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Different Estimation Methods Using Ranked Set Sampling for the Ramos–Louzada Distribution

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ABSTRACT

The data collection is a vital aspect of any scientific inquiry. Advanced data collection techniques, such as ranked set sampling (RSS), use the ranking information of the sample units to provide representative sample data. Statistical approaches based on RSS frequently produce noticeably better results when compared with similar techniques based on simple random sampling (SRS). A wonderful option for modeling positively skewed, overdispersed data with a leptokurtic shape is the one-parameter Ramos–Louzada distribution (RLD). This study examines thirteen classical estimation methodologies, including maximum and minimum spacing distances, ordinary least squares, maximum likelihood, weighted least squares, and variations of the minimum distance method for the RLD parameter using RSS and SRS approaches. A Monte Carlo simulation analysis is implemented to compare the efficacy of the resulting estimates based on several accuracy criteria. By evaluating the estimated quality for SRS and RSS based on the partial and total ranking metrics, we conclude that the maximum likelihood and maximum product spacing approaches seem very useful for both sampling processes. Analysis shows that estimates of RSS datasets have lower mean squared errors compared with SRS estimates. Thus, it can be concluded that the RSS estimates are more efficient than the SRS estimates. Furthermore, three real-world datasets are selected and examined to demonstrate the practical use of the proposed estimation techniques. This analysis demonstrates the applicability of these techniques in the real world and emphasizes their worth and efficacy for investigation and decision-making.

1 | Introduction

An experiment's cost efficiency may frequently be increased by using the sampling process known as ranked set sampling (RSS).

It works well when the ranking of the units in a small set is simple and affordable, but the quantification of the sample units is expensive or complicated. The concept of “ranked set sampling” was initially presented by McIntyre [1, 2], and Halls and Dell

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[3] used it in their work on the evaluation of forage yields in a pine-hardwood forest. The effectiveness of the RSS-based mean estimator, which is unbiased for population means, was theoretically investigated by Takahashi and Wakimoto [4]. They discovered that when the ranking is perfect, its variance is always less than the variance of the mean estimate based on simple random sampling (SRS) with the same sample size. In addition to its widespread use, RSS has demonstrated its worth in certain domains by several researchers, such as environmental studies [5–7], agriculture [8, 9], fisheries research [10], quality control [11], forest [3, 12], and reliability studies [13–16]. For more information, see [17–22].

The following outlines the core idea of sample selection under RSS: Pick t random samples, each of size t . Then, rank the units within each sample in relation to the variable of interest using a visual examination or any other inexpensive technique. Then choose $k = 1, \dots, t$ as the k -th smallest unit from the k -th sample for the actual measurement. Therefore, a measured unit is derived from each sample, for a total of t . Hence, the one-cycle RSS is represented as $W_{(k)k} = (W_{(1)1}, W_{(2)2}, \dots, W_{(t)t})$, $k = 1, \dots, t$. Before a sample of $n = tb$ measurements is collected, the process might be repeated b times. The b -cycle RSS is represented as $W_{(k)kj} = (W_{(1)11}, W_{(2)22}, \dots, W_{(t)tb})$, where $k = 1, \dots, t$, and $j = 1, 2, \dots, b$. In simplified form, we write W_{kj} in the rest of the present work. Let W_{kj} represent the order statistics of k -th sample, with $k = 1, \dots, t$ in the b cycle. According to Wolfe [23], set sizes (t) greater than five will surely produce an excessive amount of ranking errors. They thus would not significantly improve the RSS's effectiveness.

Assuming a perfect ranking, the probability density function (PDF) of W_{kj} (Arnold et al. [24]) is given by:

$$g(w_{kj}) = \frac{1}{B(k, t-k+1)} [G(w_{kj})]^{k-1} [1 - G(w_{kj})]^{t-k} g(w_{kj}); \quad w_{kj} \in R. \quad (1)$$

Numerous works on the topic of parametric inference of distributions under RSS may be found in the literature. Under RSS, Stokes [25] investigated parameter estimation for location-scale family distributions. Under modified RSS, Shaibu and Muttalak [26] investigated the estimation of parameters for gamma, exponential, and normal distributions. Bhoj [27] examined the parameter estimators for the extreme value distribution. Using RSS, the estimators of the generalized geometric distribution have been discussed by Bhoj and Ahsanullah [28]. The parameter estimators of the Gumbel distribution were examined by Yousef and Al-Subh [29]. Pedroso et al. [30] proposed the estimators for the Birnbaum-Saunders distribution. The entropy estimation for the inverse Gaussian and Laplace distributions was covered by Al-omari and Haq [31]. Taconeli and Giolo [32] proposed estimators for the weighted and power Lindley distributions. He et al. [33] investigated parameter estimation in RSS for the log-logistic distribution. Esemien and Grler [34] discussed the estimators of the generalized Rayleigh distribution. Estimators of the generalized quasi-Lindley distribution were covered by Al-Omari et al. [35]. Assessing the performance of some RSS designs using a hybrid approach has been discussed by Sabry et al. [36]. The estimators of the new Weibull-Pareto distribution were investigated by Samuh et al. [37]. A generalized

Bilal distribution with some properties and estimation under ranked set sampling has been discussed by Akhter et al. [38]. We've thought about other distributions, see exponential Pareto distribution [39], Kumaraswamy and inverted Kumaraswamy distributions [40–42], inverted Topp-Leone distribution [43], generalized unit half logistic geometric distribution [44], inverse power Cauchy distribution [45], arctan uniform distribution [46], complementary exponentiated Bell exponential distribution [47], and unit generalized Rayleigh distribution [48].

The development of flexible distributions that can address specific lifetime system issues has gained popularity as a result of the lifetime modeling of complicated research. One of the finest options for modeling overdispersed and positively skewed data with leptokurtic shape is the one-parameter Ramos–Louzada distribution (RLD) [49]. The probability density function (PDF) of the RLD is given by:

$$g(w; \lambda) = \frac{[(\lambda - 2)\lambda + w]e^{-\frac{w}{\lambda}}}{(\lambda - 1)\lambda^2}; \quad w > 0, \quad (2)$$

where $\lambda \geq 2$, is the shape parameter. It was demonstrated by Ramos and Louzada [49] that (2) performs better than the Lindley and exponential distributions in several cases. The cumulative distribution function (CDF) of the RLD is given by:

$$G(w; \lambda) = 1 - \frac{\left(\lambda + \frac{w}{\lambda} - 1\right)e^{-\frac{w}{\lambda}}}{(\lambda - 1)}; \quad w > 0. \quad (3)$$

Based on (2), it is declared to be a useful replacement for the Lindley and exponential distributions, taking on the forms of both with a shape parameter $\lambda \geq 2$. In other words, when λ approaches 2, the distribution approaches the Lindley distribution and becomes an exponential distribution for large values of λ . Figure 1 shows the PDF and hazard rate function (HRF) plots for the RLD.

Estimating the parameter(s) is a crucial aspect of analyzing any probability distribution. The maximum likelihood (ML) approach is often a very well-liked estimating technique, despite the fact that it is not necessarily the most accurate. This study examines thirteen alternative estimates, namely, ordinary least squares estimate (OLSE), Cramér-von-Mises estimator (CVME), Anderson-Darling estimate (ADE), weighted least squares estimate (WLSE), maximum product spacing estimate (MPSE), right-tail ADE (RTADE), Kolmogorov estimate (KE), minimum spacing square distance estimate (MSSDE), minimum spacing absolute log distance estimate (MSALDE), minimum spacing square log distance estimate (MSSLDE), minimum spacing Linex distance estimate (MSLNDE), minimum spacing absolute distance estimate (MSADE), in addition to ML estimate (MLE) in this study.

This work looks at the estimation of the RLD parameter within the RSS framework. The RLD is particularly significant due to its straightforward mathematical representation and its demonstrated efficacy in characterizing right-skewed data that exhibit an increasing failure rate. Using a frequentist methodology, we estimate the RLD parameter and develop recommendations for the optimal selection of RLD estimation techniques. Applied statisticians will find great value in this guide. The preferences

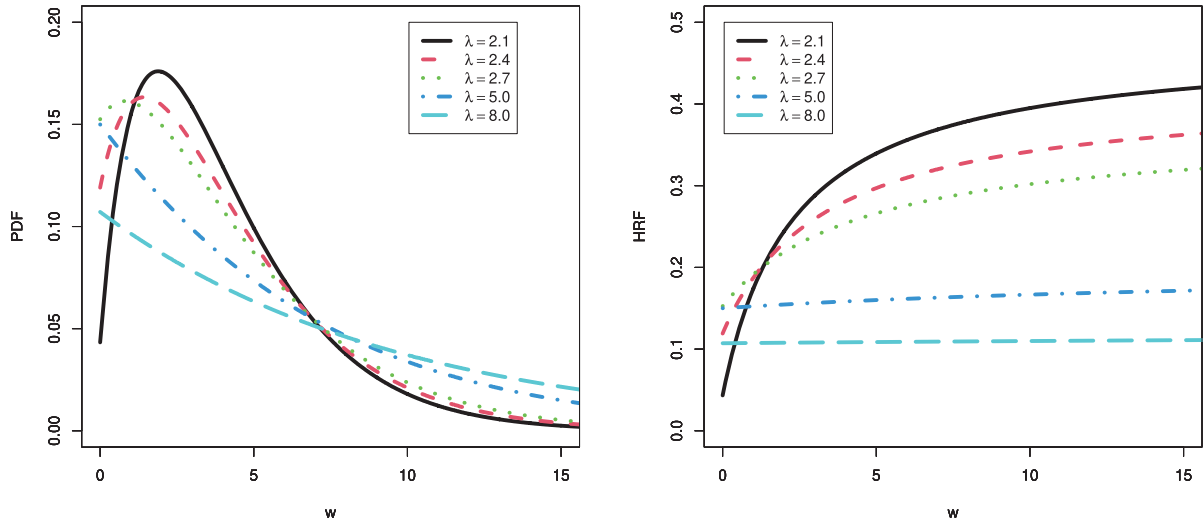


FIGURE 1 | Plots of PDF and HRF for the RLD.

of the user and the application area influence the estimating methodologies chosen. The current work is distinct since it is the first to undertake a comparative analysis of the previously discussed estimate techniques for the two-parameter RLD. Several assessment criteria and a tool for simulation are given to compare the thirteen distinct RSS estimations with the SRS approach's estimates. Finally, an analysis of three real datasets is used to highlight how the suggested estimating methods are practically applicable.

Section 2 addresses the MLE of RLD based on RSS and SRS. In Section 3, various minimum distance estimators of the RLD are covered. The RLD's minimum and maximum spacing distance estimators are provided in Section 4. Some further estimators are given in Section 5. Section 6 looks at the comparative examination of the RSS-based estimators' efficacy using a Monte Carlo simulation. Section 7 offers more details derived from genuine data. In the last part, there are some final remarks.

2 | Maximum Likelihood Estimators

Using both the RSS and SRS strategies, the MLE of λ for the RLD is provided in this section.

First, the MLE $\hat{\lambda}_1$ of λ is determined based on the SRS, to do this, let $w_1, w_2, \dots, w_n$ be an observed SRS of size n drawn from the RLD with PDF (2). The log-likelihood function (LF) of λ is given by:

$$l^S(\lambda) = -2n \log \lambda - n \log(\lambda - 1) + \sum_{r_1=0}^n \log[(\lambda - 2)\lambda + w_{r_1}] - \sum_{r_1=0}^n \frac{w_{r_1}}{\lambda}. \quad (4)$$

After taking the parameter λ into account while differentiating (4), we obtain the following nonlinear equation

$$\frac{\partial l^S(\lambda)}{\partial \lambda} = \frac{-2n}{\lambda} - \frac{n}{(\lambda - 1)} + \sum_{r_1=0}^n \frac{2\lambda - 2}{[(\lambda - 2)\lambda + w_{r_1}]} + \sum_{r_1=0}^n \frac{w_{r_1}}{\lambda^2}. \quad (5)$$

Since it is difficult to find the precise solution to (5) we will maximize it by applying optimization techniques, such as the Newton-Raphson method with Mathematica program.

Second, the MLE of λ is determined based on RSS. Assume that the RSS $W_{kj} = (W_{kj}, k = 1, \dots, t, j = 1, 2, \dots, b)$ of size $n = bt$, where b is the cycle count, and t is the set size, is obtained from the RLD. By inserting PDF (2) and CDF (3) into PDF (1), the LF of the RLD, based on RSS, can be formed as follows:

$$l(\lambda) \propto \prod_{j=1}^b \prod_{k=1}^t \left[1 - \frac{Q(w_{kj}, \lambda)}{(\lambda - 1)} \right]^{k-1} \left[\frac{Q(w_{kj}, \lambda)}{(\lambda - 1)} \right]^{t-k} \frac{[(\lambda - 2)\lambda + w_{kj}] e^{-\frac{w_{kj}}{\lambda}}}{(\lambda - 1)\lambda^2}, \quad (6)$$

where $Q(w_{kj}, \lambda) = \left(\lambda + \frac{w_{kj}}{\lambda} - 1 \right) e^{-\frac{w_{kj}}{\lambda}}$. The log-LF of (6), say $l^R(\lambda)$ is written as follows:

$$\begin{aligned} l^R(\lambda) = & -n(t - k + 1) \log(\lambda - 1) - \sum_{j=1}^b \sum_{k=1}^t \frac{w_{kj}}{\lambda} - 2n \log(\lambda) \\ & + \sum_{j=1}^b \sum_{k=1}^t \left\{ (k - 1) \log \left[1 - \frac{Q(w_{kj}, \lambda)}{(\lambda - 1)} \right] \right. \\ & \left. + (t - k) \log(Q(w_{kj}, \lambda)) + \log [(\lambda - 2)\lambda + w_{kj}] \right\}. \quad (7) \end{aligned}$$

The MLE $\hat{\lambda}_1$ of λ is yielded by maximizing (7) as follows:

$$\begin{aligned} \frac{\partial l^R(\lambda)}{\partial \lambda} = & \frac{-n(t - k + 1)}{\lambda - 1} \\ & + \sum_{j=1}^b \sum_{k=1}^t \left\{ (k - 1) \left[\frac{Q(w_{kj}, \lambda)}{(\lambda - 1)^2} - \frac{Q'(w_{kj}, \lambda)}{(\lambda - 1)} \right] \left[1 - \frac{Q(w_{kj}, \lambda)}{(\lambda - 1)} \right]^{-1} \right\} \\ & + \sum_{j=1}^b \sum_{k=1}^t \left\{ \frac{(t - k)Q'(w_{kj}, \lambda)}{Q(w_{kj}, \lambda)} + \frac{2\lambda - 2}{[(\lambda - 2)\lambda + w_{kj}]} \right\} + \sum_{j=1}^b \sum_{k=1}^t \frac{w_{kj}}{\lambda^2} - \frac{2n}{\lambda} = 0, \quad (8) \end{aligned}$$

$$\text{where } Q'(w_{kj}, \lambda) = \frac{\partial Q(w_{kj}, \lambda)}{\partial \lambda} = e^{-\frac{w_{kj}}{\lambda}} \left[1 + \frac{w_{kj}}{\lambda} - \frac{2w_{kj}}{\lambda^2} + \frac{w_{kj}^2}{\lambda^3} \right].$$

We will use optimization techniques, such as the Newton-Raphson approach with Mathematica software, to maximize (8) due to the difficulty of obtaining the solution.

3 | Minimum Distance Estimators

In this section, the different estimators, namely CVME, ADE, and RTADE, are given based on the minimization of the goodness-of-fit statistics. The basis of this statistical class is the distinction between the empirical CDF and the estimate of the CDF. These estimators are produced using RSS and SRS.

3.1 | Cramer-Von Mises Estimators

Macdonald [50] mentioned that CVM-type estimators have less bias than other minimum distance estimators, thereby supporting their use. At first, the CVME $\hat{\lambda}_2$ of λ is produced by using SRS. Given that $W_{(1)}, W_{(2)}, \dots, W_{(n)}$ be the ordered SRS of size n obtained from the RLD. The CVME $\hat{\lambda}_2$ is derived by minimizing the following function:

$$\mathbb{C}(\lambda) = \frac{1}{12n} + \sum_{r_1=1}^n \left\{ G(w_{(r_1)}|\lambda) - \frac{2r_1-1}{2n} \right\}^2. \quad (9)$$

As an alternative to (9), the following nonlinear equation can be solved to get $\hat{\lambda}_2$

$$\frac{\partial \mathbb{C}(\lambda)}{\partial \lambda} = \sum_{r_1=1}^n \left\{ 1 - \frac{e^{-\frac{w_{(r_1)}}{\lambda}}}{(\lambda-1)} \left(\lambda + \frac{w_{(r_1)}}{\lambda} - 1 \right) - \frac{2r_1-1}{2n} \right\} \delta(w_{(r_1)}|\lambda) = 0,$$

where

$$\delta(w_{(r_1)}|\lambda) = \frac{\partial G(w_{(r_1)}|\lambda)}{\partial \lambda} = \frac{e^{-\frac{w_{(r_1)}}{\lambda}}}{(\lambda-1)} \left(\frac{w_{(r_1)}^2}{\lambda^3} + \frac{w_{(r_1)}}{\lambda} - \frac{2w_{(r_1)}}{\lambda^2} + 1 \right) + \frac{e^{-\frac{w_{(r_1)}}{\lambda}}}{(\lambda+1)^2} \left(\lambda + \frac{w_{(r_1)}}{\lambda} - 1 \right). \quad (10)$$

Next, the CVME $\hat{\lambda}_2$ of λ is calculated using RSS and the previously mentioned similar approach. Assume that $W_{(1:n)}, W_{(2:n)}, \dots, W_{(n:n)}$ be an ordered RSS unit drawn from the RLD, where the sample size is $n = tb$, with set size t and cycle number b . The CVME $\hat{\lambda}_2$ is derived by solving the following nonlinear equation:

$$\frac{\partial \mathbb{C}^\circ(\lambda)}{\partial \lambda} = \sum_{r_2=1}^n \left\{ G(w_{(r_2:n)}|\lambda) - \frac{2r_2-1}{2n} \right\} \delta(w_{(r_2:n)}|\lambda) = 0,$$

where $\delta(\cdot|\lambda)$, has been given in (10).

3.2 | Anderson Darling Estimator

In the beginning, the ADE $\hat{\lambda}_3$ of λ is produced by using SRS. Consider that $W_{(1)}, W_{(2)}, \dots, W_{(n)}$ be the ordered SRS of size n taken

from the RLD. The ADE $\hat{\lambda}_3$ is derived by minimizing the following function:

$$\begin{aligned} A(\lambda) &= -n - \frac{1}{n} \sum_{r_1=1}^n (2r_1-1) \left\{ \log G(w_{(r_1)}|\lambda) + \log \bar{G}(w_{(n-r_1+1)}|\lambda) \right\} \\ &= -n - \frac{1}{n} \sum_{r_1=1}^n (2r_1-1) \left\{ \log \left[1 - \frac{e^{-\frac{w_{(r_1)}}{\lambda}}}{(\lambda-1)} \left(\lambda + \frac{w_{(r_1)}}{\lambda} - 1 \right) \right] \right. \\ &\quad \left. + \log \left[\frac{e^{-\frac{w_{(n-r_1+1)}}{\lambda}}}{(\lambda-1)} \left(\lambda + \frac{w_{(n-r_1+1)}}{\lambda} - 1 \right) \right] \right\} = 0, \end{aligned} \quad (11)$$

where $\bar{G}(\cdot|\lambda)$ is the survival function. As an alternative, the following nonlinear equation can be solved instead of using (11) to get $\hat{\lambda}_3$ of the RLD:

$$\frac{\partial A(\lambda)}{\partial \lambda} = \sum_{r_1=1}^n \frac{(2r_1-1)\delta(w_{(r_1)}|\lambda)}{G(w_{(r_1)}|\lambda)} + \frac{(2r_1-1)\delta(w_{(n-r_1+1)}|\lambda)}{\bar{G}(w_{(n-r_1+1)}|\lambda)} = 0,$$

where $\delta(w_{(n-r_1+1)}|\lambda)$, has the similar expression in (10) with an ordered sample $w_{(n-r_1+1)}$.

Next, the ADE $\hat{\lambda}_3$ of λ is established using RSS. Given that $W_{(1:n)}, W_{(2:n)}, \dots, W_{(n:n)}$ be the ordered RSS of sample size $n = bt$, taken from the RLD. Using the similar procedure discussed above, the ADE $\hat{\lambda}_3$ is derived by solving nonlinear equation:

$$\frac{\partial A^\circ(\lambda)}{\partial \lambda} = \sum_{r_2=1}^n \frac{(2r_2-1)\delta(w_{(r_2:n)}|\lambda)}{G(w_{(r_2:n)}|\lambda)} + \frac{(2r_2-1)\delta(w_{(n-r_2+1:n)}|\lambda)}{\bar{G}(w_{(n-r_2+1:n)}|\lambda)} = 0,$$

where $\delta(\cdot|\lambda)$, has the similar expression as (10).

3.3 | Right-Tail Anderson Darling Estimator

Initially, the SRS is used to construct the RTADE $\hat{\lambda}_4$ of λ . Take $W_{(1)}, W_{(2)}, \dots, W_{(n)}$ to be the ordered SRS of size n taken from the RLD. The following function is minimized to obtain the RTADE

$$\begin{aligned} R(\lambda) &= \frac{n}{2} - 2 \sum_{r_1=1}^n G(w_{(r_1)}|\lambda) - \frac{1}{n} \sum_{r_1=1}^n (2r_1-1) \log \bar{G}(w_{(n-r_1+1)}|\lambda) \\ &= \frac{n}{2} - 2 \sum_{r_1=1}^n \left[1 - \frac{e^{-\frac{w_{(r_1)}}{\lambda}}}{(\lambda-1)} \left(\lambda + \frac{w_{(r_1)}}{\lambda} - 1 \right) \right] \\ &\quad - \frac{1}{n} \sum_{r_1=1}^n (2r_1-1) \log \left[\frac{e^{-\frac{w_{(n-r_1+1)}}{\lambda}}}{(\lambda-1)} \left(\lambda + \frac{w_{(n-r_1+1)}}{\lambda} - 1 \right) \right]. \end{aligned} \quad (12)$$

As an alternative, the following nonlinear equation can be solved instead of using (12) to get $\hat{\lambda}_4$

$$\frac{\partial R(\lambda)}{\partial \lambda} = -2 \sum_{r_1=1}^n \delta(w_{(r_1)}, \lambda) - \frac{1}{n} \sum_{r_1=1}^n \frac{(2r_1-1)\delta(w_{(n-r_1+1)}, \lambda)}{\bar{G}(w_{(n-r_1+1)}, \lambda)} = 0,$$

where $\delta(\cdot|\lambda)$, has the similar expression as (10).

Following that, RSS is used to establish the RTADE $\hat{\lambda}_4$ of λ . Considering that $W_{(1:n)}, W_{(2:n)}, \dots, W_{(n:n)}$, be the ordered RSS with sample size $n = bt$ drawn from the RLD. The following function is minimized to determine the RTADE $\hat{\lambda}_4$ using the identical method to the previously explained.

$$\frac{\partial R^*(\lambda)}{\partial \lambda} = -2 \sum_{r_2=1}^n \delta(w_{(r_2:n)}|\lambda) - \frac{1}{n} \sum_{r_2=1}^n \frac{(2r_2-1)\delta(w_{(n-r_2+1:n)}|\lambda)}{\bar{G}(w_{(n-r_2+1:n)}|\lambda)} = 0,$$

where $\delta(w_{(n-r_2+1:n)}|\lambda)$, has the similar expression in (10) with an ordered sample $w_{(n-r_2+1:n)}$.

4 | Minimum and Maximum Spacing Distance Estimators

This section produces several estimators of the RLD parameter using RSS and SRS, specifically MSSDE, MSALDE, MSSLDE, MSLNDE, and MSADDE. For more about these methods, refer to [51–54].

4.1 | Minimum Spacing Absolute Distance

At first, the MSADDE $\hat{\lambda}_5$ of λ is built using SRS. Consider that $W_{(1)}, W_{(2)}, \dots, W_{(n)}$ to be the n ordered SRS that drawn from the RLD. To get the MSADDE, minimize the following function with respect to λ

$$\zeta(\lambda) = \sum_{r_1=1}^{n+1} \left| \Lambda_{r_1}(\lambda) - \frac{1}{n+1} \right|, \quad (13)$$

where $\Lambda_{r_1}(\lambda) = G(w_{(r_1)}|\lambda) - G(w_{(r_1-1)}|\lambda)$, $r_1 = 1, 2, \dots, n+1$, $G(w_{(0)}|\lambda) = 0$, $G(w_{(n+1)}|\lambda) = 1$, such that $\sum_{r_1=1}^{n+1} \Lambda_{r_1}(\lambda) = 1$.

Instead of (13), the MSADDE $\hat{\lambda}_5$ can be achieved by numerically solving the following nonlinear equation.

$$\frac{\partial \zeta(\lambda)}{\partial \lambda} = \sum_{r_1=1}^{n+1} \frac{\Lambda_{r_1}(\lambda) - \frac{1}{n+1}}{\left| \Lambda_{r_1}(\lambda) - \frac{1}{n+1} \right|} [\delta(w_{(r_1)}|\lambda) - \delta(w_{(r_1-1)}|\lambda)] = 0,$$

where $\delta(\cdot|\lambda)$, is given in (10).

After that, the MSADDE $\hat{\lambda}_5$ of λ is established using RSS. Let $W_{(1:n)}, W_{(2:n)}, \dots, W_{(n:n)}$ are ordered RSS items from the RLD with sample size $n = tb$, where t is set size and b is the cycle numbers. As provided before, the following nonlinear equation is solved numerically to yield the MSADDE

$$\frac{\partial \zeta^*(\lambda)}{\partial \lambda} = \sum_{r_1=1}^{n+1} \frac{\Lambda_{r_2}(\lambda) - \frac{1}{n+1}}{\left| \Lambda_{r_2}(\lambda) - \frac{1}{n+1} \right|} [\delta(w_{(r_2:n)}|\lambda) - \delta(w_{(r_2-1:n)}|\lambda)] = 0,$$

where $\delta(\cdot|\lambda)$, is given in (10).

4.2 | Minimum Spacing Absolute-Log Distance

At first, the SRS is used in the construction of the MSALDE $\hat{\lambda}_6$ of λ . Given that $W_{(1)}, W_{(2)}, \dots, W_{(n)}$ are the n ordered SRS that drawn

from the RLD. To get the MSALDE $\hat{\lambda}_6$, minimize the following function with respect to λ

$$\zeta_1(\lambda) = \sum_{r_1=1}^{n+1} \left| \log(\Lambda_{r_1}(\lambda)) - \log\left(\frac{1}{n+1}\right) \right|. \quad (14)$$

The following nonlinear equation can be empirically solved to provide the MSALDE $\hat{\lambda}_6$ instead of using (14).

$$\frac{\zeta_1(\lambda)}{\partial \lambda} = \sum_{r_1=1}^{n+1} \frac{\log(\Lambda_{r_1}(\lambda)) - \log\left(\frac{1}{n+1}\right)}{\left| \log(\Lambda_{r_1}(\lambda)) - \log\left(\frac{1}{n+1}\right) \right|} \left[\delta(w_{(r_1)}|\lambda) - \delta(w_{(r_1-1)}|\lambda) \right] = 0,$$

where $\delta(\cdot|\lambda)$, is given in (10).

Subsequently, the MSALDE $\hat{\lambda}_6$ of λ is established using RSS. Let $W_{(1:n)}, W_{(2:n)}, \dots, W_{(n:n)}$ be the ordered RSS items from the RLD with sample size $n = tb$, where t is set size and b are the cycle numbers. As provided before, the following function equation is solved numerically to produce the MSALDE.

$$\frac{\zeta_1^*(\lambda)}{\partial \lambda} = \sum_{r_2=1}^{n+1} \frac{\log(\Lambda_{r_2}(\lambda)) - \log\left(\frac{1}{n+1}\right)}{\left| \log(\Lambda_{r_2}(\lambda)) - \log\left(\frac{1}{n+1}\right) \right|} [\delta(w_{(r_2:n)}|\lambda) - \delta(w_{(r_2-1:n)}|\lambda)] = 0,$$

where $\delta(\cdot|\lambda)$, is given in (10).

