Syllabus for Electromagnetism subject (231)

Chapter (1): Vector algebra

1.1 Introduction
1.2 Scalar and Vectors
1.3 Unit vector
1.4 Vector addition and subtraction
1.5 Position and Distance Vectors
1.6 Vector Multiplication (Dot Product-Vector Product –Scalar triple Product –Vector Triple Product).
1.7 Components Of a Vector

The home work problems are :

(1.1, 1.3, 1.5, 1.7, 1.9 .1.11, 1.13, 1.15, 1.17 1.19, 1.21)

Chapter (2) Coordinate systems and Transformation

2.1 Introductions.

2.2 Cartesian Coordinates (X, Y, Z).

2.3 Circular Cylindrical Coordinates(ρ , φ , Z).

2.4 Spherical Coordinates (r , θ , φ) .

The home work problems are:

(2.1 , 2.3 , 2.5 , 2.7, 2.9 , 2.11 , 2.13 , 2.15 , 2.17 , 2.19 , 2.21 , 2.23)

Chapter (3) Vector Calculus

3.1 Introduction

3.2 Differential Length, Area, and Volume

Cartesian Coordinates, Cylindrical Coordinates, Spherical Coordinates.

3.3 Line, Surface and Volume integrals.

3.4 Del Operator.

3.5 Gradient of A scalar.

3.6 Divergence of a Vector and Divergence Theorem.

3.7 Curl of a Vector and Stokes's Theorem.

3.8 Laplacian of a Scalar.

3.9 Classification of a Vector Fields

The home work problems are :

(3.1 , 3.2 , 3.3 , 3.12 , 3.13 , 3.15 , 3.18 , 3.24 , 3.27, 3.33 ,3.39)

<u>Chapter (4) Electrostatic Fields</u>

4.1 Introduction

4.2Coulomb's Law and Field Intensity

4.3 Electric Fields due to continuous charge distributions.

Line Charge, Surface Charge, Volume Charge

4.4 Electric Flux Density.

4.5 Gauss' Law – Maxwell' Equation.

4.6 Application of Gauss's Law

Point Charge, Infinite Line Charge, Infinite Sheet Charge, Uniformly Charged Sphere.

4.7 Electric Potential

4.8 Relationship between **E** and **V** Maxwell' Equation.

4.9 An Electric Dipole.

4.10 Electric Flux Lines and Equipotential Surfaces.

4.11 Energy Density in Electrostatic Fields.

Study well these solved problems (4.1, 4.2, 4.4, 4.5, 4.7, 4.8, 4.11 4.12, 4.13 4.15)

The home work problems are

(4.1, 4.3, 4.9, 4.10, 4.13, 4.16, 4.26, 4.29)

Chapter (5) Electric fields in Material Space

5.1 Introduction.

5.2 Properties of Materials.

5.3 Convection current.

5.4 Conduction Current.

5.5 Polarization in Dielectrics.

5.6 Dielectric constant and strength.

5.7 Linear, Isotropic, and Homogeneous Dielectrics.

5.8 Continuity Equation and Relaxation Time.

5.9 Boundary Conditions.

A. Dielectric –Dielectric Boundary Conditions.

B. Conductor – Dielectric Boundary Conditions.

C. Conductor –Free –Space Boundary Conditions.

Study well these solved problems (5.1, 5.3 ,5.6 ,5.7 ,5.8 ,5.9)

The home work problems are

(5.1, 5.3 , 5.12 , 5.18 , 5.20 , 5.27 , 5.29)

Sheet (1) Vector algebra

1.1 Find the unit vector along the line joining point (2, 4, 4) to point (−3, 2, 2).

1.3 If

$$\mathbf{A} = 2\mathbf{a}_x + \mathbf{a}_y - 3\mathbf{a}_z$$
$$\mathbf{B} = \mathbf{a}_y - \mathbf{a}_z$$
$$\mathbf{C} = 3\mathbf{a}_x + 5\mathbf{a}_y + 7\mathbf{a}_z$$

determine:

(a) $\mathbf{A} - 2\mathbf{B} + \mathbf{C}$

(b) $\mathbf{C} - 4(\mathbf{A} + \mathbf{B})$

(c) $\dfrac{2\mathbf{A} - 3\mathbf{B}}{|\mathbf{C}|}$

(d) $\mathbf{A} \cdot \mathbf{C} - |\mathbf{B}|^2$

(e) $\frac{1}{2}\mathbf{B} \times \left(\frac{1}{3}\mathbf{A} + \frac{1}{4}\mathbf{C}\right)$

1.5 If

$$\mathbf{A} = 5\mathbf{a}_x + 3\mathbf{a}_y + 2\mathbf{a}_z$$
$$\mathbf{B} = -\mathbf{a}_x + 4\mathbf{a}_y + 6\mathbf{a}_z$$
$$\mathbf{C} = 8\mathbf{a}_x + 2\mathbf{a}_y$$

find the values of α and β such that $\alpha\mathbf{A} + \beta B + C$ is parallel to the y-axis.

