Dispersiveness of Faraday Mirrors Used to Compensate Random Polarization Distortion is Mitigated by the Randomness Itself

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Abstract: Faraday mirrors, widely used to compensate random polarization distortion in retracing fibers, as in quantum key distribution, may be operated with shorter pulses than previously thought, since their dispersion is remarkably mitigated by randomness itself. © 2024 The Author(s)

1. Introduction

Bidirectional quantum key distribution [1,2] benefits from use of a terminal Faraday mirror (FM) that converts the polarization of a forward-propagating light pulse into its orthogonal state in the backward direction. At the central wavelength of FM operation, this simple action results in passive and automatic compensation of all reciprocal and unitary polarization distortions encountered along the round trip through the optical fiber. However, away from this central wavelength, dispersion of the FM itself affects the precision of the orthogonalization process, and consequently the efficacy of polarization compensation. We show here that inherent randomness of the fiber unitary polarization transformation can assist in reclaiming the polarization-compensation condition.

2. Polarization Compensation Using Faraday Mirror

The FM consists of a Faraday rotator and a mirror in front. In the polarization-compensation process, the Faraday rotator (FR) applies a 45-degree polarization rotation about the propagation axis. This appears as a 90-degree rotation about the z axis of the Poincaré sphere, which is expressed by the operator:

$$R_f = \exp(-i\pi Z/4) = \begin{bmatrix} \exp(-i\pi/4) & 0 \\ 0 & \exp(i\pi/4) \end{bmatrix}, \qquad (1)$$

in the right/left circular polarization basis, where Z is the Pauli Z operator. After the FR, the mirror applies a symmetrical move T about the equator of the Poincaré sphere. The backward pass through the nonreciprocal FR adds another 90-degree rotation about z axis on the Poincaré sphere. This sequence of operations turns input polarization into its orthogonal state, which is mathematically expressed as $R_f T R_f$: $\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \rightarrow \begin{pmatrix} \beta^* \\ -\alpha^* \end{pmatrix}$. After retracing a fiber introducing a transformation U_{fiber}, the overall operator is

$$U_{\text{fiber}}^{-1}\, R_f T R_f\, U_{\text{fiber}} = R_f T R_f, \qquad (2)$$

which is valid for any unitary transformation U_{fiber} (because the orthogonalization is basis invariant). This means that the state of polarization received by the optical transceiver is always orthogonal to the state initially sent down the fiber [see Fig. 1(a)]. This remarkable operation is expected, as long as the light pulse has a narrow bandwidth in the vicinity of the central wavelength for which the FM is designed.

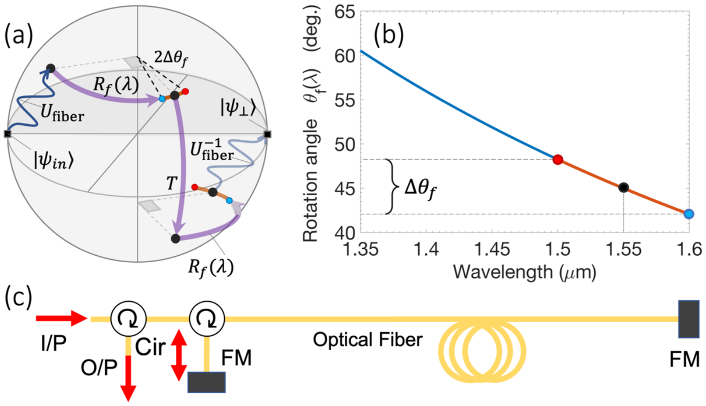

Fig. 1. (a) Evolution of the polarization-state on the Poincaré sphere after retracing a fiber terminated by one FM. The evolved states at 1.5 µm, 1.55 µm, and 1.6 µm, are denoted by red, black, and blue circles, respectively. At 1.55 µm, the state is always orthogonal to the input state. (b) Wavelength dependence of the rotation angle $\theta_f(\lambda)$ of a FR based on CeF_3 crystal. (c) Broadband compensation of polarization distortion in an optical fiber using two terminal FMs.