4.3 | Minimum Spacing Square Distance Estimator

At the beginning, the MSSDE $\hat{\lambda}_7$ of λ based on SRS is provided. Assume that $W_{(1)}, W_{(2)}, \dots, W_{(n)}$ is the n ordered SRS drawn from the RLD. To get the MSSDE $\hat{\lambda}_7$, minimize the following function with respect to λ

$$\zeta_2(\lambda) = \sum_{r_1=1}^{n+1} \left[\Lambda_{r_1}(\lambda) - \frac{1}{n+1} \right]^2. \quad (15)$$

In place of using (15), the MSSDE $\hat{\lambda}_7$ for the following nonlinear equation can be obtained empirically

$$\frac{\partial \zeta_2(\lambda)}{\partial \lambda} = \sum_{r_1=1}^{n+1} \left[\Lambda_{r_1}(\lambda) - \frac{1}{n+1} \right] [\delta(w_{(r_1)}|\lambda) - \delta(w_{(r_1-1)}|\lambda)] = 0,$$

where $\delta(\cdot|\lambda)$, is given in (10).

Next, the RSS is used to establish the MSSDE $\hat{\lambda}_7$ of λ . Let's say that there are $W_{(1:n)}, W_{(2:n)}, \dots, W_{(n:n)}$ ordered RSS items taken from the RLD with a sample size of $n = tb$, b cycle numbers, and t be the set size. The MSSDE $\hat{\lambda}_7$ is provided by solving the following nonlinear equation:

$$\frac{\partial \zeta_2^*(\lambda)}{\partial \lambda} = \sum_{r_2=1}^{n+1} \left[\Lambda_{r_2}(\lambda) - \frac{1}{n+1} \right] [\delta(w_{(r_2:n)}|\lambda) - \delta(w_{(r_2-1:n)}|\lambda)] = 0,$$

where $\delta(\cdot|\lambda)$, is given in (10).

4.4 | Minimum Spacing Square-Log Distance Estimator

Initially, the MSSLDE $\hat{\lambda}_8$ of λ based on SRS is supplied. Assume that $W_{(1)}, W_{(2)}, \dots, W_{(n)}$ the n ordered SRS that are selected from the RLD. Minimize the following function with regard to λ in order to obtain the MSSDE $\hat{\lambda}_8$.

$$\zeta_3(\lambda) = \sum_{r_1=1}^{n+1} \left[\log \Lambda_{r_1}(\lambda) - \log \left(\frac{1}{n+1} \right) \right]^2. \quad (16)$$

The following nonlinear equation can be numerically calculated instead of using (16).

$$\begin{aligned} \frac{\partial \zeta_3(\lambda)}{\partial \lambda} &= \sum_{r_1=1}^{n+1} \left[\log \Lambda_{r_1}(\lambda) - \log \left(\frac{1}{n+1} \right) \right] \\ &\quad \frac{1}{\Lambda_{r_1}(\lambda)} [\delta(w_{(r_1)}|\lambda) - \delta(w_{(r_1-1)}|\lambda)] = 0, \end{aligned}$$

where $\delta(\cdot|\lambda)$, is given in (10).

The MSSLDE $\hat{\lambda}_8$ of λ is then established using RSS. Assume that $W_{(1:n)}, W_{(2:n)}, \dots, W_{(n:n)}$ be the sorted RSS items with a sample size $n = tb$, where t set size and b cycle numbers are extracted from the RLD. In the previously mentioned way, solving the following nonlinear equation gives the MSSLDE $\hat{\lambda}_8$

$$\begin{aligned} \frac{\partial \zeta_3(\lambda)}{\partial \lambda} &= \sum_{r_2=1}^{n+1} \left[\log \Lambda_{r_2}(\lambda) - \log \left(\frac{1}{n+1} \right) \right] \\ &\quad \frac{1}{\Lambda_{r_2}(\lambda)} [\delta(w_{(r_2:n)}|\lambda) - \delta(w_{(r_2-1:n)}|\lambda)] = 0, \end{aligned}$$

where $\delta(\cdot|\lambda)$, is given in (10).

4.5 | Minimum Spacing Linex Distance Estimator

The MSLDE $\hat{\lambda}_9$ of λ is first constructed using SRS. Assume that the ordered SRS of size n , obtained from the RLD, is $W_{(1)}, W_{(2)}, \dots, W_{(n)}$. The following function is minimized to get the MSLDE $\hat{\lambda}_9$ of λ .

$$\zeta_4(\lambda) = \sum_{r_1=1}^{n+1} \left[e^{\Lambda_{r_1}(\lambda) - \frac{1}{n+1}} - \left(\Lambda_{r_1}(\lambda) - \frac{1}{n+1} \right) - 1 \right]^2.$$

It is possible to acquire quite useful results by numerically solving the following nonlinear equation

$$\begin{aligned} \frac{\partial \zeta_4(\lambda)}{\partial \lambda} &= \sum_{r_1=1}^{n+1} \left[e^{\Lambda_{r_1}(\lambda) - \frac{1}{n+1}} - \left(\Lambda_{r_1}(\lambda) - \frac{1}{n+1} \right) - 1 \right] \\ &\quad [\delta(w_{(r_1)}|\lambda) - \delta(w_{(r_1-1)}|\lambda)] \left[e^{\Lambda_{r_1}(\lambda) - \frac{1}{n+1}} - 1 \right] = 0, \end{aligned}$$

where $\delta(\cdot|\lambda)$, is given in (10).

After that, the MSLDE $\hat{\lambda}_9$ of λ is determined by using RSS. Assuming that the drawn ordered RSS from the RLD is

$W_{(1:n)}, W_{(2:n)}, \dots, W_{(n:n)}$. Using the same procedure as previously described, the following function is minimized to get the MSLDE $\hat{\lambda}_9$

$$\begin{aligned} \frac{\partial \zeta_4(\lambda)}{\partial \lambda} &= \sum_{r_2=1}^{n+1} \left[e^{\Lambda_{r_2}(\lambda) - \frac{1}{n+1}} - \left(\Lambda_{r_2}(\lambda) - \frac{1}{n+1} \right) - 1 \right] \\ &\quad [\delta(w_{(r_2:n)}|\lambda) - \delta(w_{(r_2-1:n)}|\lambda)] \left[e^{\Lambda_{r_2}(\lambda) - \frac{1}{n+1}} - 1 \right] = 0, \end{aligned}$$

where $\delta(\cdot|\lambda)$, is given in (10).

4.6 | Maximum Product of Spacings Estimator

Assume that $W_{(1)}, W_{(2)}, \dots, W_{(n)}$ be an n ordered SRS items taken from the RLD. Determining the geometric mean of the spacing, which is achieved by maximizing the function below, yields MPSE $\hat{\lambda}_{10}$ of λ

$$\Xi = \frac{1}{n+1} \sum_{r_1=1}^{n+1} \log [\Lambda_{r_1}(\lambda)].$$

The MPSE is created by numerically computing the next equation:

$$\frac{\partial \Xi}{\partial \lambda} = \frac{1}{n+1} \sum_{r_1=1}^{n+1} \frac{1}{\Lambda_{r_1}(\lambda)} [\delta(w_{(r_1)}|\lambda) - \delta(w_{(r_1-1)}|\lambda)] = 0,$$

where $\delta(\cdot|\lambda)$, is given in (10).

Later, the MSPE $\hat{\lambda}_{10}$ of λ is determined using RSS with sample size $n = bt$, where b is the cycle number and t is the set size. Assuming that the ordered RSS drawn from the RLD is $W_{(1:n)}, W_{(2:n)}, \dots, W_{(n:n)}$. Using the same procedure as previously described, solving the following nonlinear equation to get the MSPSE $\hat{\lambda}_{10}$

$$\frac{\partial \Xi}{\partial \lambda} = \frac{1}{n+1} \sum_{r_2=1}^{n+1} \frac{1}{\Lambda_{r_2}(\lambda)} [\delta(w_{(r_2:n)}|\lambda) - \delta(w_{(r_2-1:n)}|\lambda)] = 0,$$

where $\delta(\cdot|\lambda)$, is given in (10).

5 | Some Other Estimators

Several estimators of the RLD parameter, namely OLSE, WLSE, and KE, are generated in this section using RSS and SRS. The OLS and WLS methods were presented by Swain et al. [55].

5.1 | Ordinary Least Squares Estimator

Let the SRS items with sample size n be arranged in the following manner: $W_{(1)}, W_{(2)}, \dots, W_{(n)}$. Following the function's minimization with regard to λ , the OLSEs $\hat{\lambda}_{11}$ is obtained.

$$\tau = \sum_{i=1}^n \left[G(w_{(r_1)}|\lambda) - \frac{r_1}{n+1} \right]^2.$$

TABLE 1 | Numerical results of simulation for all measures when ($\lambda = 2.0$) under SRS.

<i>n</i>	Estimate	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	MSADE	MSALDE	KE	MSSDE	MSSLDE	MSLNDE
25	BIAS	$\hat{\lambda}$	0.30473 ⁽⁵⁾	0.28862 ⁽¹¹⁾	0.30064 ⁽⁴⁾	0.34455 ⁽⁹⁾	0.37047 ⁽¹⁰⁾	0.38307 ⁽¹¹⁾	0.31063 ⁽⁶⁾	0.32027 ⁽⁸⁾	0.31258 ⁽⁷⁾	0.45658 ⁽¹³⁾	0.29165 ⁽²⁾	0.4406 ⁽¹²⁾
	MSE	$\hat{\lambda}$	0.16321 ⁽¹¹⁾	0.22952 ⁽⁷⁾	0.20846 ⁽³⁾	0.28598 ⁽⁹⁾	0.29731 ⁽¹⁰⁾	0.34461 ⁽¹¹⁾	0.22325 ⁽⁵⁾	0.2464 ⁽⁸⁾	0.20075 ⁽²⁾	0.45184 ⁽¹³⁾	0.21843 ⁽⁴⁾	0.43522 ⁽¹²⁾
	MRE	$\hat{\lambda}$	0.15237 ⁽⁵⁾	0.14431 ⁽¹¹⁾	0.15032 ⁽⁴⁾	0.17228 ⁽⁹⁾	0.18524 ⁽¹⁰⁾	0.19154 ⁽¹¹⁾	0.15532 ⁽⁶⁾	0.16013 ⁽⁸⁾	0.15629 ⁽⁷⁾	0.22829 ⁽¹³⁾	0.14583 ⁽²⁾	0.2203 ⁽¹²⁾
	D_{abs}		0.06747 ⁽¹³⁾	0.02105 ⁽²⁾	0.02547 ⁽⁷⁾	0.02262 ⁽⁶⁾	0.02741 ⁽¹²⁾	0.02662 ⁽¹⁰⁾	0.02195 ⁽⁵⁾	0.02152 ⁽³⁾	0.02719 ⁽¹¹⁾	0.02656 ⁽⁹⁾	0.02068 ⁽¹¹⁾	0.02579 ⁽⁸⁾
	D_{max}		0.15954 ⁽¹³⁾	0.04148 ⁽²⁾	0.05671 ⁽¹¹⁾	0.04272 ⁽⁴⁾	0.05632 ⁽¹⁰⁾	0.0524 ⁽⁹⁾	0.04307 ⁽⁶⁾	0.04201 ⁽³⁾	0.05829 ⁽¹²⁾	0.04844 ⁽⁸⁾	0.04025 ⁽¹¹⁾	0.04716 ⁽⁷⁾
	ASAE		0.06795 ⁽¹³⁾	0.05653 ⁽⁴⁾	0.05651 ⁽³⁾	0.05685 ⁽⁶⁾	0.0567 ⁽⁵⁾	0.05502 ⁽²⁾	0.05784 ⁽⁹⁾	0.0578 ⁽⁸⁾	0.0596 ⁽¹⁰⁾	0.06243 ⁽¹¹⁾	0.05701 ⁽⁷⁾	0.06444 ⁽¹²⁾
	$\sum$ Ranks		50 ⁽⁹⁾	17 ^(1.5)	32 ⁽⁴⁾	43 ⁽⁷⁾	57 ⁽¹¹⁾	54 ⁽¹⁰⁾	37 ⁽⁵⁾	38 ⁽⁶⁾	49 ⁽⁸⁾	67 ⁽¹³⁾	17 ^(1.5)	63 ⁽¹²⁾
70	BIAS	$\hat{\lambda}$	0.14456 ⁽⁶⁾	0.12792 ⁽²⁾	0.1854 ⁽⁸⁾	0.13007 ⁽³⁾	0.19189 ⁽¹¹⁾	0.20712 ⁽¹³⁾	0.1419 ⁽⁵⁾	0.15122 ⁽⁷⁾	0.18634 ⁽⁹⁾	0.19972 ⁽¹²⁾	0.13134 ⁽⁴⁾	0.18892 ⁽¹⁰⁾
	MSE	$\hat{\lambda}$	0.03903 ⁽¹¹⁾	0.06505 ⁽⁶⁾	0.09482 ⁽¹⁰⁾	0.04295 ⁽²⁾	0.10071 ⁽¹²⁾	0.1128 ⁽¹³⁾	0.05345 ⁽⁴⁾	0.06966 ⁽⁸⁾	0.06888 ⁽⁷⁾	0.09828 ⁽¹¹⁾	0.05366 ⁽⁵⁾	0.0846 ⁽⁹⁾
	MRE	$\hat{\lambda}$	0.07228 ⁽⁶⁾	0.06396 ⁽²⁾	0.0927 ⁽⁸⁾	0.06504 ⁽³⁾	0.09594 ⁽¹¹⁾	0.10356 ⁽¹³⁾	0.07095 ⁽⁵⁾	0.07561 ⁽⁷⁾	0.09317 ⁽⁹⁾	0.09986 ⁽¹²⁾	0.06567 ⁽⁴⁾	0.09446 ⁽¹⁰⁾
	D_{abs}		0.02032 ⁽¹³⁾	0.01046 ⁽¹¹⁾	0.01634 ⁽¹¹⁾	0.01079 ⁽³⁾	0.01614 ⁽⁹⁾	0.01128 ⁽⁴⁾	0.01166 ⁽⁵⁾	0.01185 ⁽⁶⁾	0.0177 ⁽¹²⁾	0.01479 ⁽⁸⁾	0.01078 ⁽²⁾	0.01424 ⁽⁷⁾
	D_{max}		0.04684 ⁽¹³⁾	0.02111 ⁽¹¹⁾	0.03473 ⁽¹¹⁾	0.02187 ⁽²⁾	0.03372 ⁽¹⁰⁾	0.02383 ⁽⁶⁾	0.02372 ⁽⁵⁾	0.02369 ⁽⁴⁾	0.0384 ⁽¹²⁾	0.02905 ⁽⁸⁾	0.02198 ⁽³⁾	0.02794 ⁽⁷⁾
	ASAE		0.03291 ⁽¹³⁾	0.02928 ⁽⁵⁾	0.02947 ⁽⁶⁾	0.03014 ⁽⁸⁾	0.02977 ⁽⁷⁾	0.02796 ⁽¹¹⁾	0.03049 ⁽¹⁰⁾	0.02926 ⁽⁴⁾	0.03065 ⁽¹¹⁾	0.0317 ⁽¹²⁾	0.02923 ⁽³⁾	0.03022 ⁽⁹⁾
	$\sum$ Ranks		52 ^(7.5)	17 ^(1.5)	54 ⁽⁹⁾	21 ^(3.5)	60 ^(11.5)	59 ⁽¹⁰⁾	34 ⁽⁵⁾	36 ⁽⁶⁾	60 ^(11.5)	63 ⁽¹³⁾	21 ^(3.5)	52 ^(7.5)
150	BIAS	$\hat{\lambda}$	0.07233 ⁽⁶⁾	0.06811 ⁽²⁾	0.10351 ⁽¹¹⁾	0.05982 ⁽¹⁾	0.10074 ⁽¹⁰⁾	0.1256 ⁽¹²⁾	0.07615 ⁽⁷⁾	0.07184 ⁽⁵⁾	0.13096 ⁽¹³⁾	0.09853 ⁽⁹⁾	0.069 ⁽³⁾	0.09637 ⁽⁸⁾
	MSE	$\hat{\lambda}$	0.00943 ⁽³⁾	0.01552 ⁽⁷⁾	0.0223 ⁽¹¹⁾	0.00734 ⁽¹¹⁾	0.02144 ⁽¹⁰⁾	0.03612 ⁽¹³⁾	0.01257 ⁽⁵⁾	0.01295 ⁽⁶⁾	0.02908 ⁽¹²⁾	0.02039 ⁽⁸⁾	0.01154 ⁽⁴⁾	0.02101 ⁽⁹⁾
	MRE	$\hat{\lambda}$	0.03617 ⁽⁶⁾	0.03405 ⁽²⁾	0.05175 ⁽¹¹⁾	0.02991 ⁽¹⁾	0.05037 ⁽¹⁰⁾	0.0628 ⁽¹²⁾	0.03807 ⁽⁷⁾	0.03592 ⁽⁵⁾	0.06548 ⁽¹³⁾	0.04927 ⁽⁹⁾	0.0345 ⁽³⁾	0.04819 ⁽⁸⁾
	D_{abs}		0.00857 ⁽⁸⁾	0.00636 ⁽²⁾	0.01045 ⁽¹¹⁾	0.00579 ⁽¹⁾	0.00985 ⁽¹⁰⁾	0.01129 ⁽¹²⁾	0.00708 ⁽⁵⁾	0.00658 ⁽⁴⁾	0.0131 ⁽¹³⁾	0.00862 ⁽⁹⁾	0.00647 ⁽³⁾	0.00845 ⁽⁷⁾
	D_{max}		0.01899 ⁽⁹⁾	0.01325 ⁽²⁾	0.0228 ⁽¹¹⁾	0.01233 ⁽¹⁾	0.02122 ⁽¹⁰⁾	0.02379 ⁽¹²⁾	0.01493 ⁽⁵⁾	0.01379 ⁽⁴⁾	0.02848 ⁽¹³⁾	0.01788 ⁽⁸⁾	0.01366 ⁽³⁾	0.01755 ⁽⁷⁾
	ASAE		0.0191 ⁽¹²⁾	0.01779 ⁽²⁾	0.01836 ⁽⁶⁾	0.01841 ⁽⁷⁾	0.01795 ⁽⁴⁾	0.01703 ⁽¹¹⁾	0.01902 ⁽¹¹⁾	0.01845 ⁽⁸⁾	0.01942 ⁽¹³⁾	0.01862 ⁽⁹⁾	0.01814 ⁽⁵⁾	0.01876 ⁽¹⁰⁾
	$\sum$ Ranks		44 ⁽⁷⁾	17 ⁽²⁾	61 ⁽¹¹⁾	12 ⁽¹⁾	54 ⁽¹⁰⁾	62 ⁽¹²⁾	40 ⁽⁶⁾	32 ⁽⁵⁾	77 ⁽¹³⁾	52 ⁽⁹⁾	21 ⁽³⁾	49 ⁽⁸⁾
200	BIAS	$\hat{\lambda}$	0.06053 ⁽⁶⁾	0.05314 ⁽³⁾	0.08375 ⁽¹⁰⁾	0.04941 ⁽¹⁾	0.0864 ⁽¹¹⁾	0.10099 ⁽¹²⁾	0.06481 ⁽⁷⁾	0.0572 ⁽⁴⁾	0.11026 ⁽¹³⁾	0.08042 ⁽⁹⁾	0.05201 ⁽²⁾	0.08025 ⁽⁸⁾
	MSE	$\hat{\lambda}$	0.00629 ⁽⁴⁾	0.00531 ⁽²⁾	0.01293 ⁽¹⁰⁾	0.00444 ⁽¹⁾	0.01322 ⁽¹¹⁾	0.02129 ⁽¹³⁾	0.0092 ⁽⁷⁾	0.00727 ⁽⁶⁾	0.02025 ⁽¹²⁾	0.01278 ⁽⁹⁾	0.00641 ⁽⁵⁾	0.01166 ⁽⁸⁾
	MRE	$\hat{\lambda}$	0.03026 ⁽⁶⁾	0.02657 ⁽³⁾	0.04188 ⁽¹⁰⁾	0.02471 ⁽¹⁾	0.0432 ⁽¹¹⁾	0.0505 ⁽¹²⁾	0.0324 ⁽⁷⁾	0.0286 ⁽⁴⁾	0.05513 ⁽¹³⁾	0.04021 ⁽⁹⁾	0.026 ⁽²⁾	0.04013 ⁽⁸⁾
	D_{abs}		0.00694 ⁽⁷⁾	0.00529 ⁽³⁾	0.00856 ⁽¹⁰⁾	0.00485 ⁽¹⁾	0.00873 ⁽¹¹⁾	0.00969 ⁽¹²⁾	0.00614 ⁽⁶⁾	0.00546 ⁽⁴⁾	0.01124 ⁽¹³⁾	0.00738 ⁽⁸⁾	0.00505 ⁽²⁾	0.0074 ⁽⁶⁾
	D_{max}		0.01533 ⁽⁷⁾	0.0111 ⁽³⁾	0.01854 ⁽¹⁰⁾	0.01037 ⁽¹⁾	0.01887 ⁽¹¹⁾	0.02081 ⁽¹²⁾	0.01294 ⁽⁵⁾	0.01161 ⁽⁴⁾	0.02459 ⁽¹³⁾	0.01546 ⁽⁸⁾	0.01076 ⁽²⁾	0.01558 ⁽⁶⁾
	ASAE		0.01577 ⁽¹²⁾	0.01485 ⁽³⁾	0.01537 ⁽⁷⁾	0.01522 ⁽⁶⁾	0.01488 ⁽⁴⁾	0.01464 ⁽¹¹⁾	0.01576 ⁽¹¹⁾	0.01543 ⁽⁸⁾	0.01605 ⁽¹³⁾	0.01572 ⁽¹⁰⁾	0.01516 ⁽⁵⁾	0.01567 ⁽⁶⁾
	$\sum$ Ranks		42 ⁽⁶⁾	17 ⁽²⁾	57 ⁽¹⁰⁾	11 ⁽¹⁾	59 ⁽¹¹⁾	62 ⁽¹²⁾	43 ⁽⁷⁾	30 ⁽⁵⁾	77 ⁽¹³⁾	53 ⁽⁹⁾	18 ⁽³⁾	51 ⁽⁸⁾

(Continues)

TABLE 1 | (Continued)

<i>n</i>	Estimate	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	MSADE	MSALDE	KE	MSSDE	MSSLDE	MSLNDE
300	BIAS	$\hat{\lambda}$	0.04458 ⁽⁵⁾	0.04432 ⁽⁴⁾	0.0674 ⁽¹⁰⁾	0.03704 ⁽²⁾	0.07263 ⁽¹¹⁾	0.08036 ⁽¹²⁾	0.04545 ⁽⁶⁾	0.04328 ⁽³⁾	0.09153 ⁽¹³⁾	0.05733 ⁽⁸⁾	0.03686 ⁽¹⁾	0.05808 ⁽⁹⁾
	MSE	$\hat{\lambda}$	0.00348 ^(3,5)	0.00375 ⁽⁵⁾	0.00728 ⁽¹⁰⁾	0.00238 ⁽¹⁾	0.01026 ⁽¹¹⁾	0.01353 ⁽¹²⁾	0.00348 ^(3,5)	0.00398 ⁽⁶⁾	0.01381 ⁽¹³⁾	0.00599 ⁽⁹⁾	0.00258 ⁽²⁾	0.00594 ⁽⁸⁾
	MRE	$\hat{\lambda}$	0.02229 ⁽⁵⁾	0.02216 ⁽⁴⁾	0.0337 ⁽¹⁰⁾	0.01852 ⁽²⁾	0.03632 ⁽¹¹⁾	0.04018 ⁽¹²⁾	0.02325 ⁽⁷⁾	0.02164 ⁽³⁾	0.04577 ⁽¹³⁾	0.02866 ⁽⁸⁾	0.01843 ⁽¹⁾	0.02904 ⁽⁹⁾
	<i>D</i> _{abs}		0.00502 ⁽⁷⁾	0.00444 ⁽⁴⁾	0.00699 ⁽¹⁰⁾	0.00372 ⁽¹⁾	0.00741 ⁽¹¹⁾	0.00774 ⁽¹²⁾	0.00484 ⁽⁶⁾	0.00426 ⁽³⁾	0.00936 ⁽¹³⁾	0.00549 ⁽⁸⁾	0.00373 ⁽²⁾	0.00558 ⁽⁹⁾
	<i>D</i> _{max}		0.01103 ⁽⁷⁾	0.00936 ⁽⁴⁾	0.01521 ⁽¹⁰⁾	0.00802 ⁽¹⁾	0.01607 ⁽¹¹⁾	0.01654 ⁽¹²⁾	0.01049 ⁽⁶⁾	0.00911 ⁽³⁾	0.0203 ⁽¹³⁾	0.01168 ⁽⁸⁾	0.00805 ⁽²⁾	0.01189 ⁽⁹⁾
	ASAE		0.01236 ⁽¹²⁾	0.01151 ⁽³⁾	0.01154 ⁽⁴⁾	0.01158 ⁽⁵⁾	0.01191 ⁽⁷⁾	0.01125 ⁽²⁾	0.01207 ⁽⁸⁾	0.01211 ^(9,5)	0.01275 ⁽¹³⁾	0.01211 ^(9,5)	0.01173 ⁽⁶⁾	0.01226 ⁽¹¹⁾
	Σ Ranks		39.5 ⁽⁷⁾	24 ⁽³⁾	54 ⁽⁹⁾	12 ⁽¹⁾	62 ^(11,5)	62 ^(11,5)	37 ⁽⁶⁾	27.5 ⁽⁴⁾	78 ⁽¹³⁾	50.5 ⁽⁸⁾	14 ⁽²⁾	55 ⁽¹⁰⁾
400	BIAS	$\hat{\lambda}$	0.03583 ⁽⁵⁾	0.0356 ⁽⁴⁾	0.05971 ⁽¹⁰⁾	0.02829 ⁽¹⁾	0.06251 ⁽¹¹⁾	0.06571 ⁽¹²⁾	0.0374 ⁽⁶⁾	0.03505 ⁽³⁾	0.08295 ⁽¹³⁾	0.04622 ⁽⁸⁾	0.0301 ⁽²⁾	0.04854 ⁽⁹⁾
	MSE	$\hat{\lambda}$	0.00217 ⁽³⁾	0.00244 ⁽⁴⁾	0.0055 ⁽¹⁰⁾	0.00139 ⁽¹⁾	0.00603 ⁽¹¹⁾	0.0084 ⁽¹²⁾	0.00253 ⁽⁷⁾	0.00251 ⁽⁶⁾	0.01117 ⁽¹³⁾	0.00393 ⁽⁸⁾	0.00163 ⁽²⁾	0.00414 ⁽⁹⁾
	MRE	$\hat{\lambda}$	0.01791 ⁽⁵⁾	0.0178 ⁽⁴⁾	0.02985 ⁽¹⁰⁾	0.01414 ⁽¹⁾	0.03125 ⁽¹¹⁾	0.03285 ⁽¹²⁾	0.0187 ⁽⁶⁾	0.01752 ⁽³⁾	0.04148 ⁽¹³⁾	0.02311 ⁽⁸⁾	0.01505 ⁽²⁾	0.02427 ⁽⁹⁾
	<i>D</i> _{abs}		0.00397 ⁽⁶⁾	0.0036 ⁽⁴⁾	0.00624 ⁽¹⁰⁾	0.00289 ⁽¹⁾	0.00649 ⁽¹¹⁾	0.00656 ⁽¹²⁾	0.00375 ⁽⁵⁾	0.0035 ⁽³⁾	0.0085 ⁽¹³⁾	0.00452 ⁽⁸⁾	0.00309 ⁽²⁾	0.00472 ⁽⁹⁾
	<i>D</i> _{max}		0.00868 ⁽⁶⁾	0.00761 ⁽⁴⁾	0.01355 ⁽¹⁰⁾	0.00624 ⁽¹⁾	0.01412 ⁽¹¹⁾	0.01421 ⁽¹²⁾	0.00807 ⁽⁵⁾	0.00752 ⁽³⁾	0.0184 ⁽¹³⁾	0.00963 ⁽⁸⁾	0.00665 ⁽²⁾	0.01008 ⁽⁹⁾
	ASAE		0.01026 ⁽¹²⁾	0.00949 ⁽¹⁾	0.00967 ⁽⁴⁾	0.00978 ⁽⁵⁾	0.00998 ⁽⁷⁾	0.0095 ⁽²⁾	0.0101 ⁽¹⁰⁾	0.0099 ⁽⁶⁾	0.01065 ⁽¹³⁾	0.01009 ⁽⁹⁾	0.01001 ⁽⁸⁾	0.0102 ⁽¹¹⁾
	Σ Ranks		37 ⁽⁶⁾	21 ⁽³⁾	54 ⁽⁹⁾	10 ⁽¹⁾	62 ^(11,5)	62 ^(11,5)	39 ⁽⁷⁾	24 ⁽⁴⁾	78 ⁽¹³⁾	49 ⁽⁸⁾	18 ⁽²⁾	56 ⁽¹⁰⁾

TABLE 2 | Numerical results of simulation for all measures when ($\lambda = 2.0$) under RSS.

