On Dynamics of a Cannon Barrel-Recoil Mechanism with Nonlinear Hydraulic Damper and Air-Springs

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Abstract

The objective of this paper is to investigate the dynamics of the barrel assembly-recoil mechanism of military cannons when using air springs and hydraulic dampers of nonlinear characteristics in their recoil mechanisms. The nonlinear characteristics of the damper and spring and the recoil mechanism orientation introduce extra nonlinearity to the dynamic model of the system. An extremely nonlinear model of the barrel assembly is derived and solved using Runge-Kutta 4 method to provide the dynamic response of the barrel assembly upon firing. The simulation results using the data of a Howitzer M114 cannon are presented for recoil mechanism orientation ≤ 50 degrees and number of recoil elements up to 4. The performance of the recoil mechanism is evaluated through the minimum and maximum displacements of the barrel assembly, the settling time of its response upon firing and the steady-state error of its time response. The analysis shows that it is possible with the suggested recoil elements to obtain barrel assembly response similar to that of a critically damped second-order system. It is possible with proper selection of the recoil assembly parameters to decrease the maximum barrel displacement to only 14 mm without undershoot and the settling time to about 2 seconds with steady-state error as small as about 1 mm.

Keywords: Cannon recoil mechanism, Barrel assembly dynamics, Hydraulic dampers, Air springs, Nonlinear dynamic model, Barrel response upon firing, Recoil mechanism performance.

1. Introduction

Researchers pay deep attention to the analysis and design of artillery recoil mechanisms. Hogg (2000) described about 300 artillery pieces from 1900 to 2000 with full dimensions, mass, ammunitions and range details, country of origin and muzzle velocity [1]. Ahmadian, Appleton and Norris (2002) used a MR damper to control the recoil dynamics. The suggested technique for using MR dampers for free out of battery. They used a recoil demonstrator including a 0.5 caliber gun and a MR damper [2]. Slizys (2005) studied the dynamic characteristics of the plane motion of the recoil of the automatic rifle AK-4. He formulated the mathematical models of the recoil plane motion and obtained its dynamic characteristics [3]. Choi, Hoo and Wereley (2005) examined the use of a double adjustable MR damper to produce high damping force over a high speed piston range. They proposed an on-off control algorithm to improve the shock mitigation of the passive MR gun recoil system [4]. Bao-lin (2006) studied the use of a gun recoil MR damper for a gun test application. He constructed a one-dimensional parallel-plate laminar flow model for the damper based on Herchel-Bulkley shear model and obtained...
the damping characteristic curves for the damper and evaluated its performance [5]. Lin et.al. (2009) derived a mathematical model for the recoil force during firing. They claimed that their results provide a clear understanding for designing the recoil mechanism and improve its performance [6]. Xue-zheng, Jiong and Hong-sheng (2010) designed a large-stroke MR impact damper which can work effectively at the large velocities occurring in artillery recoil. They showed that the MR damper is able to effectively control the recoil dynamics in terms of recoil force and stroke [7]. Gim, Cha and barrel. They applied the numerical modal analysis, signal processing and shock response analysis techniques in their analysis [8]. Ting, Lu and Rui (2013) designed a deflection system for the breech block and anti-recoil mechanism. They realized the technical indicators detection under the recoil process using advanced hydraulic control technology [9]. Hassaan (2014) studied the dynamics of barrel assembly of a Howitzer M114 cannon upon firing when using a nonlinear hydraulic damper and a constant stiffness spring in the recoil mechanism [10] and when using a nonlinear air spring and a constant damping coefficient hydraulic damper [11].

