Concurrent Quantum State Transfer and Random Channel Sniffing

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Abstract: Harnessing the interplay between polarization and temporal degrees of freedom, the quantum polarization state of a single photon can be transferred with high fidelity through a polarization-drifting channel whose stochastic parameters are concurrently estimated. © 2020 The Author(s)

Quantum communication systems often involve the transmission of qubits through channels impaired by distortions modeled by unitary transformations with stochastic time-varying parameters. An example is the transmission of the polarization states of single photons through optical fibers with randomly drifting parameters [1]. To enhance the fidelity of state transfer, active compensation needs to be incorporated in the receiver accompanied by occasional end-to-end probing of the channel status [2]. We present here a scheme that enables high-fidelity transfer of an unknown single-photon state of polarization along with *concurrent* tomographic estimation (sniffing) of the channel stochastic parameters. The basic idea is to swap the unknown polarization qubit with a time-bin qubit [3] and to concurrently use three polarization states for tomographic reconstruction of the classical channel parameters.

The communication system is designed to transmit the polarization state $|\psi_0\rangle = (\alpha|H\rangle + \beta|V\rangle) \otimes |f(t)\rangle$ through a lossless stochastic channel represented by the time-varying unitary operator

$$\mathrm{U}(t) = \begin{bmatrix} \cos\gamma_t & -e^{i\delta_t}\sin\gamma_t \\ e^{i\phi_t}\sin\gamma_t & e^{i(\delta_t+\phi_t)}\cos\gamma_t \end{bmatrix}. \tag{1}$$

Here, $|H\rangle$ and $|V\rangle$ are polarization basis states, $|f(t)\rangle = \int dt\, f(t)|t\rangle$ is a temporal mode, and γ_t, δ_t, and ϕ_t are time-varying random variables characterizing the channel, e.g., an optical fiber. The first step in our scheme is for the transmitter (Alice) to swap the polarization state $|\psi_0\rangle$ with a time-bin state as in the state:$|\psi_1\rangle = |p\rangle \otimes [\alpha|f(t-t_1)\rangle + \beta|f(t-t_2)\rangle]$ before transmission through the channel; the receiver (Bob) will swap it back to polarization at the other end. Here, t_1 and $t_2 = t_1 + T$ are the centers of the time bins, and T is their separation. The benefit of this swapping process is attained only when the variations $x = \gamma_{t_2} - \gamma_{t_1}$, $y = \phi_{t_2} - \phi_{t_1}$ and $z = \delta_{t_2} - \delta_{t_1}$ in the channel parameters within the inter-bin time T are small. The sniffing process is enabled by use of an auxiliary polarization state $|p\rangle$ in one of the three non-orthogonal polarization settings: horizontal $|H\rangle$, diagonal $|D\rangle$, or right-circular $|R\rangle$. These states, which are selected by Alice in a time pattern known to Bob independently of the state to be transferred, enable Bob to reconstruct the channel parameters by use of a classical tomographic process akin to quantum state tomography. Implementation of the entire system is illustrated schematically in Fig. 1a. Substantial improvement of the fidelity of state transfer in comparison with the no-swap system is exemplified in Fig. 1b and 1c. The accuracy of channel parameter sniffing is instantiated in Fig. 1d.

Receiving the randomly distorted time-bin state $|\psi_2\rangle = |p\rangle \otimes [\alpha\mathrm{U}(t_1)|f(t-t_1)\rangle + \beta\mathrm{U}(t_2)|f(t-t_2)\rangle]$, Bob swaps the time-bin state back to a polarization state and subsequently to an equivalent path-mode state $|\psi_4\rangle$ [the paths c and d in Fig. 1a]. Bob further combines these two paths with beam splitters BS1, BS2, and BS3 to generate the final state $|\psi_5\rangle$ in terms of four paths i, j, l, m identified in Fig. 1a. Note that this state depends on the selected polarization state $|p\rangle$ set by Alice, and known to Bob. For very small x, y, and z, $|\psi_5\rangle \approx [\alpha|H\rangle + \beta|V\rangle] \otimes \sum_{s=\{i,j,l,m\}} \zeta_s^{(p)}|s\rangle$, so that the polarization part is identical to the original polarization state. The spatial part of the state $|\psi_5\rangle$ also provides sufficient information to compute the three channel parameters if the three settings of $|p\rangle$ are used.

