Digital Logic Design
CS 504

Imane Fahmy

References

- **Main Reference:**

  M. Morris Mano and Micheal D. Ciletti, "Digital Design", Fourth Edition, Pearson Prentice Hall, 2007

- **Secondary Reference:**

  Alan B. Marchovitz, "Introduction to Logic Design", Third Edition.

Course Evaluation

- 10% Assignment(s)
- 20% Midterm Exam
- 70% Final Exam

Regulations

- Lecture Attendance will be considered.
- No excuses for late assignments or midterm attendance.
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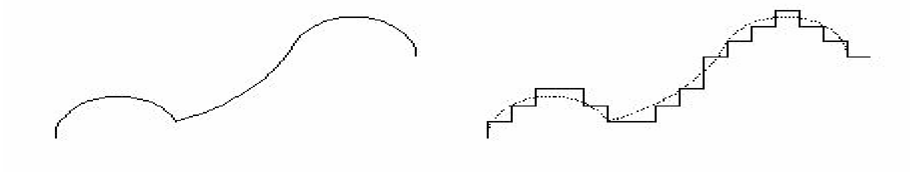
Course Outline

- Introduction
- Digital Systems and Binary Numbers
- Boolean Algebra and Logic Gates
- Gate level Minimization
- Combinatorial Logic
- Synchronous Sequential Logic
- Registers and Counters

Introduction

Why using Digital Systems?

Digital Systems Vs Analog Systems



Analog Systems

- Continuous Range
- More expensive
- More prone to environmental factors e.g., noise, weather conditions and Magnetic fields.
- Ex: Analog Signals and Vinyl Records.

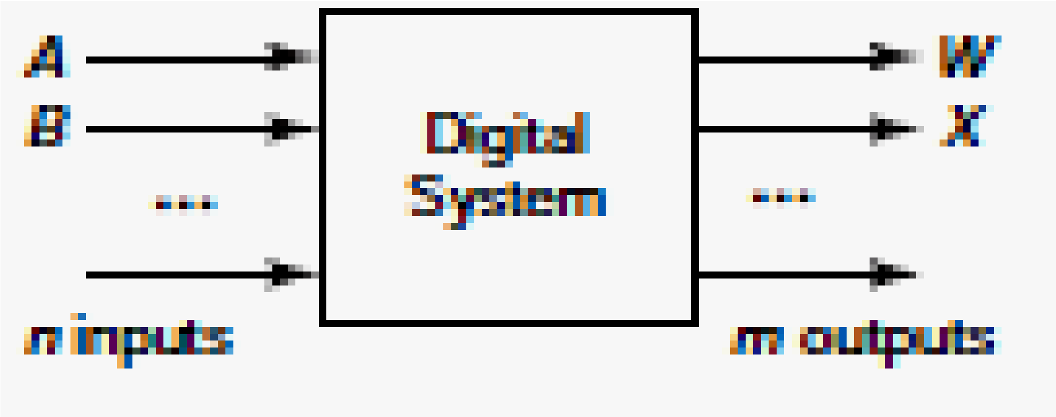
Digital Systems

- Discrete Value
- Cheaper and easier to adjust
- Resistance to noise and uses error correction techniques to produce clearer pictures and sound.
- Ex: Digital Signals, Computers,CDs and Cameras.

Digital Systems

- In digital systems, all signals are represented by discrete values usually binary (two-valued) coded with 0s and 1s strings.

- Digital systems signals are coded into strings of **bi**nary digi**ts** called **bits.**



Digital Systems

- *Question:*

  Why are commercial products made using digital circuits rather than analog?

- Answer:

  Most digital devices are programmable: By changing the program in the device, the same underlying hardware can be used for many different applications.