<i>n</i>	Estimate	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	MSADE	MSALDE	KE	MSSDE	MSSLDE	MSLNDE
25	BIAS	$\hat{\lambda}$	0.14628 ⁽¹⁾	0.21478 ⁽²⁾	0.22417 ⁽³⁾	0.3051 ⁽¹⁰⁾	0.31669 ⁽¹¹⁾	0.29761 ⁽⁸⁾	0.29308 ⁽⁷⁾	0.30057 ⁽⁹⁾	0.27753 ⁽⁶⁾	0.42434 ⁽¹³⁾	0.26998 ⁽⁵⁾	0.42079 ⁽¹²⁾
	MSE	$\hat{\lambda}$	0.05945 ⁽¹⁾	0.144 ⁽³⁾	0.13987 ⁽²⁾	0.20526 ⁽⁹⁾	0.24113 ⁽¹¹⁾	0.20032 ⁽⁸⁾	0.1989 ⁽⁷⁾	0.22816 ⁽¹⁰⁾	0.16726 ⁽⁴⁾	0.38957 ⁽¹³⁾	0.18417 ⁽⁵⁾	0.36962 ⁽¹²⁾
	MRE	$\hat{\lambda}$	0.07314 ⁽¹⁾	0.10739 ⁽²⁾	0.11209 ⁽³⁾	0.15255 ⁽¹⁰⁾	0.15835 ⁽¹¹⁾	0.1488 ⁽⁸⁾	0.14654 ⁽⁷⁾	0.15028 ⁽⁹⁾	0.13876 ⁽⁶⁾	0.21217 ⁽¹³⁾	0.13499 ⁽⁵⁾	0.2104 ⁽¹²⁾
	<i>D</i> _{abs}		0.01302 ⁽¹⁾	0.01587 ⁽²⁾	0.01862 ⁽³⁾	0.02081 ⁽⁸⁾	0.02239 ⁽⁹⁾	0.02281 ⁽¹⁰⁾	0.02042 ⁽⁷⁾	0.02021 ⁽⁶⁾	0.02372 ⁽¹¹⁾	0.02508 ⁽¹³⁾	0.01922 ⁽⁵⁾	0.02497 ⁽¹²⁾
	<i>D</i> _{max}		0.0269 ⁽¹⁾	0.03112 ⁽²⁾	0.03825 ⁽⁵⁾	0.03965 ⁽⁸⁾	0.04347 ⁽⁹⁾	0.04487 ⁽¹⁰⁾	0.03907 ⁽⁶⁾	0.03907 ⁽⁶⁾	0.04842 ⁽¹³⁾	0.04634 ⁽¹²⁾	0.03721 ⁽⁴⁾	0.04618 ⁽¹¹⁾
	ASAE		0.04669 ⁽¹⁰⁾	0.04459 ⁽⁵⁾	0.04425 ⁽³⁾	0.04423 ⁽²⁾	0.04539 ⁽⁶⁾	0.04349 ⁽¹¹⁾	0.04573 ⁽⁷⁾	0.04603 ⁽⁸⁾	0.04876 ⁽¹¹⁾	0.05153 ⁽¹³⁾	0.04614 ⁽⁹⁾	0.05033 ⁽¹²⁾
	Σ Ranks		15 ⁽¹⁾	16 ⁽²⁾	19 ⁽³⁾	47 ⁽⁸⁾	57 ⁽¹¹⁾	45 ⁽⁷⁾	42 ⁽⁶⁾	48 ⁽⁹⁾	51 ⁽¹⁰⁾	77 ⁽¹³⁾	33 ⁽⁵⁾	71 ⁽¹²⁾
70	BIAS	$\hat{\lambda}$	0.06575 ⁽¹⁾	0.09118 ⁽²⁾	0.107 ⁽⁴⁾	0.1095 ⁽⁵⁾	0.12601 ⁽⁸⁾	0.14955 ⁽¹¹⁾	0.12911 ⁽⁹⁾	0.11159 ⁽⁶⁾	0.14126 ⁽¹⁰⁾	0.18514 ⁽¹³⁾	0.11937 ⁽⁷⁾	0.1731 ⁽¹²⁾
	MSE	$\hat{\lambda}$	0.01065 ⁽¹⁾	0.02259 ⁽³⁾	0.02728 ⁽⁵⁾	0.02525 ⁽⁴⁾	0.03879 ⁽⁸⁾	0.04999 ⁽¹¹⁾	0.04064 ⁽⁹⁾	0.03274 ⁽⁶⁾	0.03856 ⁽⁷⁾	0.07714 ⁽¹³⁾	0.04089 ⁽¹⁰⁾	0.06503 ⁽¹²⁾
	MRE	$\hat{\lambda}$	0.03288 ⁽¹⁾	0.04559 ⁽²⁾	0.0535 ⁽⁴⁾	0.05475 ⁽⁵⁾	0.063 ⁽⁸⁾	0.07477 ⁽¹¹⁾	0.06455 ⁽⁹⁾	0.05579 ⁽⁶⁾	0.07063 ⁽¹⁰⁾	0.09257 ⁽¹³⁾	0.05969 ⁽⁷⁾	0.08655 ⁽¹²⁾
	<i>D</i> _{abs}		0.00655 ⁽¹⁾	0.00831 ⁽²⁾	0.01063 ⁽⁷⁾	0.00967 ^(4,5)	0.01133 ⁽⁹⁾	0.01319 ⁽¹⁰⁾	0.01094 ⁽⁸⁾	0.00967 ^(4,5)	0.01379 ⁽¹²⁾	0.01412 ⁽¹³⁾	0.01013 ⁽⁶⁾	0.0134 ⁽¹¹⁾
	<i>D</i> _{max}		0.01396 ⁽¹⁾	0.01747 ⁽²⁾	0.02268 ⁽⁸⁾	0.02005 ⁽⁵⁾	0.02356 ⁽⁹⁾	0.02721 ⁽¹¹⁾	0.02241 ⁽⁷⁾	0.01996 ⁽⁴⁾	0.02953 ⁽¹³⁾	0.02807 ⁽¹²⁾	0.02074 ⁽⁶⁾	0.02672 ⁽¹⁰⁾
	ASAE		0.02432 ⁽⁸⁾	0.02344 ⁽²⁾	0.02378 ⁽⁶⁾	0.02368 ⁽⁵⁾	0.02362 ⁽⁴⁾	0.02262 ⁽¹⁾	0.02471 ⁽¹¹⁾	0.02416 ⁽⁷⁾	0.02469 ⁽¹⁰⁾	0.02522 ⁽¹³⁾	0.02445 ⁽⁹⁾	0.02497 ⁽¹²⁾
	Σ Ranks		13 ^(1,5)	13 ^(1,5)	34 ⁽⁶⁾	28.5 ⁽⁴⁾	46 ⁽⁸⁾	55 ⁽¹⁰⁾	53 ⁽⁹⁾	33.5 ⁽⁵⁾	62 ⁽¹¹⁾	77 ⁽¹³⁾	45 ⁽⁷⁾	69 ⁽¹²⁾

(Continues)

TABLE 2 | (Continued)

<i>n</i>	Estimate	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	MSADE	MSALDE	KE	MSSDE	MSSLDE	MSLNDE
150	BIAS	$\hat{\lambda}$	0.04117 ⁽¹⁾	0.04834 ⁽²⁾	0.06711 ⁽⁷⁾	0.05757 ⁽⁴⁾	0.0754 ⁽⁹⁾	0.09103 ⁽¹⁰⁾	0.07086 ⁽⁸⁾	0.06519 ⁽⁶⁾	0.09978 ⁽¹³⁾	0.09347 ⁽¹²⁾	0.05876 ⁽⁵⁾	0.09231 ⁽¹¹⁾
	MSE	$\hat{\lambda}$	0.00294 ⁽¹⁾	0.00438 ⁽²⁾	0.00764 ⁽⁵⁾	0.00603 ⁽⁴⁾	0.00975 ⁽⁸⁾	0.01628 ⁽¹⁰⁾	0.01007 ⁽⁹⁾	0.00947 ⁽⁶⁾	0.01688 ⁽¹¹⁾	0.01706 ⁽¹²⁾	0.00952 ⁽⁷⁾	0.01717 ⁽¹³⁾
	MRE	$\hat{\lambda}$	0.02059 ⁽¹⁾	0.02417 ⁽²⁾	0.03356 ⁽⁷⁾	0.02878 ⁽⁴⁾	0.03777 ⁽⁹⁾	0.04552 ⁽¹⁰⁾	0.03543 ⁽⁸⁾	0.03259 ⁽⁶⁾	0.04989 ⁽¹³⁾	0.04674 ⁽¹²⁾	0.02938 ⁽⁵⁾	0.04615 ⁽¹¹⁾
	D_{abs}		0.00425 ⁽¹⁾	0.00481 ⁽²⁾	0.00699 ⁽⁸⁾	0.00557 ⁽⁵⁾	0.00753 ⁽⁹⁾	0.00878 ⁽¹²⁾	0.0067 ⁽⁷⁾	0.00615 ⁽⁶⁾	0.01007 ⁽¹³⁾	0.00835 ⁽¹¹⁾	0.00556 ⁽⁴⁾	0.00818 ⁽¹⁰⁾
	D_{max}		0.00917 ⁽¹⁾	0.01035 ⁽²⁾	0.01511 ⁽⁸⁾	0.01193 ⁽⁵⁾	0.0162 ⁽⁹⁾	0.01865 ⁽¹²⁾	0.01423 ⁽⁷⁾	0.01303 ⁽⁶⁾	0.02169 ⁽¹³⁾	0.01741 ⁽¹¹⁾	0.01183 ⁽³⁾	0.01704 ⁽¹⁰⁾
	ASAE		0.0151 ⁽⁸⁾	0.01443 ^(4,5)	0.01436 ⁽³⁾	0.01482 ⁽⁶⁾	0.01431 ⁽²⁾	0.01429 ⁽¹¹⁾	0.01539 ⁽¹²⁾	0.01516 ⁽⁹⁾	0.01521 ⁽¹⁰⁾	0.01557 ⁽¹³⁾	0.01485 ⁽⁷⁾	0.01524 ⁽¹¹⁾
	$\sum$ Ranks		13 ⁽¹⁾	14.5 ⁽²⁾	38 ⁽⁶⁾	28 ⁽⁴⁾	46 ⁽⁸⁾	55 ⁽¹⁰⁾	51 ⁽⁹⁾	39 ⁽⁷⁾	73 ⁽¹³⁾	71 ⁽¹²⁾	31 ⁽⁵⁾	66 ⁽¹¹⁾
	BIAS	$\hat{\lambda}$	0.03531 ⁽¹⁾	0.04423 ⁽³⁾	0.05903 ⁽⁸⁾	0.04405 ⁽²⁾	0.06581 ⁽⁹⁾	0.07721 ⁽¹²⁾	0.05482 ⁽⁷⁾	0.05286 ⁽⁶⁾	0.08939 ⁽¹³⁾	0.07032 ⁽¹⁰⁾	0.04751 ⁽⁴⁾	0.07095 ⁽¹¹⁾
	MSE	$\hat{\lambda}$	0.00215 ⁽¹⁾	0.00358 ⁽³⁾	0.00558 ⁽⁷⁾	0.00335 ⁽²⁾	0.00729 ⁽⁹⁾	0.01112 ⁽¹²⁾	0.00555 ⁽⁶⁾	0.00603 ⁽⁸⁾	0.01267 ⁽¹³⁾	0.00925 ⁽¹⁰⁾	0.00478 ⁽⁵⁾	0.00956 ⁽¹¹⁾
	MRE	$\hat{\lambda}$	0.01766 ⁽¹⁾	0.02211 ⁽³⁾	0.02952 ⁽⁸⁾	0.02203 ⁽²⁾	0.0329 ⁽⁹⁾	0.03861 ⁽¹²⁾	0.02741 ⁽⁷⁾	0.02643 ⁽⁶⁾	0.0447 ⁽¹³⁾	0.03516 ⁽¹⁰⁾	0.02376 ⁽⁴⁾	0.03548 ⁽¹¹⁾
200	D_{abs}		0.00365 ⁽¹⁾	0.00446 ⁽³⁾	0.00614 ⁽⁸⁾	0.00439 ⁽²⁾	0.00667 ⁽¹¹⁾	0.00757 ⁽¹²⁾	0.00541 ⁽⁷⁾	0.00507 ⁽⁶⁾	0.0092 ⁽¹³⁾	0.00658 ^(9,5)	0.00468 ⁽⁴⁾	0.00658 ^(9,5)
	D_{max}		0.00791 ⁽¹⁾	0.00949 ⁽³⁾	0.01329 ⁽⁸⁾	0.00944 ⁽²⁾	0.01435 ⁽¹¹⁾	0.01622 ⁽¹²⁾	0.01154 ⁽⁷⁾	0.01083 ⁽⁶⁾	0.01993 ⁽¹³⁾	0.01387 ⁽⁹⁾	0.01003 ⁽⁴⁾	0.01389 ⁽¹⁰⁾
	ASAE		0.01251 ⁽⁸⁾	0.01202 ⁽²⁾	0.01207 ⁽⁵⁾	0.01229 ⁽⁶⁾	0.01205 ⁽⁴⁾	0.01173 ⁽¹¹⁾	0.01254 ⁽⁹⁾	0.01268 ⁽¹⁰⁾	0.01287 ⁽¹¹⁾	0.01298 ⁽¹³⁾	0.01246 ⁽⁷⁾	0.01297 ⁽¹²⁾
	$\sum$ Ranks		13 ⁽¹⁾	17 ⁽³⁾	44 ⁽⁸⁾	16 ⁽²⁾	53 ⁽⁹⁾	61 ⁽¹⁰⁾	43 ⁽⁷⁾	42 ⁽⁶⁾	76 ⁽¹³⁾	61.5 ⁽¹¹⁾	28 ⁽⁵⁾	64.5 ⁽¹²⁾
	BIAS	$\hat{\lambda}$	0.02651 ⁽¹⁾	0.03498 ⁽³⁾	0.04926 ⁽⁸⁾	0.03467 ⁽²⁾	0.05228 ⁽⁹⁾	0.06078 ⁽¹²⁾	0.04432 ⁽⁷⁾	0.03833 ⁽⁶⁾	0.07409 ⁽¹³⁾	0.05349 ⁽¹⁰⁾	0.03513 ⁽⁴⁾	0.05582 ⁽¹¹⁾
	MSE	$\hat{\lambda}$	0.00117 ⁽¹⁾	0.00236 ⁽⁴⁾	0.00391 ⁽⁸⁾	0.00203 ⁽²⁾	0.0045 ⁽⁹⁾	0.00641 ⁽¹²⁾	0.00364 ⁽⁷⁾	0.00261 ⁽⁶⁾	0.00898 ⁽¹³⁾	0.00595 ⁽¹¹⁾	0.00234 ⁽³⁾	0.00556 ⁽¹⁰⁾
	MRE	$\hat{\lambda}$	0.01325 ⁽¹⁾	0.01749 ⁽³⁾	0.02463 ⁽⁸⁾	0.01733 ⁽²⁾	0.02614 ⁽⁹⁾	0.03039 ⁽¹²⁾	0.02216 ⁽⁷⁾	0.01916 ⁽⁶⁾	0.03705 ⁽¹³⁾	0.02674 ⁽¹⁰⁾	0.01756 ⁽⁴⁾	0.02791 ⁽¹¹⁾
	D_{abs}		0.00278 ⁽¹⁾	0.00354 ⁽³⁾	0.00514 ⁽⁹⁾	0.0035 ⁽²⁾	0.00536 ⁽¹¹⁾	0.00617 ⁽¹²⁾	0.0044 ⁽⁷⁾	0.00386 ⁽⁵⁾	0.00761 ⁽¹³⁾	0.00508 ⁽⁸⁾	0.00356 ⁽⁴⁾	0.00534 ⁽¹⁰⁾
	D_{max}		0.00602 ⁽¹⁾	0.00756 ^(2,5)	0.01116 ⁽⁹⁾	0.00756 ^(2,5)	0.01158 ⁽¹¹⁾	0.01329 ⁽¹²⁾	0.00943 ⁽⁷⁾	0.0083 ⁽⁵⁾	0.01645 ⁽¹³⁾	0.01081 ⁽⁸⁾	0.00772 ⁽⁴⁾	0.01138 ⁽¹⁰⁾
	ASAE		0.00977 ⁽⁸⁾	0.00913 ⁽¹⁾	0.00956 ⁽⁵⁾	0.00974 ⁽⁷⁾	0.00935 ⁽³⁾	0.00932 ⁽²⁾	0.00994 ⁽¹²⁾	0.00971 ⁽⁶⁾	0.01004 ⁽¹³⁾	0.00987 ⁽¹⁰⁾	0.00984 ⁽⁹⁾	0.00993 ⁽¹¹⁾
400	$\sum$ Ranks		13 ⁽¹⁾	16.5 ⁽²⁾	47 ^(7,5)	17.5 ⁽³⁾	52 ⁽⁹⁾	62 ⁽¹¹⁾	47 ^(7,5)	34 ⁽⁶⁾	78 ⁽¹³⁾	57 ⁽¹⁰⁾	28 ⁽⁴⁾	63 ⁽¹²⁾
	BIAS	$\hat{\lambda}$	0.02271 ⁽¹⁾	0.02752 ⁽³⁾	0.04246 ⁽⁹⁾	0.02665 ⁽²⁾	0.04317 ⁽¹⁰⁾	0.05356 ⁽¹²⁾	0.03559 ⁽⁷⁾	0.0316 ⁽⁶⁾	0.06155 ⁽¹³⁾	0.04437 ⁽¹¹⁾	0.02875 ⁽⁴⁾	0.04179 ⁽⁸⁾
	MSE	$\hat{\lambda}$	0.00085 ⁽¹⁾	0.00137 ⁽³⁾	0.00289 ⁽⁸⁾	0.00119 ⁽²⁾	0.00307 ⁽¹⁰⁾	0.00463 ⁽¹²⁾	0.00215 ⁽⁷⁾	0.00176 ⁽⁶⁾	0.00616 ⁽¹³⁾	0.00344 ⁽¹¹⁾	0.00151 ⁽⁴⁾	0.00301 ⁽⁹⁾
	MRE	$\hat{\lambda}$	0.01136 ⁽¹⁾	0.01376 ⁽³⁾	0.02123 ⁽⁹⁾	0.01332 ⁽²⁾	0.02158 ⁽¹⁰⁾	0.02678 ⁽¹²⁾	0.0178 ⁽⁷⁾	0.0158 ⁽⁶⁾	0.03077 ⁽¹³⁾	0.02218 ⁽¹¹⁾	0.01437 ⁽⁴⁾	0.02089 ⁽⁸⁾
	D_{abs}		0.00238 ⁽¹⁾	0.00282 ⁽³⁾	0.00447 ⁽¹⁰⁾	0.00273 ⁽²⁾	0.00449 ⁽¹¹⁾	0.00554 ⁽¹²⁾	0.00361 ⁽⁷⁾	0.0032 ⁽⁵⁾	0.00643 ⁽¹³⁾	0.00436 ⁽⁹⁾	0.00295 ⁽⁴⁾	0.00413 ⁽⁸⁾
	D_{max}		0.00517 ⁽¹⁾	0.00606 ⁽³⁾	0.00968 ⁽¹⁰⁾	0.00589 ⁽²⁾	0.00972 ⁽¹¹⁾	0.012 ⁽¹²⁾	0.00778 ⁽⁷⁾	0.00692 ⁽⁵⁾	0.01393 ⁽¹³⁾	0.00931 ⁽⁹⁾	0.00636 ⁽⁴⁾	0.00886 ⁽⁸⁾
	ASAE		0.00807 ⁽⁶⁾	0.00794 ⁽⁵⁾	0.00791 ⁽⁴⁾	0.00813 ⁽⁷⁾	0.00781 ⁽²⁾	0.00784 ⁽³⁾	0.00827 ⁽¹⁰⁾	0.00821 ⁽⁸⁾	0.00846 ⁽¹³⁾	0.00828 ⁽¹¹⁾	0.00826 ⁽⁹⁾	0.0084 ⁽¹²⁾
	$\sum$ Ranks		11 ⁽¹⁾	20 ⁽³⁾	50 ⁽⁸⁾	17 ⁽²⁾	54 ⁽¹⁰⁾	61 ⁽¹¹⁾	45 ⁽⁷⁾	36 ⁽⁶⁾	78 ⁽¹³⁾	62 ⁽¹²⁾	29 ⁽⁴⁾	53 ⁽⁹⁾
	BIAS	$\hat{\lambda}$	0.02271 ⁽¹⁾	0.02752 ⁽³⁾	0.04246 ⁽⁹⁾	0.02665 ⁽²⁾	0.04317 ⁽¹⁰⁾	0.05356 ⁽¹²⁾	0.03559 ⁽⁷⁾	0.0316 ⁽⁶⁾	0.06155 ⁽¹³⁾	0.04437 ⁽¹¹⁾	0.02875 ⁽⁴⁾	0.04179 ⁽⁸⁾
	MSE	$\hat{\lambda}$	0.00085 ⁽¹⁾	0.00137 ⁽³⁾	0.00289 ⁽⁸⁾	0.00119 ⁽²⁾	0.00307 ⁽¹⁰⁾	0.00463 ⁽¹²⁾	0.00215 ⁽⁷⁾	0.00176 ⁽⁶⁾	0.00616 ⁽¹³⁾	0.00344 ⁽¹¹⁾	0.00151 ⁽⁴⁾	0.00301 ⁽⁹⁾
	MRE	$\hat{\lambda}$	0.01136 ⁽¹⁾	0.01376 ⁽³⁾	0.02123 ⁽⁹⁾	0.01332 ⁽²⁾	0.02158 ⁽¹⁰⁾	0.02678 ⁽¹²⁾	0.0178 ⁽⁷⁾	0.0158 ⁽⁶⁾	0.03077 ⁽¹³⁾	0.02218 ⁽¹¹⁾	0.01437 ⁽⁴⁾	0.02089 ⁽⁸⁾

TABLE 3 | Numerical results of simulation for all measures when ($\lambda = 2.5$) under SRS.