Air springs find great attention from a lot of researchers because of its simple design and control. Presthus (2002) developed a mathematical model for air springs. He determined five of the model parameters using thermodynamics and fluid dynamic constitutions. He presented the air spring stiffness as function of the vibration frequency [12]. Wei-min, Can-hui, Ya-ling and Yan-sha (2004) studied the deformation and static characteristics of an air spring used in automotive suspension. They compared the characteristic obtained by finite elements to that obtained experimentally [13]. Deo and Sah (2006) proposed a design for a pneumatic automotive suspension system. They showed that the air spring stiffness is function of pressure, volume and temperature [14]. Silva and Costa (2008) developed empirical models relating force acting on the air spring to its deflection and internal pressure for 4 different springs [15]. Koizumi et. al. (2009) proposed an air suspension system coupled with a rotary damper to attain low dynamic stiffness and high damping coefficient. They attained an extremely low dynamic stiffness at a specific frequency [16]. Zhang and Yang (2010) established a nonlinear model for a pneumatic vibration isolator considering the volumetric compressibility of air. They showed that the system has a strong nonlinear characteristics [17]. Todkar and Joshi (2011) discussed the effect of the variation of mass ratio, air damping ratio and air spring stiffness on the motion transmissibility at the resonant frequency of the main system. They designed a control system to vary the air pressure in the damper of the absorber system [18]. Robinson (2012) investigated the use of a pneumatic suspension system containing an air spring, air flow valve and an accumulator where the air spring and accumulator provide the spring characteristics and the valve provides the damping characteristics [19]. Chen et. al. (2013) formulated a nonlinear model of a multi-axle semi-trailer with longitudinal-connected air suspension based on fluid mechanics and thermodynamics. They showed that the influence of air line diameter on load-sharing is more significant than that of he connector [20]. Razdan, Bhave and Awasare (2014) used an active pneumatic suspension with control strategy based on mass flow control for a commercially manufactured small car. They concluded that the active suspension using velocity feedback as the control strategy has better performance at resonance [21].

Hydraulic dampers find wide application in a lot of engineering industries. They are used to cushion against dynamic loading to control resulting dynamic motions. Cheng-guo, Lixin, Jin-zhao and Wen-zhang (2003) obtained a compact empirical model for the hydraulic damper relating the damping force to its piston velocity. Their model depends mainly on the trigonometric functions sin and arctan [22]. Kurino, Matsunago, Yamada and Tagami (2004) presented an ingenious passive hydraulic damper for structural control with high performance equivalent to that of a semi-active damper. Their damper controls the damping coefficient by regulating the opening of a flow control valve housed in the damper without any outer power source [23]. Wei (2006) studied the characteristics of hydraulic dampers used in automotive suspension systems. The dampers characteristics are nonlinear and covered velocity range up to 1 m/s [24]. Guzzomi, O'Neill and Tavner (2007) studied the dynamics of a Tenneco automotive hydraulic damper to predict the damper performance [25]. Salem and Galal (2009) identified the characteristics and damping coefficient of a hydraulic shock absorber used in a light weight tracked vehicle under real conditions. They showed that the hydraulic damper characteristics are nonlinear in the velocity range -0.22 to 0.22 m/s with remarkable hysteresis in the negative range of damper velocity [26]. Stawik, Czop, Krol and Wszotek (2010) identified the root cause of the temporary decrease in the damping force occurring during the early stage of the stroking cycle's compression phase of hydraulic dampers. They presented the damper characteristics as a damping force against damper displacement in an extremely nonlinear fashion [27].
Hou et al. (2011) established a detailed model of a shock absorber using the modelica language in the form of mathematical equations and object-oriented constructions [28]. Ferdek and Luczko (2012) created a physical and mathematical model for a twin-tube hydraulic damper incorporating numerical integration. They examined the effect of the amplitude and frequency range and the parameters describing the oil flow rate through the damper valves. They investigated the damper characteristics for damper-piston velocity in the range -0.6 to 0.6 m/s [29]. Kate and Jadhav (2013) presented a mathematical model for the damping force of a hydraulic shock absorber. They showed that the damping characteristics of the hydraulic shock absorber are nonlinear for the damper-rod velocity range -3 to 3 m/s [30]. Sun, Jioo, Huang and Hua (2014) filled a hydraulic damper with $5 \times 10^5$ cSt silicone oil as a non-Newtonian fluid with low velocity exponent. They presented the variation of the damper damping force with time for different impact drop height both experimentally and by simulation [31].

2. Analysis

The dynamic system of any cannon-recoil mechanism dynamic system is considered as a mass-damper-spring system. Fig.1 shows a typical Howitzer M114 155 mm cannon model [32].