1.7 (a) Show that

$$(\mathbf{A} \cdot \mathbf{B})^2 + (\mathbf{A} \times \mathbf{B})^2 = (AB)^2$$

(b) Show that

$$\mathbf{a}_x = \frac{\mathbf{a}_y \times \mathbf{a}_z}{\mathbf{a}_x \cdot \mathbf{a}_y \times \mathbf{a}_z}, \qquad \mathbf{a}_y = \frac{\mathbf{a}_z \times \mathbf{a}_x}{\mathbf{a}_x \cdot \mathbf{a}_y \times \mathbf{a}_z}, \qquad \mathbf{a}_z = \frac{\mathbf{a}_x \times \mathbf{a}_y}{\mathbf{a}_x \cdot \mathbf{a}_y \times \mathbf{a}_z}$$

1.9 Given vectors $\mathbf{T} = 2\mathbf{a}_x - 6\mathbf{a}_y + 3\mathbf{a}_z$ and $\mathbf{S} = \mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z$, find: (a) the scalar projection of **T** on **S**, (b) the vector projection of **S** on **T**, (c) the smaller angle between **T** and **S**.

1.11 Calculate the angles that vector $\mathbf{H} = 3\mathbf{a}_x + 5\mathbf{a}_y - 8\mathbf{a}_z$ makes with the x-, y-, and z-axes.

1.13 Simplify the following expressions:

(a) $\mathbf{A} \times (\mathbf{A} \times \mathbf{B})$

(b) $\mathbf{A} \times [\mathbf{A} \times (\mathbf{A} \times \mathbf{B})]$

1.15 Points $P_1(1, 2, 3)$, $P_2(-5, 2, 0)$, and $P_3(2, 7, -3)$ form a triangle in space. Calculate the area of the triangle.

1.17 Points P, Q, and R are located at $(-1, 4, 8)$, $(2, -1, 3)$, and $(-1, 2, 3)$, respectively. Determine: (a) the distance between P and Q, (b) the distance vector from P to R, (c) the angle between QP and QR, (d) the area of triangle PQR, (e) the perimeter of triangle PQR.

***1.19** (a) Prove that $\mathbf{P} = \cos\theta_1\mathbf{a}_x + \sin\theta_1\mathbf{a}_y$ and $\mathbf{Q} = \cos\theta_2\mathbf{a}_x + \sin\theta_2\mathbf{a}_y$ are unit vectors in the xy-plane respectively making angles θ_1 and θ_2 with the x-axis.

(b) By means of dot product, obtain the formula for $\cos(\theta_2 - \theta_1)$. By similarly formulating **P** and **Q**, obtain the formula for $\cos(\theta_2 + \theta_1)$.

(c) If θ is the angle between **P** and **Q**, find $\frac{1}{2}|\mathbf{P} - \mathbf{Q}|$ in terms of θ.

1.21 Given $\mathbf{A} = x^2y\mathbf{a}_x - yz\mathbf{a}_y + yz^2\mathbf{a}_z$, determine:

(a) The magnitude of **A** at point $T(2, -1, 3)$

(b) The distance vector from T to S if S is 5.6 units away from T and in the same direction as **A** at T

(c) The position vector of S

Sheet (2) Coordinate Systems and Transformation

2.1 Express the following points in Cartesian coordinates:

(a) $P(1, 60°, 2)$

(b) $Q(2, 90°, -4)$

(c) $R(, 45°, 210°)$

(d) $T(4, \pi/2, \pi/6)$

2.3 (a) If $V = xz - xy + yz$, express V in cylindrical coordinates.

(b) If $U = x^2 + 2y^2 + 3z^2$, express U in spherical coordinates.