However, for a broadband pulse, the wavelength components deviating from the FM central wavelength encounter dispersive Faraday-rotation represented by the operator

$$R_f(\lambda) = \begin{bmatrix} \exp\left(-i\theta_f(\lambda)\right) & 0 \\ 0 & \exp\left(i\theta_f(\lambda)\right) \end{bmatrix}, \qquad (3)$$

where $\theta_f(\lambda) = V(\lambda)\, B\, L$ is the rotation angle, with $V(\lambda)$ being the Verdet constant, and B is the effective magnetic field applied over the crystal length L [Fig. 1(b) shows $\theta_f(\lambda)$ of a FR with CeF_3 crystal designed such that $\theta_f(1.55\ \mu\text{m}) = 45°$ [3]]. Therefore, as the wavelength components deviate further from the FR central wavelength, Eq. (2) will no longer be valid, and the polarization state after a round trip will deviate further from perfect orthogonality. In Fig. 1(c), we suggest inserting an identical FM on the front end of the fiber, which recovers the initial polarization. The state produced after a round trip is then,

$$|\psi_{\text{out}}\rangle = R_f(\lambda)\, T\, R_f(\lambda)\;\; U_{fiber}^{-1}\, R_f(\lambda)\, T\, R_f(\lambda)\, U_{fiber}\, |\psi_{in}\rangle. \qquad (4)$$

We show next that, for a random fiber distortion: $U_{\text{fiber}} = U_r$, $|\psi_{\text{out}}\rangle$ can be closer to $|\psi_{in}\rangle$ compared with no-fiber case: $U_{\text{fiber}} = I$.

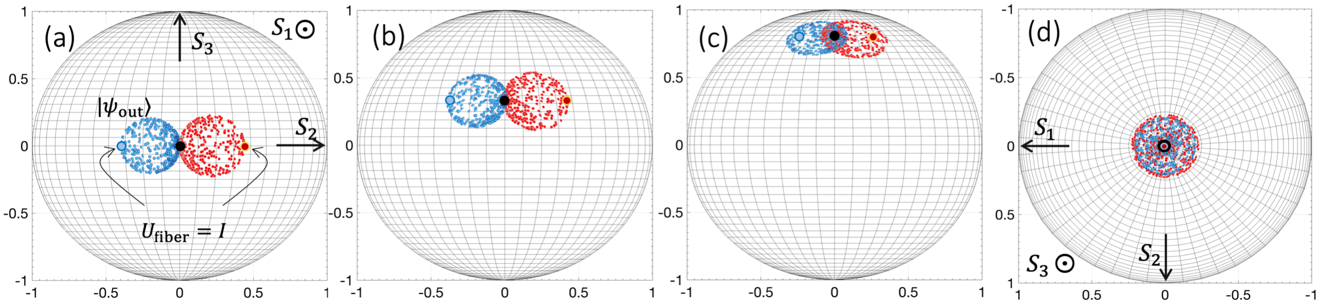

Fig. 2. Results of four numerical experiments showing the polarization state $|\psi_{\text{out}}\rangle$ based on use of two terminal CeF₃ FMs on the fiber ends. In the experiments, the fiber is retraced by initial states: (a) $|\psi_{\text{in}}\rangle = |H\rangle \equiv (|R\rangle + |L\rangle)/\sqrt{2}$, (b) $|\psi_{\text{in}}\rangle = \sqrt{2/3}|R\rangle + \sqrt{1/3}|L\rangle$, (c) $|\psi_{\text{in}}\rangle = \sqrt{9/10}|R\rangle + \sqrt{1/10}|L\rangle$, and (d) $|\psi_{\text{in}}\rangle = |R\rangle$. The sprinkled red and blue dots show polarization states of 400 runs per experiment at 1.5 µm and 1.6 µm, respectively. Every run considers random and independent setting of the fiber operator $U_{\text{fiber}} = U_r$. The circles (red and blue) denote output states (at 1.5 µm and 1.6 µm), when $U_{\text{fiber}} = I$ (in absence of optical fiber). Black circles denote the output states at 1.55 µm, which precisely overlap the initial states.

3. Mitigation of dispersive polarization distortion

The mitigation effect is demonstrated here assuming an FR using CeF₃ magneto-optical crystal, which exhibits high Verdet constant and also high transmittance in the near-infrared (NIR) range [3]. The CeF₃ FR under consideration is designed to perform a 45-degree rotation at 1.55-µm central wavelength, i.e., 90-degree rotation on the Poincaré sphere.