Digital Systems and Binary Numbers

Conversions and Arithmetic Operations

Decimal Vs Binary Coding

Decimal Coding

- Base: 10

MSD → Weights: 10^3 | 10^2 | 10^1 | 10^0 . 10^{-1} | 10^{-2} | 10^{-3} ← LSD

- Example: representing $(1936.25)_{10}$

$$1\times10^3+9\times10^2+3\times10^1+6\times10^0+2\times10^{-1}+5\times10^{-2}$$

Binary Coding

- Base: 2

MSD → Weights: 2^3 | 2^2 | 2^1 | 2^0 . 2^{-1} | 2^{-2} | 2^{-3} ← LSD

- Example: Converting binary into decimal: $(10111.01)_2 = (\ ?\ )_{10}$

$$1\times2^4+0\times2^3+1\times2^2+1\times2^1+1\times2^0+0\times2^{-1}+1\times2^{-2}=$$
$$=1\times16+0\times8+1\times4+1\times2+1\times1+0\times0.5+1\times0.25=23.25$$

Octal/Hexadecimal Conversion into Decimal

- The <u>octal</u> number system [from Greek: <u>ΟΚΤΩ</u>].
  - Its base is 8 → eight digits 0, 1, 2, 3, 4, 5, 6, 7

✓ $(236.4)_8 = (\ ?\ )_{10}$

$$2\times8^2+3\times8^1+6\times8^0+4\times8^{-1}=158.5$$

- The <u>hexadecimal</u> number system [from Greek: <u>ΔΕΚΑΕΞΙ</u>].
  - Its base is 16 → first 10 digits are borrowed from the decimal system and the letters A, B, C, D, E, F are used for the digits 10, 11, 12, 13, 14, 15

✓ $(D63FA)_{16} = (\ ?\ )_{10}$

$$13\times16^4+6\times16^3+3\times16^2+15\times16^1+10\times16^0=877562$$

Decimal Conversion into Binary

- Conversion from decimal to binary:

  Let each bit of a binary number be represented by a variable whose subscript = bit positions, i.e.,

$$(110)_2 = (a_2 a_1 a_0)_2$$

Its decimal equivalent is:

$$(1\times 2^2 + 1\times 2^1 + 0\times 2^0)_{10} = (a_2 \times 2^2 + a_1 \times 2^1 + a_0 \times 2^0)_{10}$$

It is necessary to separate the number into an integer part and a fraction: Repeatedly divide the decimal number by 2.

Decimal Conversion into Binary

- ✓ Find the binary equivalent of 37.

$$
\begin{array}{lll}
2\overline{)37} & = 18 + 0.5 & 1 \leftarrow \text{LSB} \\
2\overline{)18} & = 9 \;\; + 0 & 0 \\
2\overline{)9} & = 4 \;\; + 0.5 & 1 \\
2\overline{)4} & = 2 \;\; + 0 & 0 \\
2\overline{)2} & = 1 \;\; + 0 & 0 \\
2\overline{)1} & = 0 \;\; + 0.5 & 1 \leftarrow \text{MSB}
\end{array}
$$

$$37_{10} = 100101_2$$

- ❑ $53_{10} = \underline{\quad ? \quad}_2$ ANS: $53_{10} = 110101_2$

DECIMAL FRACTION CONVERSION INTO BINARY

- Conversion from decimal fraction to binary: same method used for integers except multiplication is used instead of division.

✓ Convert $(0.8542)_{10}$ to binary (give answer to 6 digits).

0.8542 x 2 =	1	+	0.7084	$a_{-1} = 1$	**MSB**
0.7084 x 2 =	1	+	0.4168	$a_{-2} = 1$	
0.4168 x 2 =	0	+	0.8336	$a_{-3} = 0$	
0.8336 x 2 =	1	+	0.6672	$a_{-4} = 1$	
0.6675 x 2 =	1	+	0.3344	$a_{-5} = 1$	
0.3344 x 2 =	0	+	0.6688	$a_{-6} = 0$	**LSB**

$$(0.8542)_{10} = (0.a_{-1}a_{-2}a_{-3}a_{-4}a_{-5}a_{-6})_2 = (0.110110)_2$$

- $(53.8542)_{10} = (\quad ? \quad)_2$

Decimal Conversion into Octal

- Conversion from <u>decimal</u> to <u>octal</u>:
  The decimal number is first divided by 8. The remainder is the LSB. The quotient is then divide by 8 and the remainder is the next significant bit and so on.