2.5 Convert the following vectors to cylindrical and spherical systems:

(a) $\mathbf{F} = \dfrac{x\mathbf{a}_x + y\mathbf{a}_y + 4\mathbf{a}_z}{\sqrt{x^2 + y^2 + z^2}}$

(b) $\mathbf{G} = (x^2 + y^2)\left[\dfrac{x\mathbf{a}_x}{\sqrt{x^2 + y^2 + z^2}} + \dfrac{y\mathbf{a}_y}{\sqrt{x^2 + y^2 + z^2}} + \dfrac{z\mathbf{a}_z}{\sqrt{x^2 + y^2 + z^2}}\right]$

2.7 Convert the following vectors to Cartesian coordinates:

(a) $\mathbf{C} = z \sin \phi \ \mathbf{a}_\rho - \rho \cos \phi \ \mathbf{a}_\phi + 2\rho z\mathbf{a}_z$

(b) $\mathbf{D} = \dfrac{\sin \theta}{r^2} \mathbf{a}_r + \dfrac{\cos \theta}{r^2} \mathbf{a}_\theta$

2.9 (a) Show that point transformation between cylindrical and spherical coordinates is ob tained using

$$r = \sqrt{\rho^2 + z^2}, \qquad \theta = \tan^{-1}\frac{\rho}{z}, \qquad \phi = \phi$$

or

$$\rho = r\sin\theta, \qquad z = r\cos\theta, \qquad \phi = \phi$$

(b) Show that vector transformation between cylindrical and spherical coordinates is ob tained using

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta & 0 & \cos\theta \\ \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

or

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

(*Hint:* Make use of Figures 2.5 and 2.6.)

2.11 Let $\mathbf{A} = \rho\cos\theta\,\mathbf{a}_\rho + \rho z^2 \sin\phi\,\mathbf{a}_z$

(a) Transform **A** into rectangular coordinates and calculate its magnitude at point $(3, -4, 0)$.

(b) Transform **A** into spherical system and calculate its magnitude at point $(3, -4, 0)$.

2.13 In Practice Exercise 2.2, express **A** in spherical and **B** in cylindrical coordinates. Evaluate **A** at $(10, \pi/2, 3\pi/4)$ and **B** at $(2, \pi/6, 1)$.

2.15 Describe the intersection of the following surfaces:

(a) $x = 2, \quad y = 5$
(b) $x = 2, \quad y = -1, \quad z = 10$
(c) $r = 10, \quad \theta = 30°$
(d) $\rho = 5, \quad \phi = 40°$
(e) $\phi = 60°, \quad z = 10$
(f) $r = 5, \quad \phi = 90°$

***2.17** Given vectors $\mathbf{A} = 2\mathbf{a}_x + 4\mathbf{a}_y + 10\mathbf{a}_z$ and $\mathbf{B} = -5\mathbf{a}_\rho + \mathbf{a}_\phi - 3\mathbf{a}_z$, find

(a) $\mathbf{A} + \mathbf{B}$ at $P(0, 2, -5)$
(b) The angle between **A** and **B** at P
(c) The scalar component of **A** along **B** at P

***2.19** If $\mathbf{J} = r \sin\theta \cos\phi\, \mathbf{a}_r - \cos 2\theta \sin\phi\, \mathbf{a}_\theta + \tan\frac{\theta}{2} \ln r\, \mathbf{a}_\phi$ at $T(2, \pi/2, 3\pi/2)$, determine the vector component of **J** that is

(a) Parallel to $\mathbf{a}_z$
(b) Normal to surface $\phi = 3\pi/2$
(c) Tangential to the spherical surface $r = 2$
(d) Parallel to the line $y = -2, z = 0$

***2.21** Let

$$\mathbf{A} = \rho(z^2 - 1)\mathbf{a}_\rho - \rho z \cos\phi\, \mathbf{a}_\phi + \rho^2 z^2 \mathbf{a}_z$$

and

$$\mathbf{B} = r^2 \cos\phi\, \mathbf{a}_r + 2r \sin\theta\, \mathbf{a}_\phi$$

At $T(-3, 4, 1)$, calculate: (a) **A** and **B**, (b) the vector component in cylindrical coordinates of **A** along **B** at T, (c) the unit vector in spherical coordinates perpendicular to both **A** and **B** at T.

2.23 A vector field in "mixed" coordinate variables is given by

$$\mathbf{G} = \frac{x \cos\phi}{\rho}\mathbf{a}_x + \frac{2yz}{\rho^2} + \left(1 - \frac{x^2}{\rho^2}\right)\mathbf{a}_z$$

Express **G** completely in spherical system.