![Fig.1 The Howitzer M114 155 mm cannon [32].](image)

The equivalent dynamic model is shown in Fig.2 for the barrel assembly and the recoil mechanism.

![Fig.2 Cannon equivalent dynamic system.](image)

The barrel assembly has a center of mass, G that is translates horizontally by a dynamic motion x. The recoil elements are joined to the barrel assembly and the main cannon chassis secured to the ground and has an original orientation $\theta_0$ with the ground. As the gun fires, the barrel assembly moves in the opposite direction of the projectile horizontal motion component with initial velocity depending on the projectile momentum and barrel assembly mass.

The recoil mechanism takes this barrel assembly momentum and tries to return the barrel assembly to the original position before firing to start a new firing cycle. The motion of the equivalent dynamic system upon firing is illustrated in Fig.3.
The dynamic system has the motions and orientation:

- $x$: dynamic motion of the barrel-assembly center of mass (horizontal).
- $\delta$: dynamic deflection of the recoil elements.
- $\theta$: dynamic orientation of the recoil elements.
- $\theta_o+\theta$: new orientation of the recoil mechanism centerline.

The recoil elements dynamic deflection $\delta$ is related to the barrel assembly motion $x$ through the geometrical relation (see Fig.3):

$$\delta = x \cos \theta_o$$  \hspace{1cm} (1)

The orientation change $\theta$ is related to the motion $x$ through (see Fig.3):

$$\theta = \sin^{-1} \left( x \sin \theta_o / (L_o - x) \right)$$  \hspace{1cm} (2)

where $L_o$ = initial length of the buffer elements (dampers & springs).

The dynamics of the barrel assembly-recoil mechanism depend on its differential equation. The differential equation depends on the characteristic nature of the buffer elements and its orientation angle. Fig.4 shows the free body diagram of the barrel assembly.

Using the free body diagram of Fig.4 and the second-law of motion, the differential equation of the barrel assembly of dynamic motion $x$ is:

$$Mx'' = -F_d \cos(\theta_o+\theta) - F_e \cos(\theta_o+\theta)$$  \hspace{1cm} (3)

Where:
- $M$ = barrel assembly mass in kg.
- $x'' = \frac{d^2x}{dt^2}$ = barrel assembly acceleration (m/s$^2$)
- $F_d$ = damping force of the hydraulic damper (N).
- $F_e$ = elastic force of the recoil mechanism spring.

The damping force $F_d$ of the hydraulic damper used in the present study depends on the damper velocity ($d\delta/dt$). The damping characteristics of a hydraulic damper is usually nonlinear and depends on the direction of motion of the damper piston. Using the damping data of Polach and Halzman [33], the damping characteristics are defined by the third-order polynomial model [10]:

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\[ F_d = a_1d \delta'^3 + a_2d \delta'^2 + a_3d \delta' + a_4d \]  

(4)

Where \( \delta' \) is the piston velocity and the parameters of the model (\( a_1d \) to \( a_4d \)) are given in Table 1 depending on the direction of damper-piston velocity.

### Table 1: Hydraulic damper damping parameters [10]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Positive velocity</th>
<th>Negative velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_{1d} )</td>
<td>23615.02883</td>
<td>1267.72048024</td>
</tr>
<tr>
<td>( a_{2d} )</td>
<td>-19968.573887</td>
<td>2963.66361350</td>
</tr>
<tr>
<td>( a_{3d} )</td>
<td>9490.302535</td>
<td>2532.4760920</td>
</tr>
<tr>
<td>( a_{4d} )</td>
<td>58.0478254</td>
<td>6.42175050</td>
</tr>
<tr>
<td>Correlation coefficient</td>
<td>0.99627</td>
<td>0.99754</td>
</tr>
</tbody>
</table>

The elastic characteristics of the air springs suggested for this engineering application depend on the air pressure inside the spring and the spring deflection. A typical air spring characteristics are given by Silva and Costa for spring displacement up to 100 mm and air pressure in the range 1.2 to 3.5 bar [15]. These characteristics are defined by a second-order polynomial fitted by the author in the form [11]:

\[ F_e = a_{1e} \delta'^2 + a_{2e} \delta + a_{3e} \]  

(5)

The parameters and correlation coefficient of the model in Eq.5 are given in Table 2 as function of the internal pressure of the air compressor [11].