We have calculated a measure of fidelity of the scheme under real conditions by tracing out the spatial degree of freedom $s = \{i, j, l, m\}$ in $|\psi_5\rangle$ and averaging the four elements of the reduced polarization density matrix $\rho^{(p)} = \sum_s \langle s|\psi_5\rangle\langle\psi_5|s\rangle$ over the randomness of the channel parameters. The diagonal elements $\bar{\rho}_{HH}^{(p)} = |\alpha|^2$ and $\bar{\rho}_{VV}^{(p)} = |\beta|^2$ are independent of the channel parameters and of p. The off-diagonal elements $\bar{\rho}_{HV}^{(p)} = \bar{\rho}_{VH}^{(p)*}$ do depend on p and of course on the probability distributions of x, y, z. We have determined values of these elements in terms of the standard deviations σ_x, σ_y, and σ_z assuming that they are independent Gaussian variables with zero means.

The fidelity of the transferred polarization state is

$$\mathscr{F}_p = \langle\psi_0^{(p)}|\rho^{(p)}|\psi_0^{(p)}\rangle = 1 - 2|\alpha|^2|\beta|^2\left[1 - \mathrm{Re}\left(\bar{\rho}_{HV}^{(p)}/\alpha\beta^*\right)\right]. \tag{2}$$