n	Estimate	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	MSADE	MSALDE	KE	MSSDE	MSSLDE	MSLNDE
25	BIAS	$\hat{\lambda}$	0.47113 ⁽¹¹⁾	0.56856 ⁽⁸⁾	0.56398 ⁽⁷⁾	0.58369 ⁽¹⁰⁾	0.5785 ⁽⁹⁾	0.5482 ⁽⁵⁾	0.54674 ⁽⁴⁾	0.52991 ⁽³⁾	0.59488 ⁽¹¹⁾	0.55726 ⁽⁶⁾	0.50348 ⁽²⁾	0.7792 ⁽¹³⁾
	MSE	$\hat{\lambda}$	0.39563 ⁽¹¹⁾	0.62706 ⁽⁸⁾	0.6273 ⁽⁹⁾	0.61582 ⁽⁷⁾	0.69824 ⁽¹¹⁾	0.551 ⁽⁴⁾	0.59508 ⁽⁶⁾	0.52664 ⁽³⁾	0.64734 ⁽¹⁰⁾	0.5913 ⁽⁵⁾	0.45926 ⁽²⁾	1.01067 ⁽¹³⁾
	MRE	$\hat{\lambda}$	0.18845 ⁽¹¹⁾	0.22742 ⁽⁸⁾	0.22559 ⁽⁷⁾	0.23348 ⁽¹⁰⁾	0.2314 ⁽⁹⁾	0.21928 ⁽⁵⁾	0.2187 ⁽⁴⁾	0.21196 ⁽³⁾	0.23795 ⁽¹¹⁾	0.2229 ⁽⁶⁾	0.20139 ⁽²⁾	0.31168 ⁽¹³⁾
	D_{abs}		0.0296 ⁽¹³⁾	0.02384 ⁽⁶⁾	0.02512 ⁽⁹⁾	0.02244 ⁽³⁾	0.02399 ⁽⁸⁾	0.02333 ⁽⁵⁾	0.0233 ⁽⁵⁾	0.02092 ⁽²⁾	0.02287 ⁽⁴⁾	0.02513 ⁽¹⁰⁾	0.02042 ⁽¹⁾	0.02909 ⁽¹²⁾
	D_{max}		0.06035 ⁽¹³⁾	0.04296 ⁽⁴⁾	0.04705 ⁽⁹⁾	0.04248 ⁽³⁾	0.04356 ⁽⁶⁾	0.04442 ⁽⁸⁾	0.04332 ⁽⁵⁾	0.04003 ⁽²⁾	0.04365 ⁽⁷⁾	0.04781 ⁽¹⁰⁾	0.03903 ⁽¹⁾	0.0557 ⁽¹²⁾
	ASAE		0.05647 ⁽⁹⁾	0.05201 ⁽³⁾	0.05422 ⁽⁵⁾	0.05401 ⁽⁴⁾	0.05587 ⁽⁷⁾	0.04904 ⁽¹⁾	0.05139 ⁽²⁾	0.05598 ⁽⁸⁾	0.05673 ⁽¹⁰⁾	0.05891 ⁽¹¹⁾	0.05585 ⁽⁶⁾	0.06315 ⁽¹²⁾
	Σ Ranks		38 ⁽⁷⁾	37 ^(5,5)	46 ⁽⁸⁾	37 ^(5,5)	50 ⁽¹⁰⁾	30 ⁽⁴⁾	26 ⁽³⁾	21 ⁽²⁾	53 ⁽¹¹⁾	48 ⁽⁹⁾	14 ⁽¹⁾	75 ⁽¹³⁾
	BIAS	$\hat{\lambda}$	0.28958 ⁽¹¹⁾	0.34455 ⁽⁶⁾	0.39383 ⁽¹⁰⁾	0.33136 ⁽³⁾	0.41771 ⁽¹¹⁾	0.33894 ⁽⁴⁾	0.3398 ⁽⁵⁾	0.36589 ⁽⁷⁾	0.3733 ⁽⁸⁾	0.39051 ⁽⁹⁾	0.32012 ⁽²⁾	0.46712 ⁽¹²⁾
	MSE	$\hat{\lambda}$	0.14476 ⁽¹¹⁾	0.23911 ⁽⁶⁾	0.31786 ⁽¹⁰⁾	0.18508 ⁽³⁾	0.35441 ⁽¹¹⁾	0.20152 ⁽⁴⁾	0.22633 ⁽⁵⁾	0.24333 ⁽⁷⁾	0.25074 ⁽⁸⁾	0.30036 ⁽⁹⁾	0.18431 ⁽²⁾	0.38777 ⁽¹²⁾
	MRE	$\hat{\lambda}$	0.11583 ⁽¹¹⁾	0.13782 ⁽⁶⁾	0.15753 ⁽¹⁰⁾	0.13254 ⁽³⁾	0.16708 ⁽¹¹⁾	0.13558 ⁽⁴⁾	0.13592 ⁽⁵⁾	0.14636 ⁽⁷⁾	0.14932 ⁽⁸⁾	0.15621 ⁽⁹⁾	0.12805 ⁽²⁾	0.18685 ⁽¹²⁾
70	D_{abs}		0.01286 ⁽²⁾	0.01437 ⁽⁵⁾	0.01666 ⁽¹⁰⁾	0.01279 ⁽¹⁾	0.01709 ⁽¹¹⁾	0.01449 ⁽⁷⁾	0.01414 ⁽⁴⁾	0.0147 ⁽⁸⁾	0.01443 ⁽⁶⁾	0.01653 ⁽⁹⁾	0.01297 ⁽³⁾	0.01762 ⁽¹²⁾
	D_{max}		0.02362 ⁽¹¹⁾	0.02621 ⁽⁵⁾	0.03059 ⁽⁹⁾	0.02446 ⁽³⁾	0.03171 ⁽¹¹⁾	0.02635 ⁽⁶⁾	0.02611 ⁽⁴⁾	0.02783 ⁽⁸⁾	0.02745 ⁽⁷⁾	0.03065 ⁽¹⁰⁾	0.02429 ⁽²⁾	0.03386 ⁽¹²⁾
	ASAE		0.02754 ⁽⁵⁾	0.02645 ⁽³⁾	0.02774 ⁽⁷⁾	0.02662 ⁽⁴⁾	0.02782 ⁽⁸⁾	0.02582 ⁽¹⁾	0.02628 ⁽²⁾	0.02896 ⁽¹⁰⁾	0.02894 ⁽⁹⁾	0.03105 ⁽¹¹⁾	0.02766 ⁽⁶⁾	0.03234 ⁽¹³⁾
	Σ Ranks		11 ⁽¹⁾	31 ⁽⁶⁾	56 ⁽⁹⁾	17 ^(2,5)	63 ⁽¹¹⁾	26 ⁽⁵⁾	25 ⁽⁴⁾	47 ⁽⁸⁾	46 ⁽⁷⁾	57 ⁽¹⁰⁾	17 ^(2,5)	73 ⁽¹²⁾
	BIAS	$\hat{\lambda}$	0.18468 ⁽¹¹⁾	0.23049 ⁽³⁾	0.27983 ⁽¹⁰⁾	0.21717 ⁽²⁾	0.27373 ⁽⁹⁾	0.24503 ⁽⁷⁾	0.23253 ⁽⁵⁾	0.24125 ⁽⁶⁾	0.25242 ⁽⁸⁾	0.28053 ⁽¹¹⁾	0.23068 ⁽⁴⁾	0.30267 ⁽¹²⁾
	MSE	$\hat{\lambda}$	0.0571 ⁽¹¹⁾	0.10501 ⁽⁷⁾	0.15555 ⁽¹¹⁾	0.08295 ⁽²⁾	0.14841 ⁽¹⁰⁾	0.09881 ⁽⁴⁾	0.10163 ⁽⁵⁾	0.10293 ⁽⁶⁾	0.11352 ⁽⁸⁾	0.14628 ⁽⁹⁾	0.09612 ⁽³⁾	0.1603 ⁽¹²⁾
	MRE	$\hat{\lambda}$	0.07387 ⁽¹¹⁾	0.09219 ⁽³⁾	0.11193 ⁽¹⁰⁾	0.08687 ⁽²⁾	0.10949 ⁽⁹⁾	0.09801 ⁽⁷⁾	0.09301 ⁽⁵⁾	0.0965 ⁽⁶⁾	0.10097 ⁽⁸⁾	0.11221 ⁽¹¹⁾	0.09227 ⁽⁴⁾	0.12107 ⁽¹²⁾
	D_{abs}		0.00804 ⁽¹¹⁾	0.00961 ⁽⁴⁾	0.01145 ⁽¹⁰⁾	0.00867 ⁽²⁾	0.01125 ⁽⁹⁾	0.01049 ⁽⁸⁾	0.00976 ⁽⁵⁾	0.00989 ⁽⁶⁾	0.01015 ⁽⁷⁾	0.01199 ⁽¹²⁾	0.00936 ⁽³⁾	0.01177 ⁽¹¹⁾
	D_{max}		0.01425 ⁽¹¹⁾	0.01735 ⁽⁴⁾	0.02098 ⁽¹⁰⁾	0.01601 ⁽²⁾	0.02071 ⁽⁹⁾	0.01884 ⁽⁸⁾	0.01768 ⁽⁵⁾	0.01819 ⁽⁶⁾	0.01879 ⁽⁷⁾	0.02188 ⁽¹¹⁾	0.01726 ⁽³⁾	0.02224 ⁽¹²⁾
	ASAE		0.01717 ⁽⁶⁾	0.01578 ⁽²⁾	0.01727 ⁽⁷⁾	0.01679 ⁽⁵⁾	0.01675 ⁽⁴⁾	0.01527 ⁽¹⁾	0.01608 ⁽³⁾	0.01812 ⁽¹⁰⁾	0.01728 ⁽⁸⁾	0.01874 ⁽¹¹⁾	0.01746 ⁽⁹⁾	0.01909 ⁽¹³⁾
200	Σ Ranks		11 ⁽¹⁾	23 ⁽³⁾	58 ⁽¹⁰⁾	15 ⁽²⁾	50 ⁽⁹⁾	35 ⁽⁶⁾	28 ⁽⁵⁾	40 ⁽⁷⁾	46 ⁽⁸⁾	65 ⁽¹¹⁾	26 ⁽⁴⁾	72 ⁽¹²⁾
	BIAS	$\hat{\lambda}$	0.15972 ⁽¹¹⁾	0.20199 ⁽⁴⁾	0.24095 ⁽⁹⁾	0.17919 ⁽²⁾	0.24143 ⁽¹⁰⁾	0.21737 ⁽⁷⁾	0.2119 ⁽⁶⁾	0.22094 ⁽⁸⁾	0.20831 ⁽⁵⁾	0.24742 ⁽¹²⁾	0.19855 ⁽³⁾	0.25417 ⁽¹³⁾
	MSE	$\hat{\lambda}$	0.04061 ⁽¹¹⁾	0.08319 ⁽⁶⁾	0.111759 ⁽¹¹⁾	0.05378 ⁽²⁾	0.11801 ^(12,5)	0.07881 ⁽⁵⁾	0.08565 ⁽⁸⁾	0.08397 ⁽⁷⁾	0.07473 ⁽⁴⁾	0.11801 ^(12,5)	0.06978 ⁽³⁾	0.11482 ⁽¹⁰⁾
	MRE	$\hat{\lambda}$	0.06389 ⁽¹¹⁾	0.0808 ⁽⁴⁾	0.09638 ⁽⁹⁾	0.07168 ⁽²⁾	0.09657 ⁽¹⁰⁾	0.08695 ⁽⁷⁾	0.08476 ⁽⁶⁾	0.08838 ⁽⁸⁾	0.08332 ⁽⁵⁾	0.09897 ⁽¹²⁾	0.07942 ⁽³⁾	0.10167 ⁽¹³⁾
	D_{abs}		0.007 ⁽¹⁾	0.00837 ⁽⁴⁾	0.00993 ⁽¹¹⁾	0.00724 ⁽²⁾	0.00983 ⁽¹⁰⁾	0.00922 ⁽⁸⁾	0.00878 ⁽⁶⁾	0.00902 ⁽⁷⁾	0.00849 ⁽⁵⁾	0.01054 ⁽¹³⁾	0.00972 ⁽⁹⁾	0.01007 ⁽¹²⁾
	D_{max}		0.01222 ⁽¹¹⁾	0.01507 ⁽⁴⁾	0.01803 ⁽¹⁰⁾	0.01328 ⁽²⁾	0.01789 ⁽⁹⁾	0.0165 ⁽⁷⁾	0.01579 ⁽⁶⁾	0.01658 ⁽⁸⁾	0.01555 ⁽⁵⁾	0.01917 ⁽¹³⁾	0.01476 ⁽³⁾	0.01877 ⁽¹²⁾
	ASAE		0.01411 ^(5,5)	0.01321 ⁽²⁾	0.01411 ^(5,5)	0.01392 ⁽⁴⁾	0.01435 ⁽⁷⁾	0.01264 ⁽¹⁾	0.01325 ⁽³⁾	0.01472 ⁽¹⁰⁾	0.01458 ⁽⁹⁾	0.01599 ⁽¹³⁾	0.01436 ⁽⁸⁾	0.01563 ⁽¹²⁾
	Σ Ranks		10.5 ⁽¹⁾	24 ⁽⁴⁾	55.5 ⁽⁹⁾	14 ⁽²⁾	58.5 ⁽¹⁰⁾	35 ^(6,5)	35 ^(6,5)	48 ⁽⁸⁾	33 ⁽⁵⁾	75.5 ⁽¹³⁾	23 ⁽³⁾	72 ⁽¹²⁾

(Continues)

TABLE 3 | (Continued)

<i>n</i>		Estimate	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	MSADE	MSALDE	KE	MSSDE	MSSLDE	MSLNDE
300	BIAS	$\hat{\lambda}$	0.13234 ⁽¹⁾	0.16212 ⁽⁵⁾	0.19794 ⁽¹⁰⁾	0.1433 ⁽²⁾	0.20816 ⁽¹³⁾	0.16925 ⁽⁶⁾	0.15558 ⁽³⁾	0.17828 ⁽⁸⁾	0.16971 ⁽⁷⁾	0.19862 ⁽¹¹⁾	0.19688 ⁽⁹⁾	0.16003 ⁽⁴⁾	0.20114 ⁽¹²⁾
	MSE	$\hat{\lambda}$	0.02812 ⁽¹⁾	0.0512 ⁽⁷⁾	0.07767 ⁽¹²⁾	0.03292 ⁽²⁾	0.08643 ⁽¹³⁾	0.04683 ⁽⁵⁾	0.04605 ⁽⁴⁾	0.05807 ⁽⁸⁾	0.05008 ⁽⁶⁾	0.0737 ⁽¹¹⁾	0.06552 ⁽⁹⁾	0.04289 ⁽³⁾	0.06674 ⁽¹⁰⁾
	MRE	$\hat{\lambda}$	0.05294 ⁽¹⁾	0.06485 ⁽⁵⁾	0.07918 ⁽¹⁰⁾	0.05732 ⁽²⁾	0.08326 ⁽¹³⁾	0.0677 ⁽⁶⁾	0.06223 ⁽³⁾	0.07131 ⁽⁸⁾	0.06788 ⁽⁷⁾	0.07945 ⁽¹¹⁾	0.07875 ⁽⁹⁾	0.06401 ⁽⁴⁾	0.08046 ⁽¹²⁾
	D_{abs}		0.00577 ⁽¹⁾	0.00676 ⁽⁵⁾	0.00819 ⁽¹¹⁾	0.0059 ⁽²⁾	0.00844 ⁽¹²⁾	0.00727 ⁽⁷⁾	0.00658 ⁽³⁾	0.0073 ⁽⁸⁾	0.00699 ⁽⁶⁾	0.0086 ⁽¹³⁾	0.00803 ⁽⁹⁾	0.00663 ⁽⁴⁾	0.00815 ⁽¹⁰⁾
	D_{max}		0.00997 ⁽¹⁾	0.01206 ⁽⁵⁾	0.01479 ⁽¹⁰⁾	0.01059 ⁽²⁾	0.01532 ⁽¹²⁾	0.01278 ⁽⁷⁾	0.0116 ⁽³⁾	0.01324 ⁽⁸⁾	0.0126 ⁽⁶⁾	0.01535 ⁽¹³⁾	0.01464 ⁽⁹⁾	0.01188 ⁽⁴⁾	0.01493 ⁽¹¹⁾
	ASAE		0.01073 ⁽⁴⁾	0.01056 ⁽²⁾	0.01126 ⁽⁷⁾	0.01086 ⁽⁵⁾	0.01088 ⁽⁶⁾	0.00988 ⁽¹⁾	0.01058 ⁽³⁾	0.01134 ⁽¹⁰⁾	0.01133 ⁽⁹⁾	0.01257 ⁽¹³⁾	0.01227 ⁽¹²⁾	0.01128 ⁽⁸⁾	0.01221 ⁽¹¹⁾
	Σ Ranks		9 ⁽¹⁾	29 ⁽⁵⁾	60 ⁽¹⁰⁾	15 ⁽²⁾	69 ⁽¹²⁾	32 ⁽⁶⁾	19 ⁽³⁾	50 ⁽⁸⁾	41 ⁽⁷⁾	72 ⁽¹³⁾	57 ⁽⁹⁾	27 ⁽⁴⁾	66 ⁽¹¹⁾
400	BIAS	$\hat{\lambda}$	0.11485 ⁽¹⁾	0.13454 ⁽⁴⁾	0.17681 ⁽¹³⁾	0.12369 ⁽²⁾	0.16003 ⁽⁹⁾	0.15154 ⁽⁸⁾	0.13991 ⁽⁵⁾	0.14444 ⁽⁷⁾	0.14434 ⁽⁶⁾	0.17536 ⁽¹²⁾	0.17067 ⁽¹¹⁾	0.13082 ⁽³⁾	0.17021 ⁽¹⁰⁾
	MSE	$\hat{\lambda}$	0.02083 ⁽¹⁾	0.03522 ⁽⁷⁾	0.06048 ⁽¹³⁾	0.02507 ⁽²⁾	0.05023 ⁽¹¹⁾	0.03718 ⁽⁸⁾	0.0343 ⁽⁴⁾	0.03458 ⁽⁵⁾	0.03504 ⁽⁶⁾	0.05606 ⁽¹²⁾	0.0497 ⁽¹⁰⁾	0.02816 ⁽³⁾	0.04791 ⁽⁹⁾
	MRE	$\hat{\lambda}$	0.04594 ⁽¹⁾	0.05381 ⁽⁴⁾	0.07072 ⁽¹³⁾	0.04947 ⁽²⁾	0.06401 ⁽⁹⁾	0.06061 ⁽⁸⁾	0.05597 ⁽⁵⁾	0.05778 ⁽⁷⁾	0.05773 ⁽⁶⁾	0.07014 ⁽¹²⁾	0.06827 ⁽¹¹⁾	0.05233 ⁽³⁾	0.06808 ⁽¹⁰⁾
	D_{abs}		0.00498 ⁽¹⁾	0.00562 ⁽⁴⁾	0.00731 ⁽¹²⁾	0.00518 ⁽²⁾	0.00663 ⁽⁹⁾	0.00652 ⁽⁸⁾	0.00591 ⁽⁵⁾	0.00608 ⁽⁷⁾	0.006 ⁽⁶⁾	0.00743 ⁽¹³⁾	0.00698 ⁽¹⁰⁾	0.00545 ⁽³⁾	0.00702 ⁽¹¹⁾
	D_{max}		0.00858 ⁽¹⁾	0.00996 ⁽⁴⁾	0.01315 ⁽¹²⁾	0.00918 ⁽²⁾	0.01193 ⁽⁹⁾	0.01138 ⁽⁸⁾	0.01041 ⁽⁵⁾	0.01077 ⁽⁷⁾	0.01071 ⁽⁶⁾	0.01319 ⁽¹³⁾	0.01267 ⁽¹¹⁾	0.00968 ⁽³⁾	0.01265 ⁽¹⁰⁾
	ASAE		0.00914 ⁽⁴⁾	0.00877 ⁽²⁾	0.00948 ⁽⁸⁾	0.00919 ⁽⁵⁾	0.00939 ⁽⁷⁾	0.00822 ⁽¹⁾	0.00881 ⁽³⁾	0.00965 ⁽¹⁰⁾	0.00949 ⁽⁹⁾	0.01034 ⁽¹³⁾	0.01012 ⁽¹¹⁾	0.00937 ⁽⁶⁾	0.01025 ⁽¹²⁾
	Σ Ranks		9 ⁽¹⁾	25 ⁽⁴⁾	71 ⁽¹²⁾	15 ⁽²⁾	54 ⁽⁹⁾	41 ⁽⁷⁾	27 ⁽⁵⁾	43 ⁽⁸⁾	39 ⁽⁶⁾	75 ⁽¹³⁾	64 ⁽¹¹⁾	21 ⁽³⁾	62 ⁽¹⁰⁾

TABLE 4 | Numerical results of simulation for all measures when ($\lambda = 2.5$) under RSS.

<i>n</i>		Estimate	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	MSADE	MSALDE	KE	MSSDE	MSSLDE	MSLNDE
25	BIAS	$\hat{\lambda}$	0.32695 ⁽¹⁾	0.42903 ⁽²⁾	0.43538 ⁽⁴⁾	0.52477 ⁽¹⁰⁾	0.48678 ⁽⁸⁾	0.4329 ⁽³⁾	0.43658 ⁽⁵⁾	0.51747 ⁽⁹⁾	0.59104 ⁽¹¹⁾	0.44513 ⁽⁶⁾	0.73446 ⁽¹²⁾	0.47803 ⁽⁷⁾	0.74969 ⁽¹³⁾
	MSE	$\hat{\lambda}$	0.17858 ⁽¹⁾	0.33759 ⁽⁴⁾	0.34064 ⁽⁵⁾	0.4689 ⁽⁹⁾	0.43077 ⁽⁸⁾	0.29719 ⁽²⁾	0.33422 ⁽³⁾	0.49083 ⁽¹⁰⁾	0.63176 ⁽¹¹⁾	0.35182 ⁽⁶⁾	0.90076 ⁽¹³⁾	0.41393 ⁽⁷⁾	0.89846 ⁽¹²⁾
	MRE	$\hat{\lambda}$	0.13078 ⁽¹⁾	0.17161 ⁽²⁾	0.17415 ⁽⁴⁾	0.20991 ⁽¹⁰⁾	0.19471 ⁽⁸⁾	0.17316 ⁽³⁾	0.17463 ⁽⁵⁾	0.20699 ⁽⁹⁾	0.23641 ⁽¹¹⁾	0.17805 ⁽⁶⁾	0.29378 ⁽¹²⁾	0.19121 ⁽⁷⁾	0.29988 ⁽¹³⁾
	D_{abs}		0.01521 ⁽¹⁾	0.01766 ⁽³⁾	0.01938 ⁽⁷⁾	0.01963 ⁽⁸⁾	0.01919 ⁽⁶⁾	0.01864 ⁽⁴⁾	0.01753 ⁽²⁾	0.02034 ⁽¹⁰⁾	0.02238 ⁽¹¹⁾	0.01973 ⁽⁹⁾	0.02679 ⁽¹²⁾	0.01888 ⁽⁵⁾	0.02745 ⁽¹³⁾
	D_{max}		0.02807 ⁽¹⁾	0.03374 ⁽³⁾	0.03708 ⁽⁶⁾	0.03784 ⁽⁹⁾	0.03712 ⁽⁷⁾	0.03544 ⁽⁴⁾	0.03356 ⁽²⁾	0.0393 ⁽¹⁰⁾	0.04279 ⁽¹¹⁾	0.03771 ⁽⁸⁾	0.052 ⁽¹²⁾	0.03625 ⁽⁵⁾	0.05327 ⁽¹³⁾
	ASAE		0.04515 ⁽⁹⁾	0.04198 ⁽³⁾	0.04353 ⁽⁵⁾	0.0434 ⁽⁴⁾	0.04406 ⁽⁶⁾	0.04132 ⁽²⁾	0.04123 ⁽¹⁾	0.04635 ⁽¹⁰⁾	0.04467 ⁽⁷⁾	0.04711 ⁽¹¹⁾	0.05236 ⁽¹²⁾	0.04485 ⁽⁸⁾	0.05399 ⁽¹³⁾
	Σ Ranks		14 ⁽¹⁾	17 ⁽²⁾	31 ⁽⁵⁾	50 ⁽⁹⁾	43 ⁽⁷⁾	18 ^(3,5)	18 ^(3,5)	58 ⁽¹⁰⁾	62 ⁽¹¹⁾	46 ⁽⁸⁾	73 ⁽¹²⁾	39 ⁽⁶⁾	77 ⁽¹³⁾
70	BIAS	$\hat{\lambda}$	0.21343 ⁽¹⁾	0.24222 ⁽²⁾	0.28161 ⁽⁶⁾	0.279 ⁽⁵⁾	0.3033 ⁽⁹⁾	0.26247 ⁽⁴⁾	0.26114 ⁽³⁾	0.33233 ⁽¹⁰⁾	0.33257 ⁽¹¹⁾	0.29413 ⁽⁷⁾	0.44814 ⁽¹³⁾	0.29502 ⁽⁸⁾	0.44281 ⁽¹²⁾
	MSE	$\hat{\lambda}$	0.0748 ⁽¹⁾	0.10009 ⁽²⁾	0.13602 ⁽⁶⁾	0.12515 ⁽⁵⁾	0.15974 ⁽⁹⁾	0.10563 ⁽³⁾	0.11319 ⁽⁴⁾	0.19394 ⁽¹⁰⁾	0.21275 ⁽¹¹⁾	0.14797 ⁽⁷⁾	0.343 ⁽¹³⁾	0.15779 ⁽⁸⁾	0.33064 ⁽¹²⁾
	MRE	$\hat{\lambda}$	0.08537 ⁽¹⁾	0.09689 ⁽²⁾	0.11264 ⁽⁶⁾	0.1116 ⁽⁵⁾	0.12132 ⁽⁹⁾	0.10499 ⁽⁴⁾	0.10445 ⁽³⁾	0.13293 ⁽¹⁰⁾	0.13303 ⁽¹¹⁾	0.11765 ⁽⁷⁾	0.17926 ⁽¹³⁾	0.11801 ⁽⁸⁾	0.17712 ⁽¹²⁾
	D_{abs}		0.00948 ⁽¹⁾	0.01022 ⁽²⁾	0.01201 ⁽⁷⁾	0.01096 ⁽⁴⁾	0.01224 ⁽⁸⁾	0.01149 ⁽⁵⁾	0.01081 ⁽³⁾	0.0132 ⁽¹¹⁾	0.01283 ⁽⁹⁾	0.01303 ⁽¹⁰⁾	0.01682 ⁽¹³⁾	0.01159 ⁽⁶⁾	0.01653 ⁽¹²⁾
	D_{max}		0.01698 ⁽¹⁾	0.01869 ⁽²⁾	0.02233 ⁽⁷⁾	0.02073 ⁽⁴⁾	0.02298 ⁽⁸⁾	0.02104 ⁽⁵⁾	0.02002 ⁽³⁾	0.02497 ⁽¹¹⁾	0.02445 ⁽¹⁰⁾	0.02418 ⁽⁹⁾	0.03249 ⁽¹³⁾	0.02187 ⁽⁶⁾	0.03196 ⁽¹²⁾
	ASAE		0.02232 ⁽⁶⁾	0.02163 ⁽³⁾	0.02222 ⁽⁴⁾	0.02226 ⁽⁵⁾	0.02245 ⁽⁷⁾	0.0211 ⁽²⁾	0.02104 ⁽¹⁾	0.02407 ⁽¹⁰⁾	0.02391 ⁽⁹⁾	0.02496 ⁽¹¹⁾	0.02681 ⁽¹²⁾	0.02312 ⁽⁸⁾	0.02704 ⁽¹³⁾
	Σ Ranks		11 ⁽¹⁾	13 ⁽²⁾	36 ⁽⁶⁾	28 ⁽⁵⁾	50 ⁽⁸⁾	23 ⁽⁴⁾	17 ⁽³⁾	62 ⁽¹¹⁾	61 ⁽¹⁰⁾	51 ⁽⁹⁾	77 ⁽¹³⁾	44 ⁽⁷⁾	73 ⁽¹²⁾

(Continues)

TABLE 4 | (Continued)