✓ Convert 1122 to octal.

$$
\begin{array}{lll}
8\overline{)1122} & = 140 + 0.25 & R2 \longleftarrow \text{LSB} \\
8\overline{)140} & = 17 + 0.5 & R4 \\
8\overline{)17} & = 2 + 0.125 & R1 \\
8\overline{)2} & = 0 + 0.25 & R2 \longleftarrow \text{MSB}
\end{array}
$$

$$1122_{10} = 2142_8$$

Binary Conversion into Octal/Hexadecimal

- Conversion from and to <u>binary</u>, <u>octal</u>, and <u>hexadecimal</u> plays and important part in digital computers.

since $2^3 = 8$ and $2^4 = 16$

each octal digit corresponds to 3 binary digits
and each hexa digit corresponds to 4 binary digits.

✓ $(010\ 111\ 100\ .\ 001\ 011\ 000)_2 = (274.130)_8$

✓ $(0110\ 1111\ 1101\ .\ 0001\ 0011\ 0100)_2 = (6FD.134)_{16}$

from table

Decimal	Hex	Binary	Octal
0	0	0000	00
1	1	0001	01
2	2	0010	02
3	3	0011	03
4	4	0100	04
5	5	0101	05
6	6	0110	06
7	7	0111	07
8	8	1000	10
9	9	1001	11
10	A	1010	12
11	B	1011	13
12	C	1100	14
13	D	1101	15
14	E	1110	16
15	F	1111	17

1's and 2's Complement

- 1's complement of $N = (2^n - 1) - N$ (N is a binary #)
  1's complement can be formed by changing 1's to 0's and 0's to 1's

  2's complement of a number is obtained by leaving all least significant 0's and the first 1 unchanged, and replacing 1's with 0's and 0's with 1 in all higher significant digits.

  - ✓ The 1's complement of 1101011 = 0010100
  - ✓ The 2's complement of 0110111 = 1001001

- ❑ Find the 1's and 2's-complement of 1000000

Subtraction using Complements

- Subtraction with digital hardware using complements:

  Subtraction of two n-digit unsigned numbers $M - N$ base r:

  1. Add M to the r's complement of N: $M + (r^n - N)$
  2. If $M \leq N$, the sum will produce an end carry and is equal to r^n that can be discarded. The result is then $M - N$.
  3. If $M \geq N$, the sum will not produce an end carry and is equal to $r^n - (N - M)$

Binary Subtraction using Complement

- Binary subtraction is done using the same procedure.

✓ Subtract 1010100 – 1000011 using 2's complement:

$$\begin{array}{rcr} A & = & 1010100 \\ \text{2's complement of } B & = & +\ \ 0111101 \\ \hline \text{Sum} & = & 10010001 \\ \text{Discard end carry} & = & -\ \ 10000000 \\ \hline \textit{Answer} & = & 0010001 \end{array}$$

end carry

- Subtract 1000011 – 1010100 using 2's complement:

$\textit{Answer} = -0010001$

Binary Subtraction using Complement

✓ Subtract 1010100 – 1000011 using 1's complement:

$$
\begin{array}{rr}
A = & 1010100 \\
\text{1's complement of } B = & +\ \ 0111100 \\
\hline
\text{Sum} = & 10010000 \\
\text{End-around carry} = & +\qquad\quad 1 \\
\hline
\textit{Answer} = & 0010001
\end{array}
$$

❑ Subtract 1000011 – 1010100 using 1's complement:

$\textit{Answer} = -0010001$

Arithmetic Addition

Negative numbers must be initially in 2's complement form and if the obtained sum is negative, it is in 2's complement form.

+ 6	00000110		–6	11111010
+13	00001101		+13	00001101
+19	00010011		+7	00000111

- ❑ Add –6 and –13

Answer = 11101101

Thank you