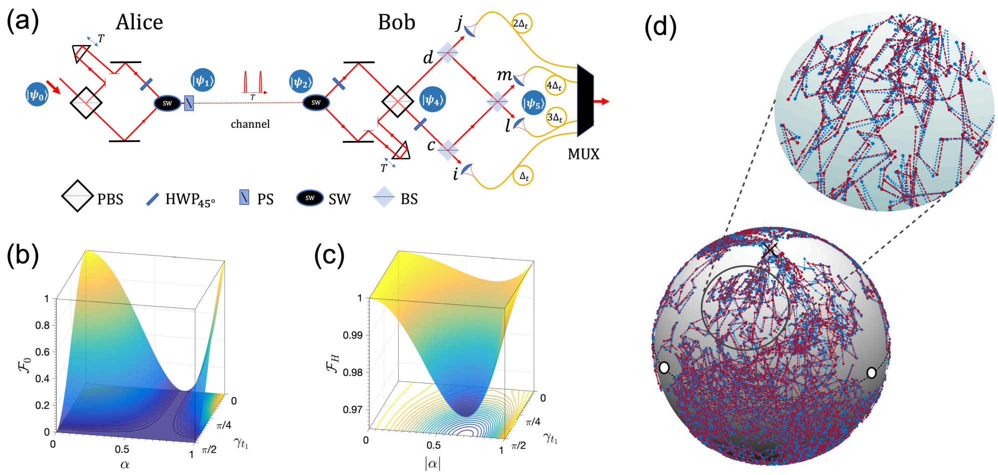

Fig. 1. (a) Schematic diagram of an optical system for state transfer and channel sniff. The initial polarization state is converted by Alice into a time-bin state using polarizing beam splitter (PBS), time delay T, half-wave plate HWP, and an optical switch (SW). The polarization of the time-bin state is controlled by a polarization switch PS. Bob converts the state back into a polarization state using a similar arrangement in reverse. The state is finally converted into a 4-path and polarization state by a set of beam splitters (BS) and an HWP in order to facilitate the sniff process. MUX is an optical multiplexer. (b) Dependence of the fidelity $\mathscr{F}_0$ of the no-swap system on the state coefficient α and the random parameter γ_{t_1} plotted over the $0-1.0$ range. Other channel parameters are assumed fixed at $\delta_{t_1} = \phi_{t_1} = \pi/2$. The standard deviations of x and y are 0.1π. (c) Same as (b) for the swap system, but plotted for values in the range $0.97-1.0$. (d) Actual values (blue line) and sniffed values (dark red line) of the channel parameters shown as points on the unit sphere. The polar angle is $2\gamma_t$ and the azimuthal angle is ϕ_t. Each random-walk step of γ_t and ϕ_t is emulated by two independent Gaussian random variables with standard deviation of 0.03π. The number of steps used is 5000 and the photon rate is 2×10^4 per step.

This is to be compared with the fidelity under the no-swap scenario $\mathscr{F}_0 = |\langle\psi_0^{(p)}|\mathrm{U}(t_1)|\psi_0^{(p)}\rangle|^2$, which is plotted in Fig. 1b. The fidelity $\mathscr{F}_H$ of the swap system for the H setting, which is plotted in Fig. 1c under the same conditions, is significantly greater (>0.97). Also, $\mathscr{F}_D$ and $\mathscr{F}_R$ exhibit similar behavior.

We finally show how measurements on the state $|\psi_5\rangle$ for $p = H, D, R$ can be used to sniff the channel parameters. The probabilities of the photon passing along each of the four paths i, j, l, m are given by

$$\begin{aligned}
&P_i^{(H)} = \tfrac{1}{2}\sin^2\gamma_{t_1}, \quad P_l^{(H)} = \tfrac{1}{4}(1-\sin\phi_{t_1}\sin 2\gamma_{t_1}),\\
&P_i^{(D)} = \tfrac{1}{4}(1+\cos\delta_{t_1}\sin 2\gamma_{t_1}), \quad P_l^{(D)} = \tfrac{1}{4}(1-\cos\delta_{t_1}\cos 2\gamma_{t_1}\sin\phi_{t_1}-\sin\delta_{t_1}\cos\phi_{t_1}),\\
&P_i^{(R)} = \tfrac{1}{4}(1+\sin\delta_{t_1}\sin 2\gamma_{t_1}), \quad P_l^{(R)} = \tfrac{1}{4}(1+\cos\delta_{t_1}\cos\phi_{t_1}-\sin\delta_{t_1}\sin\phi_{t_1}\cos 2\gamma_{t_1}),
\end{aligned} \tag{3}$$

and $P_j^{(p)} = \frac{1}{2} - P_i^{(p)}$, $P_m^{(p)} = \frac{1}{2} - P_l^{(p)}$ for $p = H, D, R$. These equations may be solved to yield the three channel parameters. Since $\gamma_t \in [0, \pi/2]$, $\phi_t \in [0, 2\pi]$, and $\delta_t \in [0, 2\pi]$, five of six equations are necessary. Of course a stream of photons must be used to build up the probabilities of detection in each of the paths.

We have run numerical Monte-Carlo simulations generating values for γ_t, δ_t, and ϕ_t following a random walk on the unit sphere. Calculated estimates of these parameters based on solving the path probabilities equations (3) are compared to the true value, as depicted on the unit sphere in Fig.1d, demonstrating precise tracking of the random-walk course. The channel parameters can therefore be tracked with accuracy as the state is transferred.

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References

1. N. Gisin et al., “Quantum cryptography,” Rev. Mod. Phys., **74**(1), 145–195 (2002).
2. J. Franson et al., “Operational system for quantum cryptography,” Electr. Lett., **31**(3), 232–234 (1995).
3. S. F. Hegazy, S. S. A. Obayya, B. E. A. Saleh, ”Transmission of polarization quantum state through a fiber optic channel by swapped time-bin state,” in *Proc. SPIE 10674, Quantum Technologies*, 2018, 106741F.