<i>n</i>	Estimate	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	MSADE	MSALDE	KE	MSSDE	MSSLDE	MSLNDE
150	BIAS	$\hat{\lambda}$	0.14721 ⁽¹⁾	0.19274 ⁽⁵⁾	0.19494 ⁽⁶⁾	0.20536 ⁽⁸⁾	0.17861 ⁽⁴⁾	0.16661 ⁽³⁾	0.21689 ⁽¹⁰⁾	0.21938 ⁽¹¹⁾	0.20607 ⁽⁹⁾	0.2956 ⁽¹³⁾	0.19562 ⁽⁷⁾	0.28822 ⁽¹²⁾
	MSE	$\hat{\lambda}$	0.03532 ⁽¹⁾	0.06057 ⁽⁵⁾	0.06643 ⁽⁶⁾	0.07059 ⁽⁸⁾	0.04967 ⁽⁴⁾	0.04684 ⁽³⁾	0.08218 ⁽¹¹⁾	0.08143 ⁽¹⁰⁾	0.07277 ⁽⁹⁾	0.15493 ⁽¹³⁾	0.06718 ⁽⁷⁾	0.14684 ⁽¹²⁾
	MRE	$\hat{\lambda}$	0.05888 ⁽¹⁾	0.0771 ⁽⁵⁾	0.07798 ⁽⁶⁾	0.08214 ⁽⁸⁾	0.07144 ⁽⁴⁾	0.06664 ⁽³⁾	0.08675 ⁽¹⁰⁾	0.08775 ⁽¹¹⁾	0.08243 ⁽⁹⁾	0.11824 ⁽¹³⁾	0.07825 ⁽⁷⁾	0.11529 ⁽¹²⁾
	<i>D</i> _{abs}	$\hat{\lambda}$	0.0063 ⁽¹⁾	0.00822 ⁽⁷⁾	0.00783 ⁽⁵⁾	0.00854 ⁽⁸⁾	0.00771 ⁽⁴⁾	0.00705 ⁽³⁾	0.00889 ⁽⁹⁾	0.00894 ⁽¹¹⁾	0.00893 ⁽¹⁰⁾	0.0115 ⁽¹³⁾	0.00792 ⁽⁶⁾	0.01107 ⁽¹²⁾
	<i>D</i> _{max}	$\hat{\lambda}$	0.01106 ⁽¹⁾	0.01472 ⁽⁷⁾	0.01445 ⁽⁵⁾	0.0155 ⁽⁸⁾	0.01373 ⁽⁴⁾	0.01254 ⁽³⁾	0.0163 ⁽¹⁰⁾	0.01643 ⁽¹¹⁾	0.01608 ⁽⁹⁾	0.02168 ⁽¹³⁾	0.01457 ⁽⁶⁾	0.0211 ⁽¹²⁾
	ASAE	$\hat{\lambda}$	0.0137 ⁽⁵⁾	0.01385 ⁽⁶⁾	0.01355 ⁽⁴⁾	0.01404 ⁽⁷⁾	0.01287 ⁽¹¹⁾	0.0135 ⁽²⁾	0.01495 ⁽¹⁰⁾	0.01475 ⁽⁹⁾	0.01547 ⁽¹¹⁾	0.01672 ⁽¹³⁾	0.01444 ⁽⁸⁾	0.01621 ⁽¹²⁾
	Σ Ranks		10 ⁽¹⁾	35 ⁽⁶⁾	32 ⁽⁵⁾	47 ⁽⁸⁾	21 ⁽⁴⁾	17 ⁽³⁾	60 ⁽¹⁰⁾	63 ⁽¹¹⁾	57 ⁽⁹⁾	78 ⁽¹³⁾	41 ⁽⁷⁾	72 ⁽¹²⁾
200	BIAS	$\hat{\lambda}$	0.13142 ⁽¹⁾	0.16382 ⁽⁶⁾	0.15029 ⁽⁴⁾	0.18003 ⁽⁷⁾	0.16191 ⁽⁵⁾	0.14292 ⁽²⁾	0.19203 ⁽⁹⁾	0.19304 ⁽¹¹⁾	0.19249 ⁽¹⁰⁾	0.24102 ⁽¹²⁾	0.18104 ⁽⁸⁾	0.24638 ⁽¹³⁾
	MSE	$\hat{\lambda}$	0.02798 ⁽¹⁾	0.04489 ⁽⁶⁾	0.03808 ⁽⁴⁾	0.05443 ⁽⁷⁾	0.04138 ⁽⁵⁾	0.03414 ⁽²⁾	0.06268 ⁽¹⁰⁾	0.06305 ⁽¹¹⁾	0.06164 ⁽⁹⁾	0.0984 ⁽¹²⁾	0.0546 ⁽⁸⁾	0.10332 ⁽¹³⁾
	MRE	$\hat{\lambda}$	0.05257 ⁽¹⁾	0.06553 ⁽⁶⁾	0.06012 ⁽⁴⁾	0.07201 ⁽⁷⁾	0.06476 ⁽⁵⁾	0.05717 ⁽²⁾	0.07681 ⁽⁹⁾	0.07722 ⁽¹¹⁾	0.077 ⁽¹⁰⁾	0.09641 ⁽¹²⁾	0.07242 ⁽⁸⁾	0.09855 ⁽¹³⁾
	<i>D</i> _{abs}	$\hat{\lambda}$	0.00563 ⁽¹⁾	0.00694 ⁽⁵⁾	0.00609 ⁽²⁾	0.00751 ⁽⁸⁾	0.007 ⁽⁶⁾	0.00613 ⁽³⁾	0.00791 ⁽¹⁰⁾	0.00782 ⁽⁹⁾	0.00829 ⁽¹¹⁾	0.00949 ⁽¹²⁾	0.00739 ⁽⁷⁾	0.00963 ⁽¹³⁾
	<i>D</i> _{max}	$\hat{\lambda}$	0.00984 ⁽¹⁾	0.01234 ⁽⁵⁾	0.0111 ⁽³⁾	0.01354 ⁽⁸⁾	0.01236 ⁽⁶⁾	0.01074 ⁽²⁾	0.01439 ⁽¹⁰⁾	0.01435 ⁽⁹⁾	0.01482 ⁽¹¹⁾	0.01773 ⁽¹²⁾	0.01349 ⁽⁷⁾	0.0181 ⁽¹³⁾
	ASAE	$\hat{\lambda}$	0.0115 ⁽⁴⁾	0.01169 ⁽⁷⁾	0.01164 ⁽⁵⁾	0.01166 ⁽⁶⁾	0.01083 ⁽¹¹⁾	0.01119 ⁽³⁾	0.01255 ⁽¹⁰⁾	0.01222 ⁽⁹⁾	0.01336 ⁽¹¹⁾	0.01378 ⁽¹³⁾	0.01207 ⁽⁸⁾	0.01361 ⁽¹²⁾
	Σ Ranks		9 ⁽¹⁾	35 ⁽⁶⁾	22 ⁽⁴⁾	43 ⁽⁷⁾	28 ⁽⁵⁾	14 ⁽²⁾	58 ⁽⁹⁾	60 ⁽¹⁰⁾	62 ⁽¹¹⁾	73 ⁽¹²⁾	46 ⁽⁸⁾	77 ⁽¹³⁾
300	BIAS	$\hat{\lambda}$	0.10498 ⁽¹⁾	0.13977 ⁽⁸⁾	0.1253 ⁽⁴⁾	0.1378 ⁽⁶⁾	0.13196 ⁽⁵⁾	0.11988 ⁽³⁾	0.16331 ⁽¹¹⁾	0.15245 ⁽⁹⁾	0.15641 ⁽¹⁰⁾	0.18643 ⁽¹³⁾	0.13827 ⁽⁷⁾	0.18391 ⁽¹²⁾
	MSE	$\hat{\lambda}$	0.01691 ⁽¹⁾	0.02224 ⁽²⁾	0.031 ⁽⁶⁾	0.03243 ⁽⁷⁾	0.02777 ⁽⁵⁾	0.02275 ⁽³⁾	0.04473 ⁽¹¹⁾	0.03998 ⁽⁹⁾	0.04009 ⁽¹⁰⁾	0.05759 ⁽¹²⁾	0.03362 ⁽⁸⁾	0.05832 ⁽¹³⁾
	MRE	$\hat{\lambda}$	0.04199 ⁽¹⁾	0.05591 ⁽⁸⁾	0.05012 ⁽⁴⁾	0.05512 ⁽⁶⁾	0.05279 ⁽⁵⁾	0.04795 ⁽³⁾	0.06532 ⁽¹¹⁾	0.06098 ⁽⁹⁾	0.06257 ⁽¹⁰⁾	0.07457 ⁽¹³⁾	0.05531 ⁽⁷⁾	0.07357 ⁽¹²⁾
	<i>D</i> _{abs}	$\hat{\lambda}$	0.00458 ⁽¹⁾	0.00498 ⁽²⁾	0.00517 ⁽⁴⁾	0.00573 ⁽⁷⁾	0.00565 ⁽⁵⁾	0.00509 ⁽³⁾	0.0067 ⁽¹⁰⁾	0.00623 ⁽⁹⁾	0.00672 ⁽¹¹⁾	0.00753 ⁽¹³⁾	0.00568 ⁽⁶⁾	0.00737 ⁽¹²⁾
	<i>D</i> _{max}	$\hat{\lambda}$	0.00784 ⁽¹⁾	0.00873 ⁽²⁾	0.01054 ⁽⁸⁾	0.01023 ⁽⁷⁾	0.0099 ⁽⁵⁾	0.0089 ⁽³⁾	0.01213 ⁽¹¹⁾	0.01127 ⁽⁹⁾	0.01183 ⁽¹⁰⁾	0.01381 ⁽¹³⁾	0.01022 ⁽⁶⁾	0.01358 ⁽¹²⁾
	ASAE	$\hat{\lambda}$	0.00897 ^(6,5)	0.00882 ^(4,5)	0.00882 ^(4,5)	0.00881 ⁽³⁾	0.00835 ⁽¹¹⁾	0.00861 ⁽²⁾	0.00968 ⁽¹⁰⁾	0.00952 ⁽⁹⁾	0.00996 ⁽¹¹⁾	0.01043 ⁽¹²⁾	0.00916 ⁽⁸⁾	0.01046 ⁽¹³⁾
	Σ Ranks		11.5 ⁽¹⁾	44.5 ⁽⁸⁾	24.5 ⁽⁴⁾	36 ⁽⁶⁾	26 ⁽⁵⁾	17 ⁽³⁾	64 ⁽¹¹⁾	54 ⁽⁹⁾	62 ⁽¹⁰⁾	76 ⁽¹³⁾	42 ⁽⁷⁾	74 ⁽¹²⁾
400	BIAS	$\hat{\lambda}$	0.0932 ⁽¹⁾	0.10554 ⁽³⁾	0.10743 ⁽⁴⁾	0.12286 ⁽⁷⁾	0.11733 ⁽⁵⁾	0.10168 ⁽²⁾	0.13684 ⁽⁹⁾	0.13934 ⁽¹⁰⁾	0.13938 ⁽¹¹⁾	0.16343 ⁽¹³⁾	0.12648 ⁽⁸⁾	0.15017 ⁽¹²⁾
	MSE	$\hat{\lambda}$	0.01334 ⁽¹⁾	0.02565 ⁽⁷⁾	0.01873 ⁽⁴⁾	0.02542 ⁽⁶⁾	0.02163 ⁽⁵⁾	0.01643 ⁽²⁾	0.03096 ⁽⁹⁾	0.03186 ⁽¹¹⁾	0.03139 ⁽¹⁰⁾	0.04504 ⁽¹³⁾	0.02649 ⁽⁸⁾	0.03756 ⁽¹²⁾
	MRE	$\hat{\lambda}$	0.03728 ⁽¹⁾	0.04222 ⁽³⁾	0.04877 ⁽⁶⁾	0.04914 ⁽⁷⁾	0.04693 ⁽⁵⁾	0.04067 ⁽²⁾	0.05474 ⁽⁹⁾	0.05574 ⁽¹⁰⁾	0.05575 ⁽¹¹⁾	0.06537 ⁽¹³⁾	0.05059 ⁽⁸⁾	0.06007 ⁽¹²⁾
	<i>D</i> _{abs}	$\hat{\lambda}$	0.00401 ⁽¹⁾	0.00452 ⁽⁴⁾	0.00512 ⁽⁶⁾	0.00514 ⁽⁷⁾	0.00499 ⁽⁵⁾	0.00439 ⁽²⁾	0.00573 ⁽⁹⁾	0.00574 ⁽¹⁰⁾	0.00594 ⁽¹¹⁾	0.00669 ⁽¹³⁾	0.00522 ⁽⁸⁾	0.0062 ⁽¹²⁾
	<i>D</i> _{max}	$\hat{\lambda}$	0.00692 ⁽¹⁾	0.00786 ⁽³⁾	0.00793 ⁽⁴⁾	0.00911 ⁽⁷⁾	0.00875 ⁽⁵⁾	0.00758 ⁽²⁾	0.01019 ⁽⁹⁾	0.0103 ⁽¹⁰⁾	0.01046 ⁽¹¹⁾	0.01211 ⁽¹³⁾	0.00934 ⁽⁸⁾	0.01114 ⁽¹²⁾
	ASAE	$\hat{\lambda}$	0.00746 ⁽⁶⁾	0.00732 ⁽³⁾	0.00744 ⁽⁵⁾	0.0076 ⁽⁷⁾	0.00703 ⁽¹¹⁾	0.00741 ⁽⁴⁾	0.0083 ⁽¹⁰⁾	0.00801 ⁽⁹⁾	0.00845 ⁽¹¹⁾	0.00888 ⁽¹³⁾	0.00792 ⁽⁸⁾	0.00856 ⁽¹²⁾
	Σ Ranks		11 ⁽¹⁾	19 ⁽³⁾	21 ⁽⁴⁾	41 ⁽⁷⁾	26 ⁽⁵⁾	14 ⁽²⁾	55 ⁽⁹⁾	60 ⁽¹⁰⁾	65 ⁽¹¹⁾	78 ⁽¹³⁾	48 ⁽⁸⁾	72 ⁽¹²⁾

TABLE 5 | Numerical results of simulation for all measures when ($\lambda = 3.0$) under SRS.

n	Estimate	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	MSADE	MSALDE	KE	MSSDE	MSSLDE	MSLNDE
25	BIAS	$\hat{\lambda}$	0.63219 ⁽¹⁾	0.65459 ⁽²⁾	0.75107 ⁽¹⁰⁾	0.68613 ⁽⁴⁾	0.7263 ⁽⁹⁾	0.69494 ⁽⁶⁾	0.70512 ⁽⁷⁾	0.75353 ⁽¹¹⁾	0.70667 ⁽⁸⁾	0.93757 ⁽¹³⁾	0.65517 ⁽³⁾	0.89887 ⁽¹²⁾
	MSE	$\hat{\lambda}$	0.63227 ⁽¹⁾	0.75924 ⁽³⁾	0.9655 ⁽¹⁰⁾	0.78801 ⁽⁵⁾	0.90917 ⁽⁸⁾	0.76077 ⁽⁴⁾	0.82136 ⁽⁷⁾	0.99714 ⁽¹¹⁾	0.91982 ⁽⁹⁾	1.39674 ⁽¹³⁾	0.73583 ⁽²⁾	1.30193 ⁽¹²⁾
	MRE	$\hat{\lambda}$	0.21073 ⁽¹⁾	0.2182 ⁽²⁾	0.25036 ⁽¹⁰⁾	0.22871 ⁽⁴⁾	0.2421 ⁽⁹⁾	0.23165 ⁽⁶⁾	0.23504 ⁽⁷⁾	0.25118 ⁽¹¹⁾	0.23556 ⁽⁸⁾	0.31252 ⁽¹³⁾	0.21839 ⁽³⁾	0.29962 ⁽¹²⁾
	D_{abs}		0.02816 ⁽¹⁰⁾	0.02493 ⁽²⁾	0.02921 ⁽¹¹⁾	0.02533 ⁽³⁾	0.02783 ⁽⁸⁾	0.0267 ⁽⁶⁾	0.02632 ⁽⁴⁾	0.02801 ⁽⁹⁾	0.02756 ⁽⁷⁾	0.03461 ⁽¹³⁾	0.02468 ⁽¹⁾	0.03296 ⁽¹²⁾
	D_{max}		0.05294 ⁽¹¹⁾	0.04316 ⁽¹⁾	0.04963 ⁽¹⁰⁾	0.04506 ⁽³⁾	0.04776 ⁽⁸⁾	0.04689 ⁽⁶⁾	0.04699 ⁽⁷⁾	0.04936 ⁽⁹⁾	0.04676 ⁽⁵⁾	0.05989 ⁽¹³⁾	0.04376 ⁽²⁾	0.05746 ⁽¹²⁾
	ASAE		0.05402 ⁽⁹⁾	0.04812 ⁽²⁾	0.0515 ⁽⁶⁾	0.05004 ⁽⁴⁾	0.05135 ⁽⁵⁾	0.04899 ⁽³⁾	0.05345 ⁽⁸⁾	0.05439 ⁽¹⁰⁾	0.05597 ⁽¹¹⁾	0.06271 ⁽¹³⁾	0.05209 ⁽⁷⁾	0.06233 ⁽¹²⁾
	$\sum$ Ranks		33 ⁽⁶⁾	12 ⁽¹⁾	57 ⁽¹⁰⁾	23 ⁽³⁾	47 ⁽⁸⁾	29 ⁽⁵⁾	40 ⁽⁷⁾	61 ⁽¹¹⁾	48 ⁽⁹⁾	78 ⁽¹³⁾	18 ⁽²⁾	72 ⁽¹²⁾
70	BIAS	$\hat{\lambda}$	0.36147 ⁽¹⁾	0.43252 ⁽³⁾	0.51108 ⁽¹¹⁾	0.43088 ⁽²⁾	0.49849 ⁽¹⁰⁾	0.45285 ⁽⁴⁾	0.4645 ⁽⁷⁾	0.46661 ⁽⁸⁾	0.49787 ⁽⁹⁾	0.57211 ⁽¹²⁾	0.45474 ⁽⁵⁾	0.61157 ⁽¹³⁾
	MSE	$\hat{\lambda}$	0.20497 ⁽¹⁾	0.31943 ⁽⁴⁾	0.42081 ⁽¹⁰⁾	0.30457 ⁽²⁾	0.4189 ⁽⁹⁾	0.30823 ⁽³⁾	0.36013 ⁽⁷⁾	0.36779 ⁽⁸⁾	0.43844 ⁽¹¹⁾	0.53394 ⁽¹²⁾	0.33221 ⁽⁵⁾	0.61308 ⁽¹³⁾
	MRE	$\hat{\lambda}$	0.12049 ⁽¹⁾	0.14417 ⁽³⁾	0.17036 ⁽¹¹⁾	0.14363 ⁽²⁾	0.16616 ⁽¹⁰⁾	0.15095 ⁽⁴⁾	0.15483 ⁽⁷⁾	0.15554 ⁽⁸⁾	0.16596 ⁽⁹⁾	0.1907 ⁽¹²⁾	0.15158 ⁽⁵⁾	0.20386 ⁽¹³⁾
	D_{abs}		0.01317 ⁽¹⁾	0.01591 ⁽³⁾	0.01897 ⁽¹¹⁾	0.0155 ⁽²⁾	0.01848 ⁽⁹⁾	0.01661 ⁽⁵⁾	0.01685 ⁽⁷⁾	0.01676 ⁽⁶⁾	0.01866 ⁽¹⁰⁾	0.02073 ⁽¹²⁾	0.01649 ⁽⁴⁾	0.0221 ⁽¹³⁾
	D_{max}		0.02508 ⁽¹⁾	0.02929 ⁽³⁾	0.03448 ⁽¹¹⁾	0.02895 ⁽²⁾	0.0335 ⁽⁹⁾	0.03098 ⁽⁵⁾	0.03143 ⁽⁸⁾	0.03126 ⁽⁶⁾	0.03353 ⁽¹⁰⁾	0.0379 ⁽¹²⁾	0.03064 ⁽⁴⁾	0.04025 ⁽¹³⁾
	ASAE		0.02518 ⁽⁵⁾	0.02462 ⁽⁴⁾	0.0253 ⁽⁶⁾	0.0245 ⁽³⁾	0.02579 ⁽⁷⁾	0.02415 ⁽²⁾	0.02742 ⁽⁹⁾	0.02752 ⁽¹⁰⁾	0.02807 ⁽¹¹⁾	0.03046 ⁽¹²⁾	0.02653 ⁽⁸⁾	0.03092 ⁽¹³⁾
	$\sum$ Ranks		10 ⁽¹⁾	20 ⁽³⁾	60 ⁽¹⁰⁵⁾	13 ⁽²⁾	54 ⁽⁹⁾	22 ⁽⁴⁾	45 ⁽⁷⁾	46 ⁽⁸⁾	60 ⁽¹⁰⁵⁾	72 ⁽¹²⁾	31 ⁽⁵⁾	78 ⁽¹³⁾
150	BIAS	$\hat{\lambda}$	0.26892 ⁽¹⁾	0.31937 ⁽⁶⁾	0.36132 ⁽⁹⁾	0.29551 ⁽³⁾	0.36567 ⁽¹⁰⁾	0.30814 ⁽⁴⁾	0.31294 ⁽⁵⁾	0.32881 ⁽⁷⁾	0.38097 ⁽¹¹⁾	0.41305 ⁽¹³⁾	0.29191 ⁽²⁾	0.39775 ⁽¹²⁾
	MSE	$\hat{\lambda}$	0.11187 ⁽¹⁾	0.16706 ⁽⁶⁾	0.20793 ⁽⁹⁾	0.13701 ⁽²⁾	0.21395 ⁽¹⁰⁾	0.14911 ⁽⁴⁾	0.15995 ⁽⁵⁾	0.16934 ⁽⁷⁾	0.23854 ⁽¹¹⁾	0.26761 ⁽¹³⁾	0.13799 ⁽³⁾	0.24889 ⁽¹²⁾
	MRE	$\hat{\lambda}$	0.08964 ⁽¹⁾	0.10646 ⁽⁶⁾	0.12044 ⁽⁹⁾	0.0985 ⁽³⁾	0.12189 ⁽¹⁰⁾	0.10271 ⁽⁴⁾	0.10431 ⁽⁵⁾	0.1096 ⁽⁷⁾	0.12699 ⁽¹¹⁾	0.13768 ⁽¹³⁾	0.0973 ⁽²⁾	0.13258 ⁽¹²⁾
	D_{abs}		0.00948 ⁽¹⁾	0.01146 ⁽⁶⁾	0.01297 ⁽⁹⁾	0.01037 ⁽³⁾	0.01318 ⁽¹⁰⁾	0.01096 ⁽⁴⁾	0.01107 ⁽⁵⁾	0.01162 ⁽⁷⁾	0.01386 ⁽¹¹⁾	0.01476 ⁽¹³⁾	0.01027 ⁽²⁾	0.01416 ⁽¹²⁾
	D_{max}		0.01862 ⁽¹⁾	0.02176 ⁽⁶⁾	0.02464 ⁽⁹⁾	0.02012 ⁽³⁾	0.0249 ⁽¹⁰⁾	0.02121 ⁽⁴⁾	0.02137 ⁽⁵⁾	0.02239 ⁽⁷⁾	0.02592 ⁽¹¹⁾	0.02785 ⁽¹³⁾	0.01997 ⁽²⁾	0.02685 ⁽¹²⁾
	ASAE		0.01555 ⁽⁶⁾	0.0146 ⁽²⁾	0.01563 ⁽⁷⁾	0.01526 ⁽⁴⁾	0.01537 ⁽⁵⁾	0.01404 ⁽¹⁾	0.0169 ⁽¹⁰⁾	0.01627 ⁽⁹⁾	0.01788 ⁽¹¹⁾	0.01877 ⁽¹³⁾	0.01616 ⁽⁸⁾	0.01853 ⁽¹²⁾
	$\sum$ Ranks		11 ⁽¹⁾	32 ⁽⁵⁾	52 ⁽⁹⁾	18 ⁽²⁾	55 ⁽¹⁰⁾	21 ⁽⁴⁾	35 ⁽⁶⁾	44 ⁽⁸⁾	66 ⁽¹¹⁾	78 ⁽¹³⁾	19 ⁽³⁾	72 ⁽¹²⁾
200	BIAS	$\hat{\lambda}$	0.23995 ⁽¹⁾	0.27694 ⁽⁴⁾	0.32129 ⁽¹⁰⁾	0.24802 ⁽²⁾	0.31731 ⁽⁹⁾	0.27798 ⁽⁵⁾	0.28305 ⁽⁶⁾	0.2862 ⁽⁷⁾	0.32568 ⁽¹¹⁾	0.34634 ⁽¹²⁾	0.24901 ⁽³⁾	0.36567 ⁽¹³⁾
	MSE	$\hat{\lambda}$	0.08715 ⁽¹⁾	0.12136 ⁽⁵⁾	0.15821 ⁽⁹⁾	0.09611 ⁽²⁾	0.15855 ⁽¹⁰⁾	0.12011 ⁽⁴⁾	0.13323 ⁽⁸⁾	0.12996 ⁽⁶⁾	0.17709 ⁽¹¹⁾	0.18656 ⁽¹²⁾	0.10065 ⁽³⁾	0.20486 ⁽¹³⁾
	MRE	$\hat{\lambda}$	0.07998 ⁽¹⁾	0.09231 ⁽⁴⁾	0.1071 ⁽¹⁰⁾	0.08267 ⁽²⁾	0.10577 ⁽⁹⁾	0.09266 ⁽⁵⁾	0.09435 ⁽⁶⁾	0.0954 ⁽⁷⁾	0.10856 ⁽¹¹⁾	0.11545 ⁽¹²⁾	0.083 ⁽³⁾	0.12189 ⁽¹³⁾
	D_{abs}		0.00837 ⁽¹⁾	0.0098 ⁽⁴⁾	0.0114 ⁽¹⁰⁾	0.00864 ⁽²⁾	0.0113 ⁽⁹⁾	0.00982 ⁽⁵⁾	0.00996 ⁽⁶⁾	0.01 ⁽⁷⁾	0.0117 ⁽¹¹⁾	0.01231 ⁽¹²⁾	0.00867 ⁽³⁾	0.01299 ⁽¹³⁾
	D_{max}		0.01661 ⁽¹⁾	0.01897 ⁽⁴⁾	0.02203 ⁽¹⁰⁾	0.01694 ⁽²⁾	0.02166 ⁽⁹⁾	0.01912 ⁽⁵⁾	0.01935 ⁽⁶⁾	0.01952 ⁽⁷⁾	0.02227 ⁽¹¹⁾	0.0235 ⁽¹²⁾	0.01706 ⁽³⁾	0.0248 ⁽¹³⁾
	ASAE		0.01256 ⁽⁵⁾	0.01197 ⁽²⁾	0.01295 ⁽⁶⁾	0.01234 ⁽⁴⁾	0.01298 ⁽⁷⁾	0.01218 ⁽³⁾	0.01433 ⁽¹⁰⁾	0.01398 ⁽⁹⁾	0.0149 ⁽¹¹⁾	0.01541 ⁽¹²⁾	0.0135 ⁽⁸⁾	0.01591 ⁽¹³⁾
	$\sum$ Ranks		10 ⁽¹⁾	23 ^(3.5)	55 ⁽¹⁰⁾	14 ⁽²⁾	53 ⁽⁹⁾	25 ⁽⁵⁾	42 ^(6.5)	43 ⁽⁸⁾	66 ⁽¹¹⁾	72 ⁽¹²⁾	23 ^(3.5)	78 ⁽¹³⁾

(Continues)

TABLE 6 | (Continued)

<i>n</i>		Estimate	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	MSADE	MSALDE	KE	MSSDE	MSSLDE	MSLNDE
150	BIAS	$\hat{\lambda}$	0.2014 ⁽¹¹⁾	0.22122 ⁽⁴⁾	0.24406 ⁽⁷⁾	0.2425 ⁽⁶⁾	0.23657 ⁽⁵⁾	0.21443 ⁽²⁾	0.21992 ⁽³⁾	0.30365 ⁽¹¹⁾	0.28912 ⁽¹⁰⁾	0.2682 ⁽⁸⁾	0.36181 ⁽¹³⁾	0.26901 ⁽⁹⁾	0.35825 ⁽¹²⁾
	MSE	$\hat{\lambda}$	0.06308 ⁽¹⁾	0.07481 ⁽³⁾	0.09216 ⁽⁶⁾	0.093 ⁽⁷⁾	0.08742 ⁽⁵⁾	0.07191 ⁽²⁾	0.07579 ⁽⁴⁾	0.15028 ⁽¹¹⁾	0.13752 ⁽¹⁰⁾	0.1145 ⁽⁸⁾	0.20926 ⁽¹³⁾	0.11852 ⁽⁹⁾	0.20124 ⁽¹²⁾
	MRE	$\hat{\lambda}$	0.06713 ⁽¹⁾	0.07374 ⁽⁴⁾	0.08135 ⁽⁷⁾	0.08083 ⁽⁶⁾	0.07886 ⁽⁵⁾	0.07148 ⁽²⁾	0.07331 ⁽³⁾	0.10122 ⁽¹¹⁾	0.09637 ⁽¹⁰⁾	0.0894 ⁽⁸⁾	0.1206 ⁽¹³⁾	0.08967 ⁽⁹⁾	0.11942 ⁽¹²⁾
	D_{abs}		0.007 ⁽¹⁾	0.00771 ⁽⁴⁾	0.00852 ⁽⁷⁾	0.00843 ⁽⁶⁾	0.00825 ⁽⁵⁾	0.00745 ⁽²⁾	0.00767 ⁽³⁾	0.01075 ⁽¹¹⁾	0.01015 ⁽¹⁰⁾	0.00944 ⁽⁹⁾	0.01283 ⁽¹³⁾	0.00939 ⁽⁸⁾	0.01266 ⁽¹²⁾
	D_{max}		0.01395 ⁽¹⁾	0.01526 ⁽⁴⁾	0.01684 ⁽⁷⁾	0.01656 ⁽⁶⁾	0.01632 ⁽⁵⁾	0.0148 ⁽²⁾	0.01517 ⁽³⁾	0.02074 ⁽¹¹⁾	0.0197 ⁽¹⁰⁾	0.01847 ⁽⁹⁾	0.02446 ⁽¹³⁾	0.01836 ⁽⁸⁾	0.02421 ⁽¹²⁾
	ASAE		0.01297 ⁽³⁾	0.01264 ⁽²⁾	0.01345 ⁽⁷⁾	0.01325 ⁽⁵⁾	0.01326 ⁽⁶⁾	0.01247 ⁽¹⁾	0.01302 ⁽⁴⁾	0.01542 ⁽¹¹⁾	0.01461 ⁽⁹⁾	0.0151 ⁽¹⁰⁾	0.01681 ⁽¹³⁾	0.01408 ⁽⁸⁾	0.01678 ⁽¹²⁾
	$\sum$ Ranks		8 ⁽¹⁾	21 ⁽⁴⁾	41 ⁽⁷⁾	36 ⁽⁶⁾	31 ⁽⁵⁾	11 ⁽²⁾	20 ⁽³⁾	66 ⁽¹¹⁾	59 ⁽¹⁰⁾	52 ⁽⁹⁾	78 ⁽¹³⁾	51 ⁽⁸⁾	72 ⁽¹²⁾
200	BIAS	$\hat{\lambda}$	0.16993 ⁽¹⁾	0.1893 ⁽³⁾	0.21831 ⁽⁷⁾	0.20987 ⁽⁵⁾	0.21656 ⁽⁶⁾	0.18711 ⁽²⁾	0.19602 ⁽⁴⁾	0.2543 ⁽¹⁰⁾	0.26361 ⁽¹¹⁾	0.24189 ⁽⁹⁾	0.32698 ⁽¹³⁾	0.2383 ⁽⁸⁾	0.30847 ⁽¹²⁾
	MSE	$\hat{\lambda}$	0.04499 ⁽¹⁾	0.05581 ⁽³⁾	0.0711 ⁽⁶⁾	0.06804 ⁽⁵⁾	0.07282 ⁽⁷⁾	0.05419 ⁽²⁾	0.05996 ⁽⁴⁾	0.10483 ⁽¹⁰⁾	0.10846 ⁽¹¹⁾	0.09628 ⁽⁹⁾	0.16805 ⁽¹³⁾	0.09489 ⁽⁸⁾	0.15129 ⁽¹²⁾
	MRE	$\hat{\lambda}$	0.05664 ⁽¹⁾	0.0631 ⁽³⁾	0.07277 ⁽⁷⁾	0.06996 ⁽⁵⁾	0.07219 ⁽⁶⁾	0.06237 ⁽²⁾	0.06534 ⁽⁴⁾	0.08477 ⁽¹⁰⁾	0.08787 ⁽¹¹⁾	0.08063 ⁽⁹⁾	0.10899 ⁽¹³⁾	0.07943 ⁽⁸⁾	0.10282 ⁽¹²⁾
	D_{abs}		0.00585 ⁽¹⁾	0.00656 ⁽³⁾	0.00755 ⁽⁷⁾	0.00726 ⁽⁵⁾	0.00752 ⁽⁶⁾	0.00645 ⁽²⁾	0.00677 ⁽⁴⁾	0.0089 ⁽¹⁰⁾	0.00918 ⁽¹¹⁾	0.00846 ⁽⁹⁾	0.01157 ⁽¹³⁾	0.00831 ⁽⁸⁾	0.01088 ⁽¹²⁾
	D_{max}		0.01175 ⁽¹⁾	0.01308 ⁽³⁾	0.01506 ⁽⁷⁾	0.01436 ⁽⁵⁾	0.01489 ⁽⁶⁾	0.01292 ⁽²⁾	0.01353 ⁽⁴⁾	0.01744 ⁽¹⁰⁾	0.018 ⁽¹¹⁾	0.01665 ⁽⁹⁾	0.02222 ⁽¹³⁾	0.01631 ⁽⁸⁾	0.02095 ⁽¹²⁾
	ASAE		0.01081 ⁽⁵⁾	0.01065 ⁽²⁾	0.01118 ⁽⁶⁾	0.01072 ⁽⁴⁾	0.01122 ⁽⁷⁾	0.01041 ⁽¹⁾	0.01068 ⁽³⁾	0.01274 ⁽¹⁰⁾	0.0123 ⁽⁹⁾	0.01297 ⁽¹¹⁾	0.01398 ⁽¹²⁾	0.01177 ⁽⁸⁾	0.01399 ⁽¹³⁾
	$\sum$ Ranks		10 ⁽¹⁾	17 ⁽³⁾	40 ⁽⁷⁾	29 ⁽⁵⁾	38 ⁽⁶⁾	11 ⁽²⁾	23 ⁽⁴⁾	60 ⁽¹⁰⁾	64 ⁽¹¹⁾	56 ⁽⁹⁾	77 ⁽¹³⁾	48 ⁽⁸⁾	73 ⁽¹²⁾
300	BIAS	$\hat{\lambda}$	0.13939 ⁽¹⁾	0.14843 ⁽²⁾	0.17057 ⁽⁷⁾	0.16329 ⁽⁵⁾	0.16881 ⁽⁶⁾	0.16151 ⁽⁴⁾	0.15781 ⁽³⁾	0.21388 ⁽¹¹⁾	0.20669 ⁽¹⁰⁾	0.20234 ⁽⁹⁾	0.26025 ⁽¹³⁾	0.19654 ⁽⁸⁾	0.25368 ⁽¹²⁾
	MSE	$\hat{\lambda}$	0.03042 ⁽¹⁾	0.03513 ⁽²⁾	0.04596 ⁽⁷⁾	0.04122 ⁽⁵⁾	0.04466 ⁽⁶⁾	0.0402 ⁽⁴⁾	0.03832 ⁽³⁾	0.07555 ⁽¹¹⁾	0.07108 ⁽¹⁰⁾	0.06971 ⁽⁹⁾	0.10563 ⁽¹³⁾	0.06105 ⁽⁸⁾	0.09942 ⁽¹²⁾
	MRE	$\hat{\lambda}$	0.04646 ⁽¹⁾	0.04948 ⁽²⁾	0.05686 ⁽⁷⁾	0.05443 ⁽⁵⁾	0.05627 ⁽⁶⁾	0.05384 ⁽⁴⁾	0.0526 ⁽³⁾	0.07129 ⁽¹¹⁾	0.0689 ⁽¹⁰⁾	0.06745 ⁽⁹⁾	0.08675 ⁽¹³⁾	0.06551 ⁽⁸⁾	0.08456 ⁽¹²⁾
	D_{abs}		0.00477 ⁽¹⁾	0.0051 ⁽²⁾	0.00589 ⁽⁷⁾	0.00564 ⁽⁵⁾	0.00581 ⁽⁶⁾	0.00554 ⁽⁴⁾	0.00541 ⁽³⁾	0.00742 ⁽¹¹⁾	0.0072 ⁽¹⁰⁾	0.007 ⁽⁹⁾	0.00907 ⁽¹³⁾	0.0068 ⁽⁸⁾	0.00883 ⁽¹²⁾
	D_{max}		0.00965 ⁽¹⁾	0.01026 ⁽²⁾	0.01181 ⁽⁷⁾	0.01121 ⁽⁵⁾	0.01164 ⁽⁶⁾	0.01117 ⁽⁴⁾	0.01091 ⁽³⁾	0.01469 ⁽¹¹⁾	0.01417 ⁽¹⁰⁾	0.01394 ⁽⁹⁾	0.01776 ⁽¹³⁾	0.01348 ⁽⁸⁾	0.01739 ⁽¹²⁾
	ASAE		0.00842 ⁽⁵⁾	0.00809 ⁽¹⁾	0.00865 ⁽⁶⁾	0.00831 ⁽⁴⁾	0.00875 ⁽⁷⁾	0.00813 ⁽²⁾	0.00822 ⁽³⁾	0.00983 ⁽¹⁰⁾	0.00952 ⁽⁹⁾	0.01018 ⁽¹¹⁾	0.01099 ⁽¹³⁾	0.00919 ⁽⁸⁾	0.01071 ⁽¹²⁾
	$\sum$ Ranks		10 ⁽¹⁾	11 ⁽²⁾	41 ⁽⁷⁾	29 ⁽⁵⁾	37 ⁽⁶⁾	22 ⁽⁴⁾	18 ⁽³⁾	65 ⁽¹¹⁾	59 ⁽¹⁰⁾	56 ⁽⁹⁾	78 ⁽¹³⁾	48 ⁽⁸⁾	72 ⁽¹²⁾
400	BIAS	$\hat{\lambda}$	0.12307 ⁽¹⁾	0.13568 ⁽³⁾	0.14698 ⁽⁶⁾	0.14183 ⁽⁵⁾	0.14943 ⁽⁷⁾	0.135 ⁽²⁾	0.13655 ⁽⁴⁾	0.1853 ⁽¹¹⁾	0.18288 ⁽¹⁰⁾	0.16716 ⁽⁹⁾	0.21775 ⁽¹³⁾	0.1637 ⁽⁸⁾	0.21601 ⁽¹²⁾
	MSE	$\hat{\lambda}$	0.02353 ⁽¹⁾	0.02888 ⁽³⁾	0.03436 ⁽⁶⁾	0.03213 ⁽⁵⁾	0.03488 ⁽⁷⁾	0.02785 ⁽²⁾	0.02966 ⁽⁴⁾	0.05606 ⁽¹¹⁾	0.05309 ⁽¹⁰⁾	0.0459 ⁽⁹⁾	0.07592 ⁽¹³⁾	0.0434 ⁽⁸⁾	0.07399 ⁽¹²⁾
	MRE	$\hat{\lambda}$	0.04102 ⁽¹⁾	0.04523 ⁽³⁾	0.04899 ⁽⁶⁾	0.04728 ⁽⁵⁾	0.04981 ⁽⁷⁾	0.045 ⁽²⁾	0.04552 ⁽⁴⁾	0.06177 ⁽¹¹⁾	0.06096 ⁽¹⁰⁾	0.05572 ⁽⁹⁾	0.07258 ⁽¹³⁾	0.05457 ⁽⁸⁾	0.072 ⁽¹²⁾
	D_{abs}		0.00419 ⁽¹⁾	0.00464 ⁽³⁾	0.00505 ⁽⁶⁾	0.00485 ⁽⁵⁾	0.00512 ⁽⁷⁾	0.00461 ⁽²⁾	0.00468 ⁽⁴⁾	0.0064 ⁽¹¹⁾	0.0063 ⁽¹⁰⁾	0.00577 ⁽⁹⁾	0.00755 ⁽¹³⁾	0.00562 ⁽⁸⁾	0.00751 ⁽¹²⁾
	D_{max}		0.00853 ⁽¹⁾	0.00937 ⁽³⁾	0.01016 ⁽⁶⁾	0.00975 ⁽⁵⁾	0.01032 ⁽⁷⁾	0.00932 ⁽²⁾	0.00944 ⁽⁴⁾	0.01276 ⁽¹¹⁾	0.01256 ⁽¹⁰⁾	0.01156 ⁽⁹⁾	0.0149 ⁽¹³⁾	0.01127 ⁽⁸⁾	0.01479 ⁽¹²⁾
	ASAE		0.00702 ⁽⁴⁾	0.00689 ⁽³⁾	0.00726 ⁽⁷⁾	0.00706 ⁽⁵⁾	0.00725 ⁽⁶⁾	0.00666 ⁽¹⁾	0.00682 ⁽²⁾	0.00838 ⁽¹¹⁾	0.00802 ⁽⁹⁾	0.00836 ⁽¹⁰⁾	0.00903 ⁽¹²⁾	0.00777 ⁽⁸⁾	0.00917 ⁽¹³⁾
	$\sum$ Ranks		9 ⁽¹⁾	18 ⁽³⁾	37 ⁽⁶⁾	30 ⁽⁵⁾	41 ⁽⁷⁾	11 ⁽²⁾	22 ⁽⁴⁾	66 ⁽¹¹⁾	59 ⁽¹⁰⁾	55 ⁽⁹⁾	77 ⁽¹³⁾	48 ⁽⁸⁾	73 ⁽¹²⁾

