within** the unstable region, then the wedge is favorable for wedge failure,

If the **line of intersection of the wedge** lies **outside** the unstable region, then the wedge is safe and unfavorable for wedge failure,

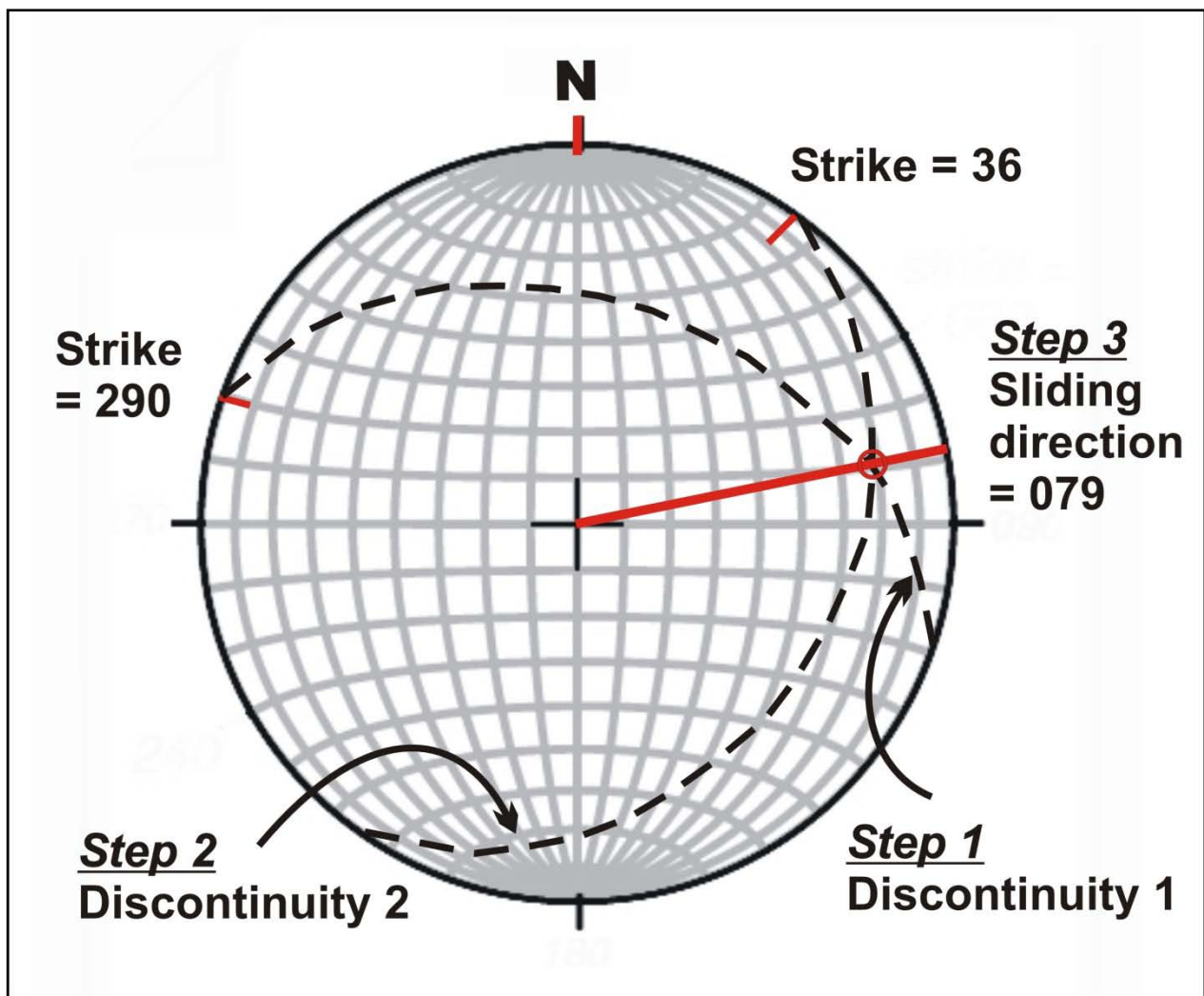
Exercises

1- Sets of **discontinuities** have mean orientations of **290°/40°N** and **036°/30°SE**.

If wedge failure occurs involving simultaneous sliding on both sets, what will be the direction of this sliding?