### Table 2: Air spring parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>1.2 bar air pressure</th>
<th>4.8 bar air pressure</th>
<th>7 bar air pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_{1e} )</td>
<td>0.0000670163</td>
<td>0.0012237762</td>
<td>0.001904761905</td>
</tr>
<tr>
<td>( a_{2e} )</td>
<td>0.0525291370</td>
<td>0.1367132860</td>
<td>0.144047619050</td>
</tr>
<tr>
<td>( a_{3e} )</td>
<td>8.9825174825</td>
<td>23.42657342657</td>
<td>32.24999999</td>
</tr>
<tr>
<td>Correlation coefficient</td>
<td>0.9989</td>
<td>0.9980</td>
<td>0.9966</td>
</tr>
</tbody>
</table>

Combining Eqs.1 and 4 gives \( F_d \) as:

\[ F_d = a_{1d}(\cos\theta_o)^3 x^3 + a_{2d}(\cos\theta_o)^2 x^2 + a_{3d}(\cos\theta_o) x' + a_{4d} \]  

(6)

As function of the displacement \( x \) of the barrel assembly, the elastic force becomes:

\[ F_e = a_{1e}(\cos\theta_o)^3 x^2 + a_{2e}(\cos\theta_o) x + a_{3e} \]  

(7)

Combining the Eqs. 3 through 7 gives the differential equation of the barrel-recoil system as:

\[ Mx'' + F_d \cos(\theta_o \theta) + F_e \cos(\theta_o \theta) = 0 \]  

(8)

### 3. Barrel Recoil Dynamics

The dynamics of the barrel-recoil mechanism of the cannon are defined by solving Eq.8 which is extremely nonlinear. The procedure is as follows:

- Transfer the second-order homogeneous equation of Eq.9 to two first-order homogeneous equations.
- The mathematical tool for this is using state variables approach to build a new state model for the system.
- Use any numerical technique to solve the state model and get the dynamic system response for a specific initial conditions [34].

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MATLAB can be used to apply Runge-Kutta 4 technique to solve the state model using its command "ode45" [35]. The dynamics can be evaluated for different recoil mechanism orientation $\theta_o$ and different number of recoil elements since they have a great effect on the dynamic system characteristics.

The state model of the barrel assembly-recoil mechanism is derived by defining the state variables $x_1$ and $x_2$ as:

$$
\begin{align*}
  x_1 &= x \\
  x_2 &= \frac{dx}{dt} = x_1'
\end{align*}
$$

(9)

Using all the above derived equations, the state model of the dynamic system is:

$$
\begin{align*}
  x_1' &= x_2 \\
  x_2' &= -\left(\frac{F_d}{M}\right)\cos(\theta_o+\theta) - \left(\frac{F_e}{M}\right)\cos(\theta_o+\theta)
\end{align*}
$$

(10)

Where $\theta$, $F_d$ and $F_e$ are given by Eqs. 2, 6 and 7 respectively.

**4. Analysis Results**

A MATLAB code is written to apply the analysis and procedures suggested in this work to assign the dynamics of the cannon barrel assembly-recoil system upon firing. The other parameters of the dynamic system are:

- Barrel assembly mass, $M$: 5600 kg
- Projectile mass: 6.86 kg
- Muzzle velocity upon firing: 564 m/s
- Initial recoil elements length, $L_o$: 3.5 m.
- Initial horizontal velocity of the barrel assembly upon firing: 0.7 m/s based on momentum conservation of the barrel – projectile rigid bodies.

The results considering the recoil mechanism initial orientation and the spring internal pressure are evaluated for an air pressure of 1.2 bar since this level has provided barrel time response upon firing similar to that of a critically damped second order dynamic system (optimal response).