TABLE 7 | Numerical results of simulation for all measures when ($\lambda = 3.5$) under SRS.

<i>n</i>	Estimate	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	MSADE	MSALDE	KE	MSSDE	MSSLDE	MSLNDE
25	BIAS	$\hat{\lambda}$	0.74142 ⁽¹⁾	0.81679 ⁽⁵⁾	0.905 ⁽¹¹⁾	0.81736 ⁽⁶⁾	0.81779 ⁽⁷⁾	0.81585 ⁽⁴⁾	0.78445 ⁽³⁾	0.87067 ⁽¹⁰⁾	0.8621 ⁽⁹⁾	1.07466 ⁽¹³⁾	0.779 ⁽²⁾	1.05604 ⁽¹²⁾
	MSE	$\hat{\lambda}$	0.86457 ⁽¹⁾	1.03211 ⁽⁵⁾	1.23948 ⁽¹¹⁾	1.09115 ⁽⁷⁾	1.04849 ⁽⁶⁾	1.01831 ⁽⁴⁾	0.98542 ⁽³⁾	1.22225 ⁽¹⁰⁾	1.16938 ⁽⁹⁾	1.76042 ⁽¹²⁾	0.98285 ⁽²⁾	1.78023 ⁽¹³⁾
	MRE	$\hat{\lambda}$	0.21183 ⁽¹⁾	0.23337 ⁽⁵⁾	0.25857 ⁽¹¹⁾	0.23353 ⁽⁶⁾	0.23365 ⁽⁷⁾	0.2331 ⁽⁴⁾	0.22413 ⁽³⁾	0.24876 ⁽¹⁰⁾	0.24631 ⁽⁹⁾	0.30705 ⁽¹³⁾	0.22257 ⁽²⁾	0.30172 ⁽¹²⁾
	D_{abs}		0.03005 ⁽⁴⁾	0.03039 ⁽⁷⁾	0.03309 ⁽¹¹⁾	0.03032 ⁽⁶⁾	0.02985 ⁽³⁾	0.03028 ⁽⁵⁾	0.02874 ⁽²⁾	0.03208 ⁽¹⁰⁾	0.03162 ⁽⁹⁾	0.03944 ⁽¹³⁾	0.02868 ⁽¹⁾	0.03877 ⁽¹²⁾
	D_{max}		0.05579 ⁽¹⁰⁾	0.05124 ⁽⁴⁾	0.05698 ⁽¹¹⁾	0.05038 ⁽³⁾	0.0513 ⁽⁵⁾	0.05168 ⁽⁶⁾	0.049 ⁽²⁾	0.05379 ⁽⁸⁾	0.05385 ⁽⁹⁾	0.06454 ⁽¹³⁾	0.04867 ⁽¹⁾	0.06316 ⁽¹²⁾
	ASAE		0.04957 ⁽⁷⁾	0.04623 ⁽²⁾	0.04942 ⁽⁶⁾	0.04715 ⁽³⁾	0.04792 ⁽⁵⁾	0.04469 ⁽¹⁾	0.05055 ⁽⁹⁾	0.05087 ⁽¹⁰⁾	0.05192 ⁽¹¹⁾	0.0596 ⁽¹²⁾	0.04975 ⁽⁸⁾	0.06015 ⁽¹³⁾
	$\sum$ Ranks		24 ^(3.5)	28 ⁽⁵⁾	61 ⁽¹¹⁾	31 ⁽⁶⁾	33 ⁽⁷⁾	24 ^(3.5)	22 ⁽²⁾	58 ⁽¹⁰⁾	56 ⁽⁹⁾	76 ⁽¹³⁾	16 ⁽¹⁾	74 ⁽¹²⁾
70	BIAS	$\hat{\lambda}$	0.45225 ⁽¹⁾	0.54595 ⁽⁸⁾	0.57934 ⁽¹⁰⁾	0.48893 ⁽²⁾	0.57197 ⁽⁹⁾	0.50102 ⁽³⁾	0.52258 ⁽⁵⁾	0.5455 ⁽⁷⁾	0.61381 ⁽¹¹⁾	0.7 ⁽¹³⁾	0.50979 ⁽⁴⁾	0.67241 ⁽¹²⁾
	MSE	$\hat{\lambda}$	0.31603 ⁽¹⁾	0.46027 ⁽⁷⁾	0.50811 ⁽¹⁰⁾	0.38674 ⁽²⁾	0.50182 ⁽⁹⁾	0.40295 ⁽³⁾	0.45328 ⁽⁶⁾	0.48706 ⁽⁸⁾	0.56817 ⁽¹¹⁾	0.78786 ⁽¹³⁾	0.41246 ⁽⁴⁾	0.70697 ⁽¹²⁾
	MRE	$\hat{\lambda}$	0.12922 ⁽¹⁾	0.15599 ⁽⁸⁾	0.16553 ⁽¹⁰⁾	0.13969 ⁽²⁾	0.16342 ⁽⁹⁾	0.14315 ⁽³⁾	0.14931 ⁽⁵⁾	0.15586 ⁽⁷⁾	0.17537 ⁽¹¹⁾	0.2 ⁽¹³⁾	0.14565 ⁽⁴⁾	0.19212 ⁽¹²⁾
	D_{abs}		0.01644 ⁽¹⁾	0.01992 ⁽⁷⁾	0.02106 ⁽¹⁰⁾	0.01814 ⁽²⁾	0.02094 ⁽⁹⁾	0.01827 ⁽³⁾	0.01924 ⁽⁵⁾	0.0202 ⁽⁸⁾	0.02251 ⁽¹¹⁾	0.02597 ⁽¹³⁾	0.01866 ⁽⁴⁾	0.02491 ⁽¹²⁾
	D_{max}		0.02951 ⁽¹⁾	0.03507 ⁽⁸⁾	0.03736 ⁽¹⁰⁾	0.031 ⁽²⁾	0.03655 ⁽⁹⁾	0.03217 ⁽³⁾	0.032 ⁽⁵⁾	0.03454 ⁽⁷⁾	0.03929 ⁽¹¹⁾	0.0436 ⁽¹³⁾	0.03255 ⁽⁴⁾	0.04199 ⁽¹²⁾
	ASAE		0.02392 ⁽⁵⁾	0.02347 ⁽⁴⁾	0.02463 ⁽⁷⁾	0.02323 ⁽²⁾	0.02462 ⁽⁶⁾	0.02216 ⁽¹⁾	0.02653 ⁽¹⁰⁾	0.0258 ⁽⁹⁾	0.02711 ⁽¹¹⁾	0.03034 ⁽¹³⁾	0.0251 ⁽⁸⁾	0.03033 ⁽¹²⁾
	$\sum$ Ranks		10 ⁽¹⁾	42 ⁽⁷⁾	57 ⁽¹⁰⁾	12 ⁽²⁾	51 ⁽⁹⁾	16 ⁽³⁾	36 ⁽⁶⁾	46 ⁽⁸⁾	66 ⁽¹¹⁾	78 ⁽¹³⁾	28 ⁽⁴⁾	72 ⁽¹²⁾
150	BIAS	$\hat{\lambda}$	0.3028 ⁽¹⁾	0.36439 ⁽⁶⁾	0.4243 ⁽¹⁰⁾	0.3354 ⁽²⁾	0.41581 ⁽⁹⁾	0.34134 ⁽³⁾	0.35893 ⁽⁵⁾	0.38057 ⁽⁷⁾	0.46227 ⁽¹²⁾	0.45627 ⁽¹¹⁾	0.3548 ⁽⁴⁾	0.48435 ⁽¹³⁾
	MSE	$\hat{\lambda}$	0.14315 ⁽¹⁾	0.20343 ⁽⁵⁾	0.27534 ⁽¹⁰⁾	0.17551 ⁽²⁾	0.25714 ⁽⁹⁾	0.1807 ⁽³⁾	0.20984 ⁽⁶⁾	0.23366 ⁽⁸⁾	0.3198 ⁽¹¹⁾	0.34239 ⁽¹²⁾	0.20063 ⁽⁴⁾	0.36829 ⁽¹³⁾
	MRE	$\hat{\lambda}$	0.08651 ⁽¹⁾	0.10411 ⁽⁶⁾	0.12123 ⁽¹⁰⁾	0.09583 ⁽²⁾	0.1188 ⁽⁹⁾	0.09753 ⁽³⁾	0.10255 ⁽⁵⁾	0.10874 ⁽⁷⁾	0.13208 ⁽¹²⁾	0.13036 ⁽¹¹⁾	0.10137 ⁽⁴⁾	0.13839 ⁽¹³⁾
	D_{abs}		0.01103 ⁽¹⁾	0.01333 ⁽⁶⁾	0.0155 ⁽¹⁰⁾	0.01247 ⁽²⁾	0.01512 ⁽⁹⁾	0.01248 ⁽³⁾	0.01319 ⁽⁵⁾	0.01409 ⁽⁸⁾	0.01686 ⁽¹¹⁾	0.01693 ⁽¹²⁾	0.01309 ⁽⁴⁾	0.01798 ⁽¹³⁾
	D_{max}		0.01975 ⁽¹⁾	0.02356 ⁽⁶⁾	0.02745 ⁽¹⁰⁾	0.0215 ⁽²⁾	0.0269 ⁽⁹⁾	0.02216 ⁽³⁾	0.0231 ⁽⁵⁾	0.02438 ⁽⁷⁾	0.02986 ⁽¹²⁾	0.02896 ⁽¹¹⁾	0.02282 ⁽⁴⁾	0.0307 ⁽¹³⁾
	ASAE		0.01454 ⁽⁵⁾	0.01405 ⁽²⁾	0.01516 ⁽⁷⁾	0.01432 ⁽⁴⁾	0.01503 ⁽⁶⁾	0.0136 ⁽¹⁾	0.01648 ⁽¹⁰⁾	0.0159 ⁽⁹⁾	0.01729 ⁽¹¹⁾	0.0183 ⁽¹²⁾	0.01539 ⁽⁸⁾	0.01833 ⁽¹³⁾
	$\sum$ Ranks		10 ⁽¹⁾	31 ⁽⁵⁾	57 ⁽¹⁰⁾	14 ⁽²⁾	51 ⁽⁹⁾	16 ⁽³⁾	36 ⁽⁶⁾	46 ⁽⁸⁾	69 ^(11.5)	69 ^(11.5)	28 ⁽⁴⁾	78 ⁽¹³⁾
200	BIAS	$\hat{\lambda}$	0.27206 ⁽¹⁾	0.31542 ⁽⁴⁾	0.35772 ⁽¹⁰⁾	0.28563 ⁽²⁾	0.34717 ⁽⁹⁾	0.30713 ⁽³⁾	0.32676 ⁽⁶⁾	0.33298 ⁽⁸⁾	0.41478 ⁽¹³⁾	0.38369 ⁽¹¹⁾	0.31706 ⁽⁵⁾	0.39776 ⁽¹²⁾
	MSE	$\hat{\lambda}$	0.11859 ⁽¹⁾	0.15383 ⁽⁴⁾	0.19607 ⁽¹⁰⁾	0.12959 ⁽²⁾	0.18763 ⁽⁹⁾	0.14412 ⁽³⁾	0.1765 ⁽⁷⁾	0.18345 ⁽⁸⁾	0.2566 ⁽¹³⁾	0.23804 ⁽¹¹⁾	0.16547 ⁽⁶⁾	0.25267 ⁽¹²⁾
	MRE	$\hat{\lambda}$	0.07773 ⁽¹⁾	0.09012 ⁽⁴⁾	0.10221 ⁽¹⁰⁾	0.08161 ⁽²⁾	0.09919 ⁽⁹⁾	0.08775 ⁽³⁾	0.09336 ⁽⁶⁾	0.09514 ⁽⁸⁾	0.11851 ⁽¹³⁾	0.10963 ⁽¹¹⁾	0.09059 ⁽⁵⁾	0.11364 ⁽¹²⁾
	D_{abs}		0.00998 ⁽¹⁾	0.01151 ⁽⁴⁾	0.01305 ⁽¹⁰⁾	0.01058 ⁽²⁾	0.01273 ⁽⁹⁾	0.01127 ⁽³⁾	0.01201 ⁽⁶⁾	0.01236 ⁽⁸⁾	0.01525 ⁽¹³⁾	0.01424 ⁽¹¹⁾	0.01171 ⁽⁵⁾	0.01477 ⁽¹²⁾
	D_{max}		0.01771 ⁽¹⁾	0.02046 ⁽⁵⁾	0.02322 ⁽¹⁰⁾	0.01839 ⁽²⁾	0.02247 ⁽⁹⁾	0.01993 ⁽³⁾	0.02107 ⁽⁶⁾	0.02132 ⁽⁷⁾	0.02675 ⁽¹³⁾	0.02457 ⁽¹¹⁾	0.02038 ⁽⁴⁾	0.02535 ⁽¹²⁾
	ASAE		0.01214 ⁽⁵⁾	0.01188 ⁽⁴⁾	0.01249 ⁽⁶⁾	0.01162 ⁽²⁾	0.01266 ⁽⁷⁾	0.01148 ⁽¹⁾	0.01408 ⁽¹¹⁾	0.01327 ⁽⁹⁾	0.01398 ⁽¹⁰⁾	0.01512 ⁽¹²⁾	0.01279 ⁽⁸⁾	0.01521 ⁽¹³⁾
	$\sum$ Ranks		10 ⁽¹⁾	25 ⁽⁴⁾	56 ⁽¹⁰⁾	12 ⁽²⁾	52 ⁽⁹⁾	16 ⁽³⁾	42 ⁽⁷⁾	48 ⁽⁸⁾	75 ⁽¹³⁾	67 ⁽¹¹⁾	33 ⁽⁵⁾	73 ⁽¹²⁾

(Continues)

TABLE 7 | (Continued)

n	Estimate	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	MSADE	MSALDE	KE	MSSDE	MSSLDE	MSLNDE
300	BIAS	$\hat{\lambda}$	0.22213 ⁽¹⁾	0.27409 ⁽⁷⁾	0.29436 ⁽⁹⁾	0.22539 ⁽²⁾	0.30338 ⁽¹⁰⁾	0.24481 ⁽³⁾	0.27473 ⁽⁸⁾	0.27219 ⁽⁶⁾	0.33408 ⁽¹³⁾	0.33256 ⁽¹²⁾	0.25816 ⁽⁵⁾	0.3227 ⁽¹¹⁾
	MSE	$\hat{\lambda}$	0.07523 ⁽¹⁾	0.11422 ⁽⁶⁾	0.13353 ⁽⁹⁾	0.08181 ⁽²⁾	0.14198 ⁽¹⁰⁾	0.09462 ⁽³⁾	0.12324 ⁽⁸⁾	0.11711 ⁽⁷⁾	0.1658 ⁽¹²⁾	0.17386 ⁽¹³⁾	0.10731 ⁽⁵⁾	0.16515 ⁽¹¹⁾
	MRE	$\hat{\lambda}$	0.06347 ⁽¹⁾	0.07831 ⁽⁷⁾	0.0841 ⁽⁹⁾	0.0644 ⁽²⁾	0.08668 ⁽¹⁰⁾	0.06995 ⁽³⁾	0.07849 ⁽⁸⁾	0.07777 ⁽⁶⁾	0.09545 ⁽¹³⁾	0.09502 ⁽¹²⁾	0.07376 ⁽⁵⁾	0.0922 ⁽¹¹⁾
	D_{abs}		0.00819 ⁽¹⁾	0.01011 ⁽⁷⁾	0.01082 ⁽⁹⁾	0.00835 ⁽²⁾	0.0111 ⁽¹⁰⁾	0.00896 ⁽³⁾	0.01016 ⁽⁸⁾	0.0101 ⁽⁶⁾	0.01225 ⁽¹²⁾	0.01232 ⁽¹³⁾	0.00956 ⁽⁵⁾	0.01197 ⁽¹¹⁾
	D_{max}		0.01443 ⁽¹⁾	0.01774 ^(7.5)	0.01906 ⁽⁹⁾	0.01454 ⁽²⁾	0.01969 ⁽¹⁰⁾	0.01591 ⁽³⁾	0.01774 ^(7.5)	0.01753 ⁽⁶⁾	0.02164 ⁽¹³⁾	0.02136 ⁽¹²⁾	0.01663 ⁽⁴⁾	0.02072 ⁽¹¹⁾
	ASAE		0.00913 ⁽⁵⁾	0.00902 ^(1.5)	0.00982 ⁽⁶⁾	0.00906 ⁽³⁾	0.01006 ⁽⁸⁾	0.00902 ^(1.5)	0.01077 ⁽¹⁰⁾	0.01039 ⁽⁹⁾	0.01106 ⁽¹¹⁾	0.01173 ⁽¹²⁾	0.00998 ⁽⁷⁾	0.01187 ⁽¹³⁾
	Σ Ranks		10 ⁽¹⁾	36 ⁽⁶⁾	51 ⁽⁹⁾	13 ⁽²⁾	58 ⁽¹⁰⁾	16.5 ⁽³⁾	49.5 ⁽⁸⁾	40 ⁽⁷⁾	74 ^(12.5)	74 ^(12.5)	31 ⁽⁵⁾	68 ⁽¹¹⁾
400	BIAS	$\hat{\lambda}$	0.19418 ⁽¹⁾	0.23233 ⁽⁷⁾	0.27454 ⁽¹¹⁾	0.20545 ⁽²⁾	0.26718 ⁽⁹⁾	0.21474 ⁽³⁾	0.24542 ⁽⁸⁾	0.22313 ⁽⁵⁾	0.29877 ⁽¹³⁾	0.27401 ⁽¹⁰⁾	0.21779 ⁽⁴⁾	0.27677 ⁽¹²⁾
	MSE	$\hat{\lambda}$	0.05926 ⁽¹⁾	0.08323 ⁽⁶⁾	0.11739 ⁽¹¹⁾	0.06431 ⁽²⁾	0.10931 ⁽⁹⁾	0.07178 ⁽³⁾	0.09835 ⁽⁸⁾	0.0797 ⁽⁵⁾	0.1338 ⁽¹³⁾	0.11676 ⁽¹⁰⁾	0.07658 ⁽⁴⁾	0.12697 ⁽¹²⁾
	MRE	$\hat{\lambda}$	0.05548 ⁽¹⁾	0.06638 ⁽⁷⁾	0.07844 ⁽¹¹⁾	0.0587 ⁽²⁾	0.07634 ⁽⁹⁾	0.06135 ⁽³⁾	0.07012 ⁽⁸⁾	0.06375 ⁽⁵⁾	0.08536 ⁽¹³⁾	0.07829 ⁽¹⁰⁾	0.06223 ⁽⁴⁾	0.07908 ⁽¹²⁾
	D_{abs}		0.00718 ⁽¹⁾	0.00854 ⁽⁷⁾	0.01009 ⁽¹⁰⁾	0.00762 ⁽²⁾	0.00978 ⁽⁹⁾	0.00791 ⁽³⁾	0.00906 ⁽⁸⁾	0.00829 ⁽⁵⁾	0.01088 ⁽¹³⁾	0.01018 ⁽¹¹⁾	0.00808 ⁽⁴⁾	0.01026 ⁽¹²⁾
	D_{max}		0.01262 ⁽¹⁾	0.01509 ⁽⁷⁾	0.01779 ^(11.5)	0.0133 ⁽²⁾	0.01736 ⁽⁹⁾	0.01394 ⁽³⁾	0.01588 ⁽⁸⁾	0.01441 ⁽⁵⁾	0.01944 ⁽¹³⁾	0.01763 ⁽¹⁰⁾	0.01406 ⁽⁴⁾	0.01779 ^(11.5)
	ASAE		0.00766 ⁽⁴⁾	0.00746 ⁽²⁾	0.00824 ⁽⁷⁾	0.00769 ⁽⁵⁾	0.00805 ⁽⁶⁾	0.00728 ⁽¹⁾	0.00909 ⁽¹⁰⁾	0.00873 ⁽⁹⁾	0.00946 ⁽¹¹⁾	0.00963 ⁽¹²⁾	0.00833 ⁽⁸⁾	0.00984 ⁽¹³⁾
	Σ Ranks		9 ⁽¹⁾	36 ⁽⁷⁾	61.5 ⁽¹⁰⁾	15 ⁽²⁾	51 ⁽⁹⁾	16 ⁽³⁾	50 ⁽⁸⁾	34 ^(5.5)	76 ⁽¹³⁾	63 ⁽¹¹⁾	28 ⁽⁴⁾	72.5 ⁽¹²⁾

TABLE 8 | Numerical results of simulation for all measures when ($\lambda = 3.5$) under RSS.