Solution

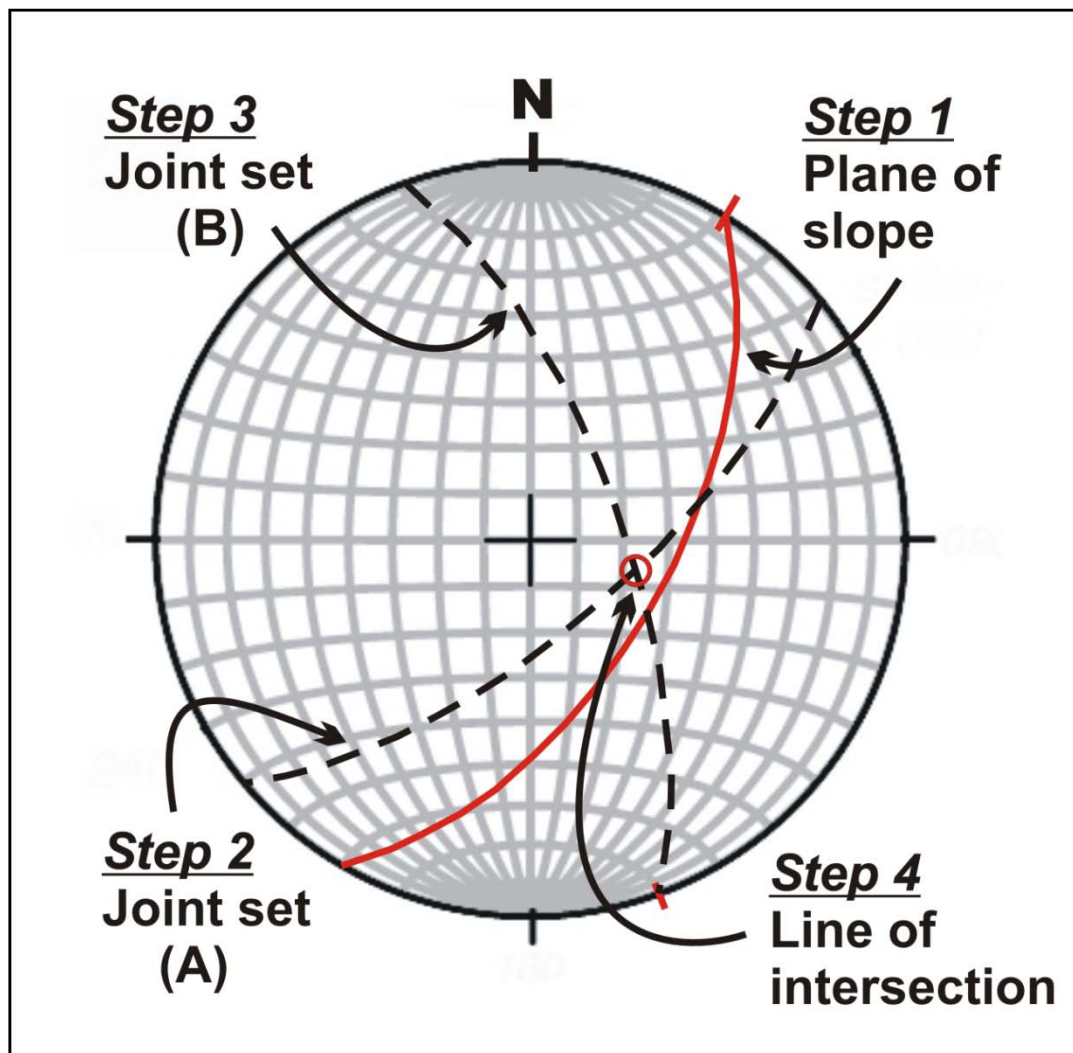
- 079°



2- A **quarry face** whose orientation is **030°/60°SE** is cut by **two joint sets**:
 set A: **050°/70°SE** (i.e. dip = 70°, dip direction = 140°) and
 set B: **160°/70°NE** (i.e. dip = 70°, dip direction = 70°).
 Plot these features on a stereogram and discuss the possibility of wedge failure.