****

Sheet (3) Vector Calculus

3.1 Using the differential length dl, find the length of each of the following curves:

(a) $\rho = 3, \pi/4 < \phi < \pi/2, z = \text{constant}$

(b) $r = 1, \theta = 30°, 0 < \phi < 60°$

(c) $r = 4, 30° < \theta < 90°, \phi = \text{constant}$

3.2 Calculate the areas of the following surfaces using the differential surface area dS:

(a) $\rho = 2, 0 < z < 5, \pi/3 < \phi < \pi/2$

(b) $z = 1, 1 < \rho < 3, 0 < \phi < \pi/4$

(c) $r = 10, \pi/4 < \theta < 2\pi/3, 0 < \phi < 2\pi$

(d) $0 < r < 4, 60° < \theta < 90°, \phi = \text{constant}$

3.3 Use the differential volume dv to determine the volumes of the following regions:

(a) $0 < x < 1, 1 < y < 2, -3 < z < 3$

(b) $2 < \rho < 5, \pi/3 < \phi < \pi, -1 < z < 4$

(c) $1 < r < 3, \pi/2 < \theta < 2\pi/3, \pi/6 < \phi < \pi/2$

3.12 **Find the gradient of the these scalar fields:**

(a) $U = 4xz^2 + 3yz$

(b) $W = 2\rho(z^2 + 1)\cos\phi$

(c) $H = r^2 \cos\theta \cos\phi$

3.13 Determine the gradient of the following fields and compute its value at the specified point.

(a) $V = e^{(2x+3y)} \cos 5z, (0.1, -0.2, 0.4)$

(b) $T = 5\rho e^{-2z} \sin\phi, (2, \pi/3, 0)$

(c) $Q = \dfrac{\sin\theta \sin\phi}{r^2}, (1, \pi/6, \pi/2)$

3.15 The temperature in an auditorium is given by $T = x^2 + y^2 - z$. A mosquito located at (1, 1, 2) in the auditorium desires to fly in such a direction that it will get warm as soon as possible. In what direction must it fly?

3.18 The heat flow vector $\mathbf{H} = k\nabla T$, where T is the temperature and k is the thermal conductivity. Show that where

$$T = 50 \sin \frac{\pi x}{2} \cosh \frac{\pi y}{2}$$

then $\nabla \cdot \mathbf{H} = 0$.

3.24 Evaluate ∇V, $\nabla \cdot \nabla V$, and $\nabla \times \nabla V$ if:

(a) $V = 3x^2y + xz$

(b) $V = \rho z \cos \phi$

(c) $V = 4r^2 \cos \theta \sin \phi$

3.27 If $\mathbf{r}$ and r are as defined in the previous problem, prove that:

(a) $\nabla (\ln r) = \dfrac{\mathbf{r}}{r}$

(b) $\nabla^2 (\ln r) = \dfrac{1}{r^2}$

3.33 If $\mathbf{F} = x^2\mathbf{a}_x + y^2\mathbf{a}_y + (z^2 - 1)\,\mathbf{a}_z$, find $\oint_S \mathbf{F} \cdot d\mathbf{S}$, where S is defined by $\rho = 2, 0 < z < 2, 0 \leq \phi \leq 2\pi$.

3.39 Find the flux of the curl of field

$$\mathbf{T} = \frac{1}{r^2}\cos\theta\,\mathbf{a}_r + r\sin\theta\cos\phi\,\mathbf{a}_\theta + \cos\theta\,\mathbf{a}_\phi$$

through the hemisphere $r = 4$, $z \leq 0$.

Sheet (4) Electrostatic Fields

4.1 Point charges $Q_1 = 5\ \mu$C and $Q_2 = -4\ \mu$C are placed at (3, 2, 1) and (−4, 0, 6), respectively. Determine the force on Q_1.

4.3 Point charges Q_1 and Q_2 are, respectively, located at (4, 0, −3) and (2, 0, 1). If $Q_2 = 4$ nC, find Q_1 such that

(a) The **E** at (5, 0, 6) has no z-component

(b) The force on a test charge at (5, 0, 6) has no x-component.

4.9 A circular disk of radius a carries charge $\rho_S = \dfrac{1}{\rho}$ C/m². Calculate the potential at $(0, 0, h)$.

4.10 A ring placed along $y^2 + z^2 = 4$, $x = 0$ carries a uniform charge of 5 μC/m.

(a) Find **D** at $P(3, 0, 0)$.

(b) If two identical point charges Q are placed at (0, −3, 0) and (0, 3, 0) in addition to the ring, find the value of Q such that $\mathbf{D} = 0$ at P.