n	Estimate	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	MSADE	MSALDE	KE	MSSDE	MSSLDE	MSLNDE
25	BIAS	$\hat{\lambda}$	0.53057 ⁽¹⁾	0.54538 ⁽²⁾	0.61975 ⁽⁶⁾	0.72442 ⁽¹⁰⁾	0.60715 ⁽⁵⁾	0.59237 ⁽⁴⁾	0.71866 ⁽⁹⁾	0.82953 ⁽¹¹⁾	0.63915 ⁽⁷⁾	0.98068 ⁽¹³⁾	0.68039 ⁽⁸⁾	0.96849 ⁽¹²⁾
	MSE	$\hat{\lambda}$	0.43931 ⁽¹⁾	0.48275 ⁽²⁾	0.59641 ⁽⁶⁾	0.81559 ⁽⁹⁾	0.57284 ⁽⁵⁾	0.552 ⁽⁴⁾	0.82082 ⁽¹⁰⁾	1.15505 ⁽¹¹⁾	0.62813 ⁽⁷⁾	1.49198 ⁽¹²⁾	0.74129 ⁽⁸⁾	1.51049 ⁽¹³⁾
	MRE	$\hat{\lambda}$	0.15159 ⁽¹⁾	0.15582 ⁽²⁾	0.17707 ⁽⁶⁾	0.20698 ⁽¹⁰⁾	0.17347 ⁽⁵⁾	0.16516 ⁽³⁾	0.20533 ⁽⁹⁾	0.23701 ⁽¹¹⁾	0.18261 ⁽⁷⁾	0.28019 ⁽¹³⁾	0.1944 ⁽⁸⁾	0.27671 ⁽¹²⁾
	D_{abs}		0.01924 ⁽¹⁾	0.02005 ⁽²⁾	0.02282 ⁽⁶⁾	0.0269 ⁽¹⁰⁾	0.02219 ⁽⁵⁾	0.02168 ⁽⁴⁾	0.0264 ⁽⁹⁾	0.03059 ⁽¹¹⁾	0.02356 ⁽⁷⁾	0.03604 ⁽¹³⁾	0.02506 ⁽⁸⁾	0.03569 ⁽¹²⁾
	D_{max}		0.03453 ⁽¹⁾	0.03497 ⁽²⁾	0.03985 ⁽⁶⁾	0.04465 ⁽⁹⁾	0.03869 ⁽⁵⁾	0.03774 ⁽⁴⁾	0.04512 ⁽¹⁰⁾	0.05067 ⁽¹¹⁾	0.04075 ⁽⁷⁾	0.05898 ⁽¹³⁾	0.0427 ⁽⁸⁾	0.05808 ⁽¹²⁾
	ASAE		0.04102 ⁽⁴⁾	0.04114 ⁽⁵⁾	0.0425 ⁽⁷⁾	0.04027 ⁽²⁾	0.04237 ⁽⁶⁾	0.04065 ⁽³⁾	0.04453 ⁽⁹⁾	0.04469 ⁽¹⁰⁾	0.0469 ⁽¹¹⁾	0.05296 ⁽¹²⁾	0.04308 ⁽⁸⁾	0.05416 ⁽¹³⁾
	Σ Ranks		9 ⁽¹⁾	15 ⁽²⁾	37 ⁽⁶⁾	50 ⁽⁹⁾	31 ⁽⁵⁾	23 ⁽⁴⁾	56 ⁽¹⁰⁾	65 ⁽¹¹⁾	46 ⁽⁷⁾	76 ⁽¹³⁾	48 ⁽⁸⁾	74 ⁽¹²⁾
70	BIAS	$\hat{\lambda}$	0.32549 ⁽¹⁾	0.33615 ⁽²⁾	0.37309 ⁽⁶⁾	0.39686 ⁽⁷⁾	0.36935 ⁽⁵⁾	0.34387 ⁽³⁾	0.45533 ⁽¹⁰⁾	0.47859 ⁽¹¹⁾	0.42059 ⁽⁹⁾	0.63371 ⁽¹³⁾	0.41867 ⁽⁸⁾	0.58722 ⁽¹²⁾
	MSE	$\hat{\lambda}$	0.16781 ⁽¹⁾	0.17337 ⁽²⁾	0.21688 ⁽⁶⁾	0.2498 ⁽⁷⁾	0.21062 ⁽⁵⁾	0.18463 ⁽³⁾	0.34832 ⁽¹⁰⁾	0.36813 ⁽¹¹⁾	0.26717 ⁽⁸⁾	0.63568 ⁽¹³⁾	0.28692 ⁽⁹⁾	0.54199 ⁽¹²⁾
	MRE	$\hat{\lambda}$	0.093 ⁽¹⁾	0.09604 ⁽²⁾	0.1066 ⁽⁶⁾	0.11339 ⁽⁷⁾	0.10553 ⁽⁵⁾	0.09825 ⁽³⁾	0.13009 ⁽¹⁰⁾	0.13674 ⁽¹¹⁾	0.12017 ⁽⁹⁾	0.18106 ⁽¹³⁾	0.11962 ⁽⁸⁾	0.16778 ⁽¹²⁾
	D_{abs}		0.01189 ⁽¹⁾	0.01229 ⁽²⁾	0.01365 ⁽⁶⁾	0.01478 ⁽⁷⁾	0.01358 ⁽⁵⁾	0.01263 ⁽³⁾	0.01678 ⁽¹⁰⁾	0.01777 ⁽¹¹⁾	0.01538 ⁽⁸⁾	0.02357 ⁽¹³⁾	0.01539 ⁽⁹⁾	0.0218 ⁽¹²⁾
	D_{max}		0.02118 ⁽¹⁾	0.02174 ⁽²⁾	0.02422 ⁽⁶⁾	0.02523 ⁽⁷⁾	0.02389 ⁽⁵⁾	0.02223 ⁽³⁾	0.02906 ⁽¹⁰⁾	0.0304 ⁽¹¹⁾	0.02724 ⁽⁹⁾	0.03942 ⁽¹³⁾	0.02675 ⁽⁸⁾	0.03685 ⁽¹²⁾
	ASAE		0.02105 ⁽⁵⁾	0.02084 ⁽³⁾	0.02186 ⁽⁷⁾	0.02102 ⁽⁴⁾	0.02157 ⁽⁶⁾	0.02051 ⁽²⁾	0.02406 ⁽¹⁰⁾	0.02356 ⁽⁹⁾	0.02425 ⁽¹¹⁾	0.02707 ⁽¹²⁾	0.02253 ⁽⁸⁾	0.02759 ⁽¹³⁾
	Σ Ranks		10 ⁽¹⁾	13 ⁽²⁾	37 ⁽⁶⁾	39 ⁽⁷⁾	31 ⁽⁵⁾	17 ⁽³⁾	60 ⁽¹⁰⁾	64 ⁽¹¹⁾	54 ⁽⁹⁾	77 ⁽¹³⁾	50 ⁽⁸⁾	73 ⁽¹²⁾

(Continues)

TABLE 8 | (Continued)

<i>n</i>	Estimate	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	MSADE	MSALDE	KE	MSSDE	MSSLDE	MSLNDE
150	BIAS	$\hat{\lambda}$	0.22071 ⁽¹¹⁾	0.24147 ⁽⁴⁾	0.26875 ⁽⁶⁾	0.27793 ⁽⁷⁾	0.22323 ⁽²⁾	0.24101 ⁽³⁾	0.32417 ⁽¹⁰⁾	0.33044 ⁽¹¹⁾	0.29692 ⁽⁹⁾	0.41203 ⁽¹²⁾	0.29588 ⁽⁸⁾	0.4141 ⁽¹³⁾
	MSE	$\hat{\lambda}$	0.0751 ⁽¹⁾	0.09118 ⁽⁴⁾	0.11106 ⁽⁶⁾	0.12266 ⁽⁷⁾	0.07971 ⁽²⁾	0.09062 ⁽³⁾	0.16672 ⁽¹⁰⁾	0.17827 ⁽¹¹⁾	0.13428 ⁽⁸⁾	0.27277 ⁽¹³⁾	0.14172 ⁽⁹⁾	0.26466 ⁽¹²⁾
	MRE	$\hat{\lambda}$	0.06306 ⁽¹⁾	0.06899 ⁽⁴⁾	0.07679 ⁽⁶⁾	0.07941 ⁽⁷⁾	0.06378 ⁽²⁾	0.06886 ⁽³⁾	0.09262 ⁽¹⁰⁾	0.09441 ⁽¹¹⁾	0.08483 ⁽⁹⁾	0.11772 ⁽¹²⁾	0.08454 ⁽⁸⁾	0.11832 ⁽¹³⁾
	D_{abs}	$\hat{\lambda}$	0.00812 ⁽¹⁾	0.00887 ⁽³⁾	0.00988 ⁽⁶⁾	0.01035 ⁽⁷⁾	0.0082 ⁽²⁾	0.00888 ⁽⁴⁾	0.01192 ⁽¹⁰⁾	0.01221 ⁽¹¹⁾	0.01086 ⁽⁸⁾	0.01531 ⁽¹²⁾	0.01099 ⁽⁹⁾	0.01543 ⁽¹³⁾
	D_{max}	$\hat{\lambda}$	0.01434 ⁽¹⁾	0.01566 ⁽⁴⁾	0.0175 ⁽⁶⁾	0.01784 ⁽⁷⁾	0.01451 ⁽²⁾	0.01561 ⁽³⁾	0.02094 ⁽¹⁰⁾	0.02116 ⁽¹¹⁾	0.01934 ⁽⁹⁾	0.02621 ⁽¹²⁾	0.01901 ⁽⁸⁾	0.02636 ⁽¹³⁾
	ASAE	$\hat{\lambda}$	0.01262 ⁽³⁾	0.01266 ⁽⁴⁾	0.0135 ⁽⁷⁾	0.01296 ⁽⁵⁾	0.01242 ⁽¹⁾	0.01251 ⁽²⁾	0.0154 ⁽¹¹⁾	0.01469 ⁽⁹⁾	0.01497 ⁽¹⁰⁾	0.01718 ⁽¹²⁾	0.01409 ⁽⁸⁾	0.01722 ⁽¹³⁾
	$\sum$ Ranks		8 ⁽¹⁾	23 ⁽⁴⁾	37 ⁽⁶⁾	40 ⁽⁷⁾	11 ⁽²⁾	18 ⁽³⁾	61 ⁽¹⁰⁾	64 ⁽¹¹⁾	53 ⁽⁹⁾	73 ⁽¹²⁾	50 ⁽⁸⁾	77 ⁽¹³⁾
200	BIAS	$\hat{\lambda}$	0.1834 ⁽¹⁾	0.1989 ⁽³⁾	0.22769 ⁽⁶⁾	0.22912 ⁽⁷⁾	0.19721 ⁽²⁾	0.1997 ⁽⁴⁾	0.29294 ⁽¹¹⁾	0.28052 ⁽¹⁰⁾	0.25345 ⁽⁸⁾	0.35402 ⁽¹²⁾	0.25749 ⁽⁹⁾	0.37492 ⁽¹³⁾
	MSE	$\hat{\lambda}$	0.05552 ⁽¹⁾	0.06213 ⁽³⁾	0.08055 ⁽⁶⁾	0.08485 ⁽⁷⁾	0.06309 ⁽⁴⁾	0.06002 ⁽²⁾	0.13874 ⁽¹¹⁾	0.12216 ⁽¹⁰⁾	0.10048 ⁽⁸⁾	0.19904 ⁽¹²⁾	0.10829 ⁽⁹⁾	0.2192 ⁽¹³⁾
	MRE	$\hat{\lambda}$	0.0524 ⁽¹⁾	0.05683 ⁽³⁾	0.06505 ⁽⁶⁾	0.06546 ⁽⁷⁾	0.05634 ⁽²⁾	0.05706 ⁽⁴⁾	0.0837 ⁽¹¹⁾	0.08015 ⁽¹⁰⁾	0.07241 ⁽⁸⁾	0.10115 ⁽¹²⁾	0.07357 ⁽⁹⁾	0.10712 ⁽¹³⁾
	D_{abs}	$\hat{\lambda}$	0.00677 ⁽¹⁾	0.00729 ^(2,5)	0.0084 ⁽⁶⁾	0.00851 ⁽⁷⁾	0.00729 ^(2,5)	0.00737 ⁽⁴⁾	0.01084 ⁽¹¹⁾	0.0104 ⁽¹⁰⁾	0.00932 ⁽⁸⁾	0.01316 ⁽¹²⁾	0.00951 ⁽⁹⁾	0.01395 ⁽¹³⁾
	D_{max}	$\hat{\lambda}$	0.01193 ⁽¹⁾	0.01294 ⁽³⁾	0.0148 ⁽⁷⁾	0.01476 ⁽⁶⁾	0.01281 ⁽²⁾	0.01296 ⁽⁴⁾	0.01886 ⁽¹¹⁾	0.01805 ⁽¹⁰⁾	0.01649 ⁽⁸⁾	0.02263 ⁽¹²⁾	0.01659 ⁽⁹⁾	0.02393 ⁽¹³⁾
	ASAE	$\hat{\lambda}$	0.01053 ⁽³⁾	0.0107 ⁽⁵⁾	0.01127 ⁽⁷⁾	0.01057 ⁽⁴⁾	0.01022 ⁽¹⁾	0.01048 ⁽²⁾	0.0128 ⁽¹¹⁾	0.01218 ⁽⁹⁾	0.0126 ⁽¹⁰⁾	0.01427 ⁽¹²⁾	0.01193 ⁽⁸⁾	0.0146 ⁽¹³⁾
	$\sum$ Ranks		8 ⁽¹⁾	19.5 ⁽³⁾	38 ^(6,5)	38 ^(6,5)	13.5 ⁽²⁾	20 ⁽⁴⁾	66 ⁽¹¹⁾	59 ⁽¹⁰⁾	50 ⁽⁸⁾	72 ⁽¹²⁾	53 ⁽⁹⁾	78 ⁽¹³⁾
300	BIAS	$\hat{\lambda}$	0.16207 ⁽¹⁾	0.16793 ⁽²⁾	0.18566 ⁽⁵⁾	0.18825 ⁽⁶⁾	0.16844 ⁽⁴⁾	0.16796 ⁽³⁾	0.23345 ⁽¹¹⁾	0.22465 ⁽¹⁰⁾	0.21983 ⁽⁸⁾	0.29147 ⁽¹³⁾	0.22198 ⁽⁹⁾	0.27969 ⁽¹²⁾
	MSE	$\hat{\lambda}$	0.0407 ⁽¹⁾	0.04376 ⁽²⁾	0.05345 ⁽⁵⁾	0.05589 ⁽⁶⁾	0.04441 ⁽⁴⁾	0.04403 ⁽³⁾	0.0878 ⁽¹¹⁾	0.08077 ⁽¹⁰⁾	0.07446 ⁽⁸⁾	0.1336 ⁽¹³⁾	0.0777 ⁽⁹⁾	0.1195 ⁽¹²⁾
	MRE	$\hat{\lambda}$	0.0463 ⁽¹⁾	0.04798 ⁽²⁾	0.05305 ⁽⁵⁾	0.05379 ⁽⁶⁾	0.04812 ⁽⁴⁾	0.04799 ⁽³⁾	0.0667 ⁽¹¹⁾	0.06419 ⁽¹⁰⁾	0.06281 ⁽⁸⁾	0.08328 ⁽¹³⁾	0.06342 ⁽⁹⁾	0.07991 ⁽¹²⁾
	D_{abs}	$\hat{\lambda}$	0.00598 ⁽¹⁾	0.00621 ⁽³⁾	0.00686 ⁽⁵⁾	0.00701 ⁽⁶⁾	0.00622 ⁽⁴⁾	0.0062 ⁽²⁾	0.00864 ⁽¹¹⁾	0.00834 ⁽¹⁰⁾	0.00812 ⁽⁸⁾	0.01087 ⁽¹³⁾	0.00822 ⁽⁹⁾	0.01042 ⁽¹²⁾
	D_{max}	$\hat{\lambda}$	0.01054 ⁽¹⁾	0.0109 ^(2,5)	0.01206 ⁽⁵⁾	0.01213 ⁽⁶⁾	0.01094 ⁽⁴⁾	0.0109 ^(2,5)	0.01511 ⁽¹¹⁾	0.01449 ⁽¹⁰⁾	0.01427 ⁽⁸⁾	0.01868 ⁽¹³⁾	0.01434 ⁽⁹⁾	0.01797 ⁽¹²⁾
	ASAE	$\hat{\lambda}$	0.00823 ⁽³⁾	0.00832 ⁽⁴⁾	0.00866 ⁽⁷⁾	0.00837 ⁽⁵⁾	0.00818 ⁽²⁾	0.00816 ⁽¹⁾	0.01002 ⁽¹¹⁾	0.00952 ⁽⁹⁾	0.00971 ⁽¹⁰⁾	0.01097 ⁽¹³⁾	0.0093 ⁽⁸⁾	0.01077 ⁽¹²⁾
	$\sum$ Ranks		8 ⁽¹⁾	15.5 ⁽³⁾	32 ⁽⁵⁾	35 ⁽⁶⁾	22 ⁽⁴⁾	14.5 ⁽²⁾	66 ⁽¹¹⁾	59 ⁽¹⁰⁾	50 ⁽⁸⁾	78 ⁽¹³⁾	53 ⁽⁹⁾	72 ⁽¹²⁾
400	BIAS	$\hat{\lambda}$	0.1285 ⁽¹⁾	0.13953 ⁽²⁾	0.1607 ⁽⁶⁾	0.16233 ⁽⁷⁾	0.13969 ⁽³⁾	0.14673 ⁽⁴⁾	0.20779 ⁽¹¹⁾	0.1926 ⁽⁹⁾	0.19275 ⁽¹⁰⁾	0.24051 ⁽¹²⁾	0.18636 ⁽⁸⁾	0.24761 ⁽¹³⁾
	MSE	$\hat{\lambda}$	0.02575 ⁽¹⁾	0.03049 ⁽²⁾	0.03953 ⁽⁶⁾	0.0418 ⁽⁷⁾	0.03054 ⁽³⁾	0.03387 ⁽⁴⁾	0.07096 ⁽¹¹⁾	0.05942 ⁽¹⁰⁾	0.05632 ⁽⁸⁾	0.09224 ⁽¹²⁾	0.05692 ⁽⁹⁾	0.09486 ⁽¹³⁾
	MRE	$\hat{\lambda}$	0.03672 ⁽¹⁾	0.03987 ⁽²⁾	0.04592 ⁽⁶⁾	0.04638 ⁽⁷⁾	0.03991 ⁽³⁾	0.04192 ⁽⁴⁾	0.05937 ⁽¹¹⁾	0.05503 ⁽⁹⁾	0.05507 ⁽¹⁰⁾	0.06872 ⁽¹²⁾	0.05325 ⁽⁸⁾	0.07075 ⁽¹³⁾
	D_{abs}	$\hat{\lambda}$	0.00475 ⁽¹⁾	0.00516 ⁽²⁾	0.00593 ⁽⁶⁾	0.00604 ⁽⁷⁾	0.00517 ⁽³⁾	0.00542 ⁽⁴⁾	0.00768 ⁽¹¹⁾	0.00716 ⁽¹⁰⁾	0.00709 ⁽⁹⁾	0.00894 ⁽¹²⁾	0.00691 ⁽⁸⁾	0.00992 ⁽¹³⁾
	D_{max}	$\hat{\lambda}$	0.00836 ⁽¹⁾	0.00906 ⁽²⁾	0.01045 ⁽⁶⁾	0.01049 ⁽⁷⁾	0.00908 ⁽³⁾	0.00953 ⁽⁴⁾	0.01345 ⁽¹¹⁾	0.01243 ⁽⁹⁾	0.01254 ⁽¹⁰⁾	0.01547 ⁽¹²⁾	0.01204 ⁽⁸⁾	0.01594 ⁽¹³⁾
	ASAE	$\hat{\lambda}$	0.0069 ⁽⁵⁾	0.00683 ⁽⁴⁾	0.00728 ⁽⁶⁾	0.00679 ^(2,5)	0.00664 ⁽¹⁾	0.00679 ^(2,5)	0.00844 ⁽¹¹⁾	0.00791 ⁽⁹⁾	0.00839 ⁽¹⁰⁾	0.00909 ⁽¹²⁾	0.00771 ⁽⁸⁾	0.00926 ⁽¹³⁾
	$\sum$ Ranks		10 ⁽¹⁾	14 ⁽²⁾	36 ⁽⁶⁾	37.5 ⁽⁷⁾	16 ⁽³⁾	22.5 ⁽⁴⁾	66 ⁽¹¹⁾	56 ⁽⁹⁾	57 ⁽¹⁰⁾	72 ⁽¹²⁾	49 ⁽⁸⁾	78 ⁽¹³⁾

TABLE 9 | Numerical results of simulation for all measures when ($\lambda = 4.0$) under SRS.

n	Estimate	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	MSADE	MSALDE	KE	MSSDE	MSSLDE	MSLNDE
25	BIAS	$\hat{\lambda}$	0.83772 ⁽¹⁾	0.96187 ⁽⁶⁾	1.02128 ⁽¹¹⁾	0.90799 ⁽³⁾	1.00604 ⁽¹⁰⁾	0.99318 ⁽⁸⁾	0.86975 ⁽²⁾	0.98079 ⁽⁷⁾	1.00551 ⁽⁹⁾	1.20078 ⁽¹²⁾	0.92774 ⁽⁴⁾	1.20824 ⁽¹³⁾
	MSE	$\hat{\lambda}$	1.10465 ⁽¹⁾	1.45383 ⁽⁶⁾	1.59031 ⁽¹¹⁾	1.34938 ⁽⁴⁾	1.5194 ⁽⁸⁾	1.55287 ⁽¹⁰⁾	1.23664 ⁽²⁾	1.51375 ⁽⁷⁾	1.52715 ⁽⁹⁾	2.22173 ⁽¹²⁾	1.36647 ⁽⁵⁾	2.27191 ⁽¹³⁾
	MRE	$\hat{\lambda}$	0.20943 ⁽¹⁾	0.24047 ⁽⁶⁾	0.25532 ⁽¹¹⁾	0.227 ⁽³⁾	0.25151 ⁽¹⁰⁾	0.24829 ⁽⁸⁾	0.21744 ⁽²⁾	0.2452 ⁽⁷⁾	0.25138 ⁽⁹⁾	0.3002 ⁽¹²⁾	0.23193 ⁽⁴⁾	0.30206 ⁽¹³⁾
	D_{abs}		0.03148 ⁽²⁾	0.03499 ⁽⁶⁾	0.037 ⁽¹¹⁾	0.03307 ⁽³⁾	0.03622 ⁽¹⁰⁾	0.03613 ⁽⁹⁾	0.03136 ⁽¹⁾	0.03548 ⁽⁷⁾	0.03576 ⁽⁸⁾	0.04275 ⁽¹²⁾	0.03372 ⁽⁴⁾	0.04308 ⁽¹³⁾
	D_{max}		0.05376 ⁽³⁾	0.05735 ⁽⁷⁾	0.06064 ⁽¹¹⁾	0.05268 ⁽²⁾	0.05963 ⁽⁹⁾	0.05835 ⁽⁸⁾	0.05143 ⁽¹⁾	0.05689 ⁽⁶⁾	0.06007 ⁽¹⁰⁾	0.06791 ⁽¹²⁾	0.05473 ⁽⁴⁾	0.06811 ⁽¹³⁾
	ASAE		0.04832 ⁽⁷⁾	0.04562 ⁽³⁾	0.04703 ⁽⁵⁾	0.04601 ⁽⁴⁾	0.04753 ⁽⁶⁾	0.04544 ⁽²⁾	0.04967 ⁽⁹⁾	0.05024 ⁽¹⁰⁾	0.05078 ⁽¹¹⁾	0.05821 ⁽¹³⁾	0.04859 ⁽⁸⁾	0.0579 ⁽¹²⁾
	$\sum$ Ranks		15 ⁽¹⁾	34 ⁽⁶⁾	60 ⁽¹¹⁾	19 ⁽³⁾	53 ⁽⁹⁾	45 ⁽⁸⁾	17 ⁽²⁾	44 ⁽⁷⁾	56 ⁽¹⁰⁾	73 ⁽¹²⁾	29 ⁽⁵⁾	77 ⁽¹³⁾
70	BIAS	$\hat{\lambda}$	0.51727 ⁽¹⁾	0.59305 ⁽⁶⁾	0.65645 ⁽¹⁰⁾	0.52843 ⁽²⁾	0.65584 ⁽⁹⁾	0.6173 ⁽⁸⁾	0.57184 ⁽⁵⁾	0.61517 ⁽⁷⁾	0.69529 ⁽¹¹⁾	0.76813 ⁽¹²⁾	0.55661 ⁽³⁾	0.77186 ⁽¹³⁾
	MSE	$\hat{\lambda}$	0.4101 ⁽¹⁾	0.53793 ⁽⁶⁾	0.65261 ⁽¹⁰⁾	0.46207 ⁽²⁾	0.64936 ⁽⁹⁾	0.58216 ⁽⁷⁾	0.52794 ⁽⁵⁾	0.59915 ⁽⁸⁾	0.72195 ⁽¹¹⁾	0.96297 ⁽¹³⁾	0.50312 ⁽⁴⁾	0.94004 ⁽¹²⁾
	MRE	$\hat{\lambda}$	0.12932 ⁽¹⁾	0.14826 ⁽⁶⁾	0.16411 ⁽¹⁰⁾	0.13211 ⁽²⁾	0.16396 ⁽⁹⁾	0.15433 ⁽⁸⁾	0.14296 ⁽⁵⁾	0.15379 ⁽⁷⁾	0.17382 ⁽¹¹⁾	0.19203 ⁽¹²⁾	0.13915 ⁽³⁾	0.19296 ⁽¹³⁾
	D_{abs}		0.01928 ⁽¹⁾	0.02195 ⁽⁶⁾	0.02413 ⁽⁹⁾	0.01978 ⁽²⁾	0.02417 ⁽¹⁰⁾	0.02267 ⁽⁷⁾	0.02116 ⁽⁵⁾	0.02288 ⁽⁸⁾	0.02525 ⁽¹¹⁾	0.02814 ⁽¹²⁾	0.02071 ⁽³⁾	0.02823 ⁽¹³⁾
	D_{max}		0.03174 ⁽²⁾	0.03598 ⁽⁶⁾	0.03984 ⁽¹⁰⁾	0.03159 ⁽¹⁾	0.03975 ⁽⁹⁾	0.03756 ⁽⁸⁾	0.03451 ⁽⁴⁾	0.03681 ⁽⁷⁾	0.04244 ⁽¹¹⁾	0.04506 ⁽¹²⁾	0.03336 ⁽³⁾	0.04553 ⁽¹³⁾
	ASAE		0.02311 ⁽⁴⁾	0.02313 ⁽⁵⁾	0.02475 ⁽⁸⁾	0.0226 ⁽²⁾	0.02427 ⁽⁶⁾	0.02302 ⁽³⁾	0.0265 ⁽¹⁰⁾	0.02538 ⁽⁹⁾	0.02708 ⁽¹¹⁾	0.02993 ⁽¹³⁾	0.02441 ⁽⁷⁾	0.02929 ⁽¹²⁾
	$\sum$ Ranks		10 ⁽¹⁾	35 ⁽⁶⁾	57 ⁽¹⁰⁾	11 ⁽²⁾	52 ⁽⁹⁾	41 ⁽⁷⁾	34 ⁽⁵⁾	46 ⁽⁸⁾	66 ⁽¹¹⁾	74 ⁽¹²⁾	23 ⁽⁴⁾	76 ⁽¹³⁾
150	BIAS	$\hat{\lambda}$	0.34713 ⁽¹⁾	0.40011 ⁽⁴⁾	0.45993 ⁽¹⁰⁾	0.36318 ⁽²⁾	0.43451 ⁽⁹⁾	0.40382 ⁽⁵⁾	0.42915 ⁽⁸⁾	0.41048 ⁽⁷⁾	0.5241 ⁽¹³⁾	0.51791 ⁽¹²⁾	0.40496 ⁽⁶⁾	0.50026 ⁽¹¹⁾
	MSE	$\hat{\lambda}$	0.19258 ⁽¹⁾	0.25529 ⁽⁴⁾	0.3267 ⁽¹⁰⁾	0.214 ⁽²⁾	0.29082 ⁽⁸⁾	0.25548 ⁽⁵⁾	0.29516 ⁽⁹⁾	0.27188 ⁽⁷⁾	0.41181 ⁽¹²⁾	0.42901 ⁽¹³⁾	0.2615 ⁽⁶⁾	0.39754 ⁽¹¹⁾
	MRE	$\hat{\lambda}$	0.08678 ⁽¹⁾	0.10003 ⁽⁴⁾	0.11498 ⁽¹⁰⁾	0.0908 ⁽²⁾	0.10863 ⁽⁹⁾	0.10095 ⁽⁵⁾	0.10729 ⁽⁸⁾	0.10262 ⁽⁷⁾	0.13103 ⁽¹³⁾	0.12948 ⁽¹²⁾	0.10124 ⁽⁶⁾	0.12507 ⁽¹¹⁾
	D_{abs}		0.01309 ⁽¹⁾	0.01497 ⁽⁴⁾	0.01712 ⁽¹⁰⁾	0.01369 ⁽²⁾	0.0163 ⁽⁹⁾	0.01515 ⁽⁵⁾	0.01612 ⁽⁸⁾	0.01547 ⁽⁷⁾	0.0195 ⁽¹³⁾	0.01936 ⁽¹²⁾	0.01522 ⁽⁶⁾	0.01868 ⁽¹¹⁾
	D_{max}		0.02126 ⁽¹⁾	0.02445 ⁽⁵⁾	0.0282 ⁽¹⁰⁾	0.02197 ⁽²⁾	0.02649 ⁽⁹⁾	0.02465 ⁽⁶⁾	0.02608 ⁽⁸⁾	0.02483 ⁽⁷⁾	0.03202 ⁽¹³⁾	0.03103 ⁽¹²⁾	0.02444 ⁽⁴⁾	0.03002 ⁽¹¹⁾
	ASAE		0.01389 ⁽³⁾	0.01423 ⁽⁵⁾	0.0151 ⁽⁷⁾	0.0138 ⁽²⁾	0.01445 ⁽⁶⁾	0.01394 ⁽⁴⁾	0.01654 ⁽¹⁰⁾	0.0158 ⁽⁹⁾	0.01696 ⁽¹¹⁾	0.01811 ⁽¹²⁾	0.01521 ⁽⁸⁾	0.01816 ⁽¹³⁾
	$\sum$ Ranks		8 ⁽¹⁾	26 ⁽⁴⁾	57 ⁽¹⁰⁾	12 ⁽²⁾	50 ⁽⁸⁾	30 ⁽⁵⁾	51 ⁽⁹⁾	44 ⁽⁷⁾	75 ⁽¹³⁾	73 ⁽¹²⁾	36 ⁽⁶⁾	68 ⁽¹¹⁾
200	BIAS	$\hat{\lambda}$	0.3118 ⁽¹⁾	0.34435 ⁽⁵⁾	0.395 ⁽⁹⁾	0.31476 ⁽²⁾	0.40972 ⁽¹⁰⁾	0.35348 ⁽⁷⁾	0.34813 ⁽⁶⁾	0.37542 ⁽⁸⁾	0.43343 ⁽¹¹⁾	0.44757 ⁽¹³⁾	0.33486 ⁽⁴⁾	0.43467 ⁽¹²⁾
	MSE	$\hat{\lambda}$	0.15197 ⁽¹⁾	0.18855 ⁽⁵⁾	0.23692 ⁽⁹⁾	0.15673 ⁽²⁾	0.26208 ⁽¹⁰⁾	0.19432 ⁽⁶⁾	0.20357 ⁽⁷⁾	0.22254 ⁽⁸⁾	0.28398 ⁽¹¹⁾	0.31492 ⁽¹³⁾	0.1768 ⁽⁴⁾	0.30717 ⁽¹²⁾
	MRE	$\hat{\lambda}$	0.07795 ⁽¹⁾	0.08609 ⁽⁵⁾	0.09875 ⁽⁹⁾	0.07869 ⁽²⁾	0.10243 ⁽¹⁰⁾	0.08837 ⁽⁷⁾	0.08703 ⁽⁶⁾	0.09385 ⁽⁸⁾	0.10836 ⁽¹¹⁾	0.11189 ⁽¹³⁾	0.08371 ⁽⁴⁾	0.10867 ⁽¹²⁾
	D_{abs}		0.01182 ⁽¹⁾	0.01302 ⁽⁵⁾	0.01482 ⁽⁹⁾	0.01195 ⁽²⁾	0.01539 ⁽¹⁰⁾	0.01334 ⁽⁷⁾	0.01311 ⁽⁶⁾	0.01417 ⁽⁸⁾	0.01624 ⁽¹¹⁾	0.01679 ⁽¹³⁾	0.01265 ⁽⁴⁾	0.01632 ⁽¹²⁾
	D_{max}		0.01908 ⁽²⁾	0.02101 ⁽⁵⁾	0.02415 ⁽⁹⁾	0.019 ⁽¹⁾	0.02502 ⁽¹⁰⁾	0.02159 ⁽⁷⁾	0.02124 ⁽⁶⁾	0.02266 ⁽⁸⁾	0.02656 ⁽¹²⁾	0.02692 ⁽¹³⁾	0.02032 ⁽⁴⁾	0.02607 ⁽¹¹⁾
	ASAE		0.01154 ⁽³⁾	0.01164 ⁽⁴⁾	0.01231 ⁽⁶⁾	0.01138 ⁽²⁾	0.01254 ⁽⁷⁾	0.01198 ⁽⁵⁾	0.01347 ⁽¹⁰⁾	0.0132 ⁽⁹⁾	0.01382 ⁽¹¹⁾	0.01477 ⁽¹²⁾	0.01298 ⁽⁸⁾	0.01523 ⁽¹³⁾
	$\sum$ Ranks		9 ⁽¹⁾	29 ⁽⁵⁾	51 ⁽⁹⁾	11 ⁽²⁾	57 ⁽¹⁰⁾	39 ⁽⁶⁾	41 ⁽⁷⁾	49 ⁽⁸⁾	67 ⁽¹¹⁾	77 ⁽¹³⁾	28 ⁽⁴⁾	72 ⁽¹²⁾