Solution

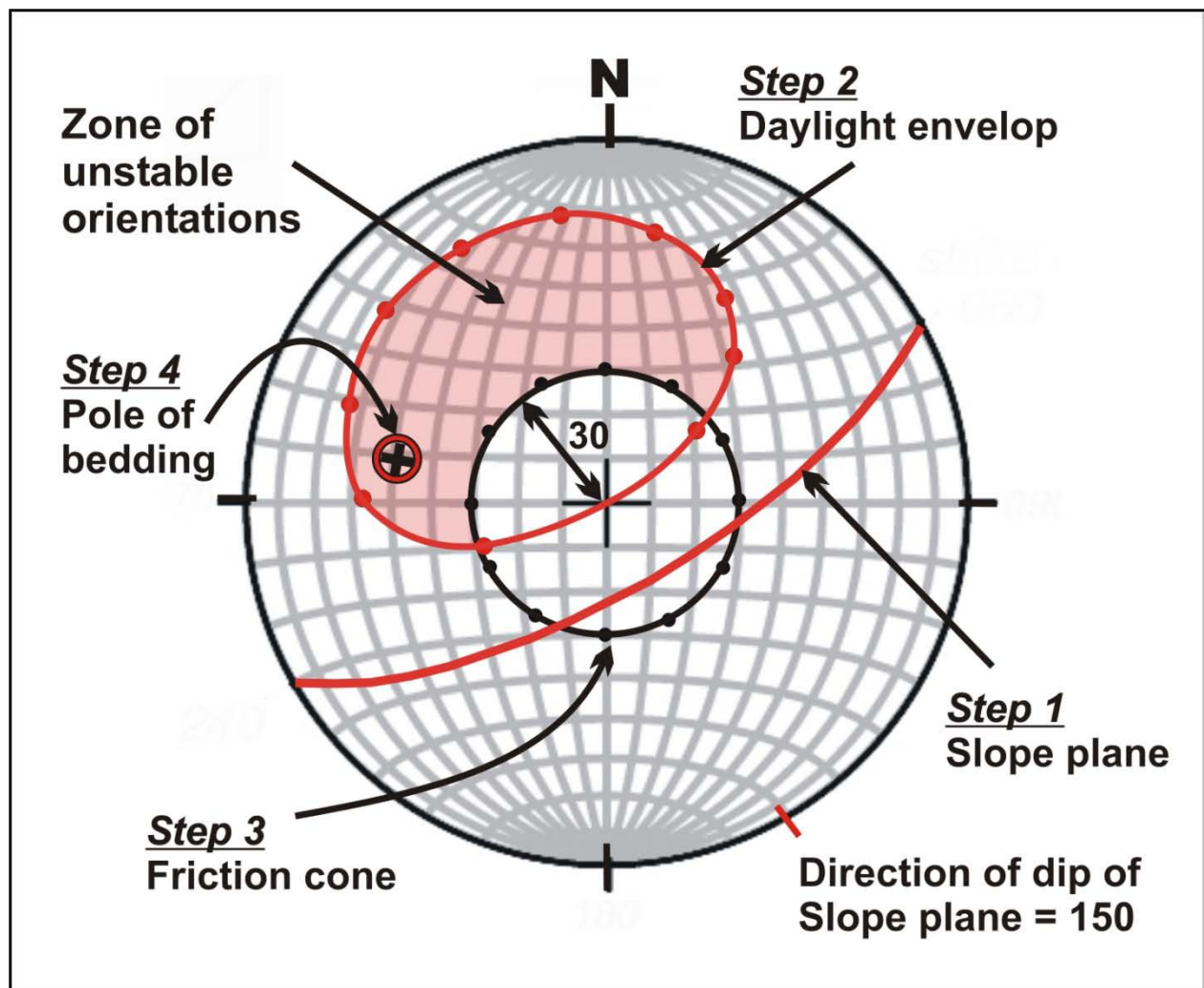
- No wedge failure possible; line of intersection of joint sets A and B does not daylight on the slope. The line of intersection plunges in the same direction of slope **at higher angle**.



3- The face of a railway cutting in sandstone has a **slope** of 70° and its slope is towards direction 150° . A structural survey of the site reveals that bedding in the sandstone constitutes important planes of weakness. The **bedding** attitude is $010^\circ/50^\circ$ E and the angle of **friction** is 30° . Indicate the type of failure that may occur and give your reasons

Solution

- Plane failure along bedding. Reasons: the pole to bedding lies in the daylighting region of the stereogram and also lies outside the friction cone.



- 4- A rock **slope** with an angle of slope **60°** and sloping towards the direction **052°** is intersected by two sets of **joints** (set A and set B) with orientations **124°/50°NE** (i.e. dip = 50°, dip direction = 034°) and **036°/50°SE** (dip = 50°, dip direction = 126°) respectively. Assuming an angle of friction of **33°**, discuss the possibility of wedge failure of the rock slope.

Solution

- Wedge failure is kinematically possible. The line of intersection of the two joint sets lies within the unstable region.