We have conducted four numerical experiments [4] with different settings of input polarization state $|\psi_{\text{in}}\rangle$ (see Fig. 2). Each experiment involves 400 runs processed at each of the wavelengths: 1.5 µm, 1.55 µm, and 1.6 µm. Every run of the numerical experiments included a forward pass through the fiber (represented numerically by a random and independent operator $U_{\text{fiber}} = U_r$) followed by reflection from the first CeF₃ FM, then backward fiber pass (operator U_{fiber}^{-1}), and finally reflection from a second CeF₃ FM. This yields an output state $|\psi_{\text{out}}\rangle$ as in Eq. (4).

As shown in Fig. 2, it is evident that, compared with the no-fiber case: $U_{\text{fiber}} = I$, the randomness of the fiber operator U_{fiber} leads to output states $|\psi_{\text{out}}\rangle$ that are often closer to the ideal dispersion-free state (at 1.55 µm). Remarkably, fiber randomness mitigates the distortion introduced by the FR dispersion.

This mitigation effect is quantitatively elucidated in Fig. 3 for wavelengths 1.5 µm and 1.6 µm. The fidelity of the output state $|\psi_{\text{out}}\rangle$ to an input state $|\psi_{\text{in}}\rangle$: $\langle\psi_{\text{in}}|\psi_{\text{out}}\rangle$ is numerically obtained in every run for random and independent setting U_{fiber}. It is then compared to the fidelity of the output state in absence of the optical fiber, i.e., with $U_{\text{fiber}} = I$, which is obtained from the geometry on the Poincaré sphere as

$$F(|\psi_{\text{in}}\rangle, |\psi_{\text{out}}\rangle) = |\langle\psi_{\text{in}}|\psi_{\text{out}}\rangle| = \left|\cos\left\{\sin^{-1}\left[\sin\left(4\theta_f(\lambda) - \pi\right)\sin\theta\right]\right\}\right|, \quad (5)$$

where θ is the polar angle of $|\psi_{\text{in}}\rangle$ on the Poincaré sphere. It is evident from Fig. 3 that the mean fidelity in the case of a random fiber matrix, $U_{\text{fiber}} = U_r$, is higher compared to the no-fiber case, $U_{\text{fiber}} = I$, over a wide range of polar angles. The role of the fiber randomness in mitigating FR dispersion is similar to that of unitary optical elements inserted between two FRs designed for broadband FR operation [5].

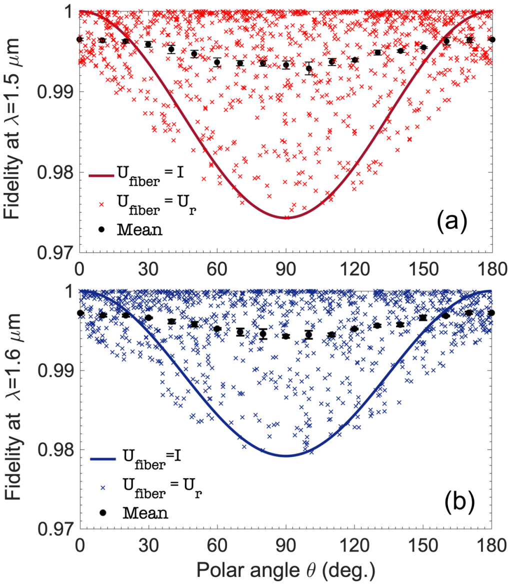

Fig. 3. Fidelity of the output state $\langle\psi_{\text{in}}|\psi_{\text{out}}\rangle$ after a round trip including the two FMs plotted versus the polar angle of the input state $|\psi_{\text{in}}\rangle$ on the Poincaré sphere. (a) $\lambda = 1.5\,\mu m$, (b) $\lambda = 1.6\,\mu m$. The optical fiber operator U_{fiber} is either substituted by I (no-fiber case), or U_r which is random and independent for each of the 400 runs. The mean fidelity (black bars) is obtained for the output polarization states when $U_{\text{fiber}} = U_r$.This mean fidelity exceeds the no-fiber fidelity ($U_{\text{fiber}} = I$) over a wide span of polar angles. This demonstrates the mitigated dispersiveness of the FM due to the random optical-fiber distortion.

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