4.13 Line $x = 3, z = -1$ carries charge 20 nC/m while plane $x = -2$ carries charge 4 nC/m². Find the force on a point charge −5 mC located at the origin.

4.16 Determine the charge density due to each of the following electric flux densities:

(a) $\mathbf{D} = 8xy\mathbf{a}_x + 4x^2\mathbf{a}_y$ C/m²

(b) $\mathbf{D} = \rho \sin\phi\ \mathbf{a}_\rho + 2\rho\cos\phi\ \mathbf{a}_\phi + 2z^2\mathbf{a}_z$ C/m²

(c) $\mathbf{D} = \dfrac{2\cos\theta}{r^3}\mathbf{a}_r + \dfrac{\sin\theta}{r^3}\mathbf{a}_\theta$ C/m²

4.26 Given that the electric field in a certain region is

$$\mathbf{E} = (z + 1)\sin\phi\, \mathbf{a}_\rho + (z + 1)\cos \mathbf{a}_\phi + \rho \sin\phi\, \mathbf{a}_z \text{ V/m}$$

determine the work done in moving a 4-nC charge from

(a) $A(1, 0, 0)$ to $B(4, 0, 0)$

(b) $B(4, 0, 0)$ to $C(4, 30°, 0)$

(c) $C(4, 30°, 0)$ to $D(4, 30°, -2)$

(d) A to D

4.29 Determine the electric field due to the following potentials:

(a) $V = x^2 + 2y^2 + 4z^2$

(b) $V = \sin(x^2 + y^2 + z^2)^{1/2}$

(c) $V = \rho^2(z + 1)\sin\phi$

(d) $V = e^{-r}\sin\theta\cos 2\phi$

Sheet (5) Electric Fields In Material Space

5.1 In a certain region, $\mathbf{J} = 3r^2 \cos\theta\, \mathbf{a}_r - r^2 \sin\theta\, \mathbf{a}_\theta$ A/m, find the current crossing the surface defined by $\theta = 30°$, $0 < \phi < 2\pi$, $0 < r < 2$ m.

5.3 The current density in a cylindrical conductor of radius a is

$$\mathbf{J} = 10e^{-(1-\rho/a)}\mathbf{a}_z \text{ A/m}^2$$

Find the current through the cross section of the conductor.

5.12 A dielectric material contains 2×10^{19} polar molecules/m^3, each of dipole moment 1.8×10^{-27} C/m. Assuming that all the dipoles are aligned in the direction of the electric field $\mathbf{E} = 10^5\, \mathbf{a}_x$ V/m, find $\mathbf{P}$ and ε_r.

5.18 At the center of a hollow dielectric sphere ($\varepsilon = \varepsilon_0\varepsilon_r$) is placed a point charge Q. If the sphere has inner radius a and outer radius b, calculate **D**, **E**, and **P**.

5.20 For static (time-independent) fields, which of the following current densities are possible?

(a) $\mathbf{J} = 2x^3y\mathbf{a}_x + 4x^2z^2\mathbf{a}_y - 6x^2yz\mathbf{a}_z$

(b) $\mathbf{J} = xy\mathbf{a}_x + y(z+1)\mathbf{a}_y + 2y\mathbf{a}_z$

(c) $\mathbf{J} = \dfrac{z^2}{\rho}\mathbf{a}_\rho + z\cos\phi\, \mathbf{a}_z$

(d) $\mathbf{J} = \dfrac{\sin\theta}{r^2}\mathbf{a}_r$

5.27 Region 1 ($z < 0$) contains a dielectric for which $\varepsilon_r = 2.5$, while region 2 ($z > 0$) is characterized by $\varepsilon_r = 4$. Let $\mathbf{E}_1 = -30\mathbf{a}_x + 50\mathbf{a}_y + 70\mathbf{a}_z$ V/m and find: (a) $\mathbf{D}_2$, (b) $\mathbf{P}_2$, (c) the angle between $\mathbf{E}_1$ and the normal to the surface.

5.29 Two homogeneous dielectric regions 1 ($\rho \leq 4$ cm) and 2 ($\rho \geq 4$ cm) have dielectric constants 3.5 and 1.5, respectively. If $\mathbf{D}_2 = 12\mathbf{a}_\rho - 6\mathbf{a}_\phi + 9\mathbf{a}_z$ nC/m^2, calculate: (a) $\mathbf{E}_1$ and $\mathbf{D}_1$, (b) $\mathbf{P}_2$ and ρ_{pv2}, (c) the energy density for each region.