- For zero and 30 degrees recoil mechanism orientation using one damper and air spring: Figs.6 and 7.

![Fig.6 Dynamic response with $\theta_o = 0$ and $n=1$.](image1)

![Fig.7 Dynamic response with $\theta_o = 30^\circ$ and $n=1$.](image2)

- For 0 and 30 degrees recoil mechanism orientation with $n = 2$: Figs.8 and 9.
Fig. 8  Dynamic response with $\theta_{o} = 0$ and n=2.

For 0 and 30 degrees recoil mechanism orientation with n = 3: Figs. 10 and 11.

Fig. 10  Dynamic response with $\theta_{o} = 0^\circ$ and n=3.

Fig. 11  Dynamic response with $\theta_{o} = 30^\circ$ and n=3.

Fig. 12  Dynamic response with $\theta_{o} = 0^\circ$ and n=4.

Fig. 13  Dynamic response with $\theta_{o} = 30^\circ$ and n=4.
5. Characteristics of the Barrel Assembly Dynamic Motion

- The maximum barrel displacement upon firing, $x_{\text{max}}$, increases nonlinearly as the recoil elements orientation increases for the two air pressure levels of the air spring as shown in Fig.14.
- Because the dynamic response is of an oscillating nature for some of the number of dampers and air springs, it has a minimum value $x_{\text{min}}$ (maximum undershoot) which is the maximum displacement in the forward direction after firing.
- $x_{\text{min}}$ is function of the recoil elements orientation and the number of recoil elements as shown in Fig.15.

![Fig.14. Maximum barrel assembly displacement.](image1)

![Fig.15 Minimum barrel assembly displacement.](image2)

- The settling time of the barrel assembly is affected by both the recoil mechanism orientation and the number of dampers and air springs in the recoil mechanism as shown in Fig.16.
- Because of the nonlinearity nature of the dynamic system under study, a steady-state error appears at the end of the transient motion which is a deviation from the zero displacement position of the barrel assembly. This is illustrated in Fig.17.

![Fig.16 Settling time of the barrel assembly.](image3)

![Fig.17 Steady-state error of the barrel assembly.](image4)
6. Conclusions
- Nonlinearities in both elements of the recoil mechanism were considered.
- The effect of the number of recoil elements on the dynamics of the barrel assembly was investigated in details for up to 4 units of each element.
- The nonlinear characteristics of the recoil elements were modeled by a third-order polynomial for the hydraulic damper and a second-order polynomial for the air-spring.
- Only one level of air pressure was considered (1.2 bar).
- The time response of the barrel assembly upon firing had an oscillating nature before settling for a number of recoil elements \( \leq 3 \).
- The time response of the barrel assembly upon firing did not oscillate for a number of recoil elements = 4. However, it had larger steady state error.
- The maximum barrel assembly displacement increased with increasing the recoil mechanism orientation.
- The maximum barrel assembly displacement decreased with increasing the number of recoil mechanism elements.
- The undershoot increased with increasing the recoil mechanism orientation, and decreased with increasing the number of recoil mechanism elements.
- The undershoot was almost zero with 4 recoil elements providing a time response of the barrel assembly upon firing similar to that of a critically damped second-order dynamic system under the same initial conditions.
- The settling time increased with the increase of the recoil orientation and decreased with the increase of the number of recoil elements.
- The settling time with 4 recoil elements was almost constant at the level of 2 seconds.
- The steady-state error decreased in a nonlinear fashion with the increase in the recoil elements orientation between -1.1 and -1.8 mm.
- The settling time was almost independent of the number of recoil elements.

References


[32] en.wikipedia.org/wiki/M114_115_mm_howitzer


**BIOGRAPHY:**

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An emeritus professor in the Department of Mechanical Design and Production, Faculty of Engineering, Cairo University, EGYPT. He got his Ph.D. from Bradford University, UK in 1979 under the supervision of the great professor John Parnaby. He published 10's of research papers in various International Journals. He is the author of books on Experimental Systems Control and History of Mechanical Engineering. His current research is in Mechanical Vibrations, Automatic Control, Mechanism Synthesis and History of Mechanical Engineering.