(Continues)

TABLE 9 | (Continued)

<i>n</i>		Estimate	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	MSADE	MSALDE	KE	MSSDE	MSSLDE	MSLNDE
300	BIAS	$\hat{\lambda}$	0.24525 ⁽¹⁾	0.30162 ⁽⁶⁾	0.32732 ⁽¹⁰⁾	0.25362 ⁽²⁾	0.32347 ⁽⁹⁾	0.27186 ⁽³⁾	0.30257 ⁽⁷⁾	0.29321 ⁽⁵⁾	0.31227 ⁽⁸⁾	0.36822 ⁽¹³⁾	0.35003 ⁽¹¹⁾	0.27837 ⁽⁴⁾	0.36088 ⁽¹²⁾
	MSE	$\hat{\lambda}$	0.09381 ⁽¹⁾	0.13983 ⁽⁵⁾	0.16606 ⁽¹⁰⁾	0.10248 ⁽²⁾	0.15886 ⁽⁹⁾	0.11592 ⁽³⁾	0.14633 ⁽⁷⁾	0.14238 ⁽⁶⁾	0.15469 ⁽⁸⁾	0.20524 ⁽¹²⁾	0.19848 ⁽¹¹⁾	0.12282 ⁽⁴⁾	0.20841 ⁽¹³⁾
	MRE	$\hat{\lambda}$	0.06131 ⁽¹⁾	0.07541 ⁽⁶⁾	0.08183 ⁽¹⁰⁾	0.06341 ⁽²⁾	0.08087 ⁽⁹⁾	0.06797 ⁽³⁾	0.07564 ⁽⁷⁾	0.0733 ⁽⁵⁾	0.07807 ⁽⁸⁾	0.09206 ⁽¹³⁾	0.08751 ⁽¹¹⁾	0.06959 ⁽⁴⁾	0.09022 ⁽¹²⁾
	<i>D</i> _{abs}	$\hat{\lambda}$	0.00933 ⁽¹⁾	0.01114 ⁽⁶⁾	0.01236 ⁽¹⁰⁾	0.00964 ⁽²⁾	0.01222 ⁽⁹⁾	0.01029 ⁽³⁾	0.01142 ⁽⁷⁾	0.01111 ⁽⁵⁾	0.01183 ⁽⁸⁾	0.01381 ⁽¹³⁾	0.01322 ⁽¹¹⁾	0.01056 ⁽⁴⁾	0.0136 ⁽¹²⁾
	<i>D</i> _{max}	$\hat{\lambda}$	0.01504 ⁽¹⁾	0.01848 ⁽⁶⁾	0.02004 ⁽¹⁰⁾	0.01538 ⁽²⁾	0.01983 ⁽⁹⁾	0.01665 ⁽³⁾	0.01852 ⁽⁷⁾	0.01787 ⁽⁵⁾	0.01893 ⁽⁸⁾	0.02262 ⁽¹³⁾	0.02127 ⁽¹¹⁾	0.01692 ⁽⁴⁾	0.02189 ⁽¹²⁾
	ASAE	$\hat{\lambda}$	0.00883 ⁽²⁾	0.00918 ⁽⁵⁾	0.0098 ⁽⁷⁾	0.00896 ⁽³⁾	0.00962 ⁽⁶⁾	0.00864 ⁽¹⁾	0.00915 ⁽⁴⁾	0.01058 ⁽¹⁰⁾	0.01012 ⁽⁹⁾	0.01116 ⁽¹¹⁾	0.01185 ⁽¹³⁾	0.00983 ⁽⁸⁾	0.01157 ⁽¹²⁾
$\sum$ Ranks			7 ⁽¹⁾	34 ⁽⁵⁾	57 ⁽¹⁰⁾	13 ⁽²⁾	51 ⁽⁹⁾	16 ⁽³⁾	39 ⁽⁷⁾	36 ⁽⁶⁾	49 ⁽⁸⁾	75 ⁽¹³⁾	68 ⁽¹¹⁾	28 ⁽⁴⁾	73 ⁽¹²⁾
400	BIAS	$\hat{\lambda}$	0.20811 ⁽¹⁾	0.25303 ⁽⁶⁾	0.27557 ⁽⁸⁾	0.21874 ⁽²⁾	0.27792 ⁽¹⁰⁾	0.23614 ⁽³⁾	0.25685 ⁽⁷⁾	0.27661 ⁽⁹⁾	0.24528 ⁽⁵⁾	0.30704 ⁽¹³⁾	0.30225 ⁽¹¹⁾	0.24358 ⁽⁴⁾	0.3026 ⁽¹²⁾
	MSE	$\hat{\lambda}$	0.06731 ⁽¹⁾	0.10001 ⁽⁶⁾	0.11943 ⁽⁸⁾	0.07445 ⁽²⁾	0.12093 ⁽⁹⁾	0.08742 ⁽³⁾	0.10266 ⁽⁷⁾	0.12352 ⁽¹⁰⁾	0.09826 ⁽⁵⁾	0.14795 ⁽¹³⁾	0.14378 ⁽¹¹⁾	0.09512 ⁽⁴⁾	0.14603 ⁽¹²⁾
	MRE	$\hat{\lambda}$	0.05203 ⁽¹⁾	0.06326 ⁽⁶⁾	0.06889 ⁽⁸⁾	0.05468 ⁽²⁾	0.06948 ⁽¹⁰⁾	0.05903 ⁽³⁾	0.06421 ⁽⁷⁾	0.06915 ⁽⁹⁾	0.06132 ⁽⁵⁾	0.07676 ⁽¹³⁾	0.07556 ⁽¹¹⁾	0.06089 ⁽⁴⁾	0.07565 ⁽¹²⁾
	<i>D</i> _{abs}	$\hat{\lambda}$	0.00791 ⁽¹⁾	0.0096 ⁽⁶⁾	0.01042 ⁽⁸⁾	0.00832 ⁽²⁾	0.01052 ⁽¹⁰⁾	0.00895 ⁽³⁾	0.00975 ⁽⁷⁾	0.01048 ⁽⁹⁾	0.00932 ⁽⁵⁾	0.01162 ⁽¹³⁾	0.01147 ^(11.5)	0.00936 ⁽⁴⁾	0.01147 ^(11.5)
	<i>D</i> _{max}	$\hat{\lambda}$	0.01274 ⁽¹⁾	0.01546 ⁽⁶⁾	0.01688 ⁽⁹⁾	0.01332 ⁽²⁾	0.01703 ⁽¹⁰⁾	0.01446 ⁽³⁾	0.01571 ⁽⁷⁾	0.01685 ⁽⁸⁾	0.01495 ⁽⁵⁾	0.01879 ⁽¹³⁾	0.0183 ⁽¹¹⁾	0.01483 ⁽⁴⁾	0.01836 ⁽¹²⁾
	ASAE	$\hat{\lambda}$	0.00746 ^(3.5)	0.00746 ^(3.5)	0.00801 ⁽⁶⁾	0.0074 ⁽²⁾	0.00802 ⁽⁷⁾	0.0073 ⁽¹⁾	0.0075 ⁽⁵⁾	0.00902 ⁽¹¹⁾	0.00847 ⁽⁹⁾	0.00901 ⁽¹⁰⁾	0.00974 ⁽¹³⁾	0.00815 ⁽⁸⁾	0.00963 ⁽¹²⁾
$\sum$ Ranks			8.5 ⁽¹⁾	33.5 ⁽⁵⁾	47 ⁽⁸⁾	12 ⁽²⁾	56 ^(9.5)	16 ⁽³⁾	40 ⁽⁷⁾	56 ^(9.5)	34 ⁽⁶⁾	75 ⁽¹³⁾	68.5 ⁽¹¹⁾	28 ⁽⁴⁾	71.5 ⁽¹²⁾

TABLE 10 | Numerical results of simulation for all measures when ($\lambda = 4.0$) under RSS.

<i>n</i>		Estimate	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	MSADE	MSALDE	KE	MSSDE	MSSLDE	MSLNDE
25	BIAS	$\ddot{\lambda}$	0.58773 ⁽¹⁾	0.59159 ⁽²⁾	0.64698 ⁽⁶⁾	0.74574 ⁽¹⁰⁾	0.64232 ⁽⁵⁾	0.61043 ⁽³⁾	0.62567 ⁽⁴⁾	0.71466 ⁽⁸⁾	0.89419 ⁽¹¹⁾	0.69858 ⁽⁷⁾	1.06007 ⁽¹²⁾	0.73556 ⁽⁹⁾	1.09472 ⁽¹³⁾
	MSE	$\ddot{\lambda}$	0.55247 ⁽¹⁾	0.54826 ⁽²⁾	0.64579 ⁽⁵⁾	0.90377 ⁽¹⁰⁾	0.64707 ⁽⁶⁾	0.58805 ⁽³⁾	0.62478 ⁽⁴⁾	0.83817 ⁽⁸⁾	1.33681 ⁽¹¹⁾	0.74597 ⁽⁷⁾	1.81874 ⁽¹²⁾	0.89084 ⁽⁹⁾	1.87936 ⁽¹³⁾
	MRE	$\ddot{\lambda}$	0.14693 ⁽¹⁾	0.1479 ⁽²⁾	0.16175 ⁽⁶⁾	0.18644 ⁽¹⁰⁾	0.16058 ⁽⁵⁾	0.15261 ⁽³⁾	0.15642 ⁽⁴⁾	0.17866 ⁽⁸⁾	0.22355 ⁽¹¹⁾	0.17465 ⁽⁷⁾	0.26502 ⁽¹²⁾	0.18389 ⁽⁹⁾	0.27368 ⁽¹³⁾
	<i>D</i> _{abs}	$\ddot{\lambda}$	0.02174 ⁽¹⁾	0.02213 ⁽²⁾	0.02403 ⁽⁶⁾	0.02735 ⁽¹⁰⁾	0.02379 ⁽⁵⁾	0.02257 ⁽³⁾	0.02324 ⁽⁴⁾	0.02627 ⁽⁸⁾	0.03225 ⁽¹¹⁾	0.02556 ⁽⁷⁾	0.03786 ⁽¹²⁾	0.0269 ⁽⁹⁾	0.03933 ⁽¹³⁾
	<i>D</i> _{max}	$\ddot{\lambda}$	0.0361 ⁽²⁾	0.03574 ⁽¹⁾	0.03957 ⁽⁶⁾	0.04349 ⁽⁹⁾	0.03865 ⁽⁵⁾	0.03673 ⁽³⁾	0.03747 ⁽⁴⁾	0.04241 ⁽⁸⁾	0.05169 ⁽¹¹⁾	0.04227 ⁽⁷⁾	0.0601 ⁽¹²⁾	0.04378 ⁽¹⁰⁾	0.06176 ⁽¹³⁾
	ASAE	$\ddot{\lambda}$	0.04094 ⁽⁵⁾	0.04026 ⁽³⁾	0.04265 ⁽⁶⁾	0.03998 ⁽²⁾	0.04294 ⁽⁷⁾	0.03977 ⁽¹⁾	0.04063 ⁽⁴⁾	0.04562 ⁽¹⁰⁾	0.04463 ⁽⁹⁾	0.04652 ⁽¹¹⁾	0.05349 ⁽¹³⁾	0.04333 ⁽⁸⁾	0.0534 ⁽¹²⁾
$\sum$ Ranks			11 ⁽¹⁾	12 ⁽²⁾	35 ⁽⁶⁾	51 ⁽⁹⁾	33 ⁽⁵⁾	16 ⁽³⁾	24 ⁽⁴⁾	50 ⁽⁸⁾	64 ⁽¹¹⁾	46 ⁽⁷⁾	73 ⁽¹²⁾	54 ⁽¹⁰⁾	77 ⁽¹³⁾
70	BIAS	$\ddot{\lambda}$	0.34803 ⁽¹⁾	0.36041 ⁽³⁾	0.39747 ⁽⁶⁾	0.43367 ⁽⁷⁾	0.39209 ⁽⁵⁾	0.3683 ⁽⁴⁾	0.35986 ⁽²⁾	0.47481 ⁽¹⁰⁾	0.51442 ⁽¹¹⁾	0.45621 ⁽⁸⁾	0.64148 ⁽¹²⁾	0.46411 ⁽⁹⁾	0.66642 ⁽¹³⁾
	MSE	$\ddot{\lambda}$	0.19061 ⁽¹⁾	0.20224 ⁽²⁾	0.24806 ⁽⁶⁾	0.30389 ⁽⁷⁾	0.23804 ⁽⁵⁾	0.21011 ⁽⁴⁾	0.20727 ⁽³⁾	0.36629 ⁽¹⁰⁾	0.44397 ⁽¹¹⁾	0.32349 ⁽⁸⁾	0.65766 ⁽¹²⁾	0.34596 ⁽⁹⁾	0.70933 ⁽¹³⁾
	MRE	$\ddot{\lambda}$	0.08701 ⁽¹⁾	0.0901 ⁽³⁾	0.09937 ⁽⁶⁾	0.10842 ⁽⁷⁾	0.09802 ⁽⁵⁾	0.09207 ⁽⁴⁾	0.08997 ⁽²⁾	0.1187 ⁽¹⁰⁾	0.12861 ⁽¹¹⁾	0.11405 ⁽⁸⁾	0.16037 ⁽¹²⁾	0.11603 ⁽⁹⁾	0.16661 ⁽¹³⁾
	<i>D</i> _{abs}	$\ddot{\lambda}$	0.01312 ⁽¹⁾	0.01357 ⁽³⁾	0.01496 ⁽⁶⁾	0.01634 ⁽⁷⁾	0.01473 ⁽⁵⁾	0.01388 ⁽⁴⁾	0.01354 ⁽²⁾	0.01767 ⁽¹⁰⁾	0.01918 ⁽¹¹⁾	0.01693 ⁽⁸⁾	0.02359 ⁽¹²⁾	0.01739 ⁽⁹⁾	0.02457 ⁽¹³⁾
	<i>D</i> _{max}	$\ddot{\lambda}$	0.02136 ⁽¹⁾	0.02196 ^(2.5)	0.02432 ⁽⁶⁾	0.02602 ⁽⁷⁾	0.02395 ⁽⁵⁾	0.02246 ⁽⁴⁾	0.02196 ^(2.5)	0.02868 ⁽¹⁰⁾	0.03077 ⁽¹¹⁾	0.0281 ⁽⁹⁾	0.03794 ⁽¹²⁾	0.02795 ⁽⁸⁾	0.03928 ⁽¹³⁾
	ASAE	$\ddot{\lambda}$	0.02112 ⁽⁵⁾	0.02062 ⁽²⁾	0.02176 ⁽⁶⁾	0.02063 ⁽³⁾	0.02181 ⁽⁷⁾	0.02032 ⁽¹⁾	0.02107 ⁽⁴⁾	0.02422 ⁽¹⁰⁾	0.02423 ⁽¹¹⁾	0.02394 ⁽⁹⁾	0.02789 ⁽¹²⁾	0.02252 ⁽⁸⁾	0.02806 ⁽¹³⁾
$\sum$ Ranks			10 ⁽¹⁾	15.5 ^(2.5)	36 ⁽⁶⁾	38 ⁽⁷⁾	32 ⁽⁵⁾	21 ⁽⁴⁾	15.5 ^(2.5)	60 ⁽¹⁰⁾	66 ⁽¹¹⁾	50 ⁽⁸⁾	72 ⁽¹²⁾	52 ⁽⁹⁾	78 ⁽¹³⁾

(Continues)

TABLE 10 | (Continued)

<i>n</i>	Estimate	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	MSADE	MSALDE	KE	MSSDE	MSSLDE	MSLNDE
150	BIAS	$\hat{\lambda}$	0.24128 ⁽²⁾	0.25881 ⁽⁵⁾	0.27896 ⁽⁷⁾	0.27747 ⁽⁶⁾	0.24336 ⁽³⁾	0.25231 ⁽⁴⁾	0.35223 ⁽¹¹⁾	0.35198 ⁽¹⁰⁾	0.30264 ⁽⁸⁾	0.42528 ⁽¹²⁾	0.3218 ⁽⁹⁾	0.46048 ⁽¹³⁾
	MSE	$\hat{\lambda}$	0.09299 ⁽²⁾	0.10584 ⁽⁵⁾	0.12294 ⁽⁷⁾	0.11869 ⁽⁶⁾	0.09339 ⁽³⁾	0.10084 ⁽⁴⁾	0.2014 ⁽¹¹⁾	0.19473 ⁽¹⁰⁾	0.1435 ⁽⁸⁾	0.29602 ⁽¹²⁾	0.16983 ⁽⁹⁾	0.3409 ⁽¹³⁾
	MRE	$\hat{\lambda}$	0.06032 ⁽²⁾	0.0647 ⁽⁵⁾	0.06974 ⁽⁷⁾	0.06937 ⁽⁶⁾	0.06084 ⁽³⁾	0.06308 ⁽⁴⁾	0.08806 ⁽¹¹⁾	0.088 ⁽¹⁰⁾	0.07566 ⁽⁸⁾	0.10632 ⁽¹²⁾	0.08045 ⁽⁹⁾	0.11512 ⁽¹³⁾
	D_{abs}	$\hat{\lambda}$	0.00882 ⁽¹⁾	0.00981 ⁽⁵⁾	0.01054 ⁽⁷⁾	0.01053 ⁽⁶⁾	0.00925 ⁽³⁾	0.00956 ⁽⁴⁾	0.01327 ⁽¹⁰⁾	0.01332 ⁽¹¹⁾	0.01149 ⁽⁸⁾	0.016 ⁽¹²⁾	0.01215 ⁽⁹⁾	0.01721 ⁽¹³⁾
	D_{max}	$\hat{\lambda}$	0.01424 ⁽¹⁾	0.01588 ⁽⁵⁾	0.01689 ⁽⁶⁾	0.01696 ⁽⁷⁾	0.01487 ⁽³⁾	0.01545 ⁽⁴⁾	0.02144 ⁽¹¹⁾	0.02129 ⁽¹⁰⁾	0.01856 ⁽⁸⁾	0.02546 ⁽¹²⁾	0.01949 ⁽⁹⁾	0.0275 ⁽¹³⁾
200	ASAE	0.01263 ⁽²⁾	0.01271 ⁽³⁾	0.01347 ⁽⁶⁾	0.0128 ⁽⁴⁾	0.01359 ⁽⁷⁾	0.01242 ⁽¹⁾	0.01293 ⁽⁵⁾	0.01528 ⁽¹¹⁾	0.01463 ⁽⁹⁾	0.0149 ⁽¹⁰⁾	0.01728 ⁽¹²⁾	0.0142 ⁽⁸⁾	0.01749 ⁽¹³⁾
	$\sum$ Ranks	7 ⁽¹⁾	13 ⁽²⁾	31 ⁽⁵⁾	38 ^(6,5)	38 ^(6,5)	16 ⁽³⁾	25 ⁽⁴⁾	65 ⁽¹¹⁾	60 ⁽¹⁰⁾	50 ⁽⁸⁾	72 ⁽¹²⁾	53 ⁽⁹⁾	78 ⁽¹³⁾
	BIAS	$\hat{\lambda}$	0.19839 ⁽¹⁾	0.22737 ⁽⁵⁾	0.23795 ⁽⁷⁾	0.23187 ⁽⁶⁾	0.21752 ⁽⁴⁾	0.21657 ⁽³⁾	0.31804 ⁽¹¹⁾	0.30093 ⁽¹⁰⁾	0.25872 ⁽⁸⁾	0.37822 ⁽¹²⁾	0.27776 ⁽⁹⁾	0.40001 ⁽¹³⁾
	MSE	$\hat{\lambda}$	0.06198 ⁽¹⁾	0.08142 ⁽⁵⁾	0.08969 ⁽⁷⁾	0.08404 ⁽⁶⁾	0.07437 ⁽⁴⁾	0.07222 ⁽³⁾	0.16233 ⁽¹¹⁾	0.14627 ⁽¹⁰⁾	0.10525 ⁽⁸⁾	0.23178 ⁽¹²⁾	0.12469 ⁽⁹⁾	0.25057 ⁽¹³⁾
	MRE	$\hat{\lambda}$	0.0496 ⁽¹⁾	0.05335 ⁽²⁾	0.05949 ⁽⁷⁾	0.05797 ⁽⁶⁾	0.05438 ⁽⁴⁾	0.05414 ⁽³⁾	0.07951 ⁽¹¹⁾	0.07523 ⁽¹⁰⁾	0.06468 ⁽⁸⁾	0.09456 ⁽¹²⁾	0.06944 ⁽⁹⁾	0.1 ⁽¹³⁾
300	D_{abs}	$\hat{\lambda}$	0.00755 ⁽¹⁾	0.00862 ⁽⁵⁾	0.00904 ⁽⁷⁾	0.00881 ⁽⁶⁾	0.00826 ⁽⁴⁾	0.00825 ⁽³⁾	0.01204 ⁽¹¹⁾	0.01136 ⁽¹⁰⁾	0.00982 ⁽⁸⁾	0.01423 ⁽¹²⁾	0.01053 ⁽⁹⁾	0.01507 ⁽¹³⁾
	D_{max}	$\hat{\lambda}$	0.01217 ⁽¹⁾	0.01393 ⁽⁵⁾	0.01445 ⁽⁷⁾	0.01417 ⁽⁶⁾	0.01333 ⁽⁴⁾	0.01325 ⁽³⁾	0.01936 ⁽¹¹⁾	0.01823 ⁽¹⁰⁾	0.01588 ⁽⁸⁾	0.0228 ⁽¹²⁾	0.01684 ⁽⁹⁾	0.02405 ⁽¹³⁾
	ASAE	0.01058 ⁽⁴⁾	0.01064 ⁽⁵⁾	0.01119 ⁽⁶⁾	0.01052 ⁽³⁾	0.01123 ⁽⁷⁾	0.01027 ⁽¹⁾	0.0104 ⁽²⁾	0.01307 ⁽¹¹⁾	0.01226 ⁽¹⁰⁾	0.01217 ⁽⁹⁾	0.01403 ⁽¹²⁾	0.01184 ⁽⁸⁾	0.01472 ⁽¹³⁾
	$\sum$ Ranks	9 ⁽¹⁾	15 ⁽²⁾	31 ⁽⁵⁾	38 ⁽⁷⁾	37 ⁽⁶⁾	21 ⁽⁴⁾	17 ⁽³⁾	66 ⁽¹¹⁾	60 ⁽¹⁰⁾	49 ⁽⁸⁾	72 ⁽¹²⁾	53 ⁽⁹⁾	78 ⁽¹³⁾
	BIAS	$\hat{\lambda}$	0.16211 ⁽¹⁾	0.18332 ⁽⁵⁾	0.19659 ⁽⁷⁾	0.18142 ⁽⁴⁾	0.17593 ⁽³⁾	0.17311 ⁽²⁾	0.24698 ⁽¹⁰⁾	0.25492 ⁽¹¹⁾	0.21502 ⁽⁸⁾	0.32243 ⁽¹³⁾	0.23287 ⁽⁹⁾	0.32209 ⁽¹²⁾
400	MSE	$\hat{\lambda}$	0.04239 ⁽¹⁾	0.05146 ⁽⁵⁾	0.06073 ⁽⁷⁾	0.05123 ⁽⁴⁾	0.04855 ⁽³⁾	0.04746 ⁽²⁾	0.09951 ⁽¹⁰⁾	0.10006 ⁽¹¹⁾	0.07251 ⁽⁸⁾	0.16567 ⁽¹²⁾	0.08856 ⁽⁹⁾	0.16679 ⁽¹³⁾
	MRE	$\hat{\lambda}$	0.04053 ⁽¹⁾	0.04583 ⁽⁵⁾	0.04915 ⁽⁷⁾	0.04768 ⁽⁶⁾	0.04398 ⁽³⁾	0.04328 ⁽²⁾	0.06174 ⁽¹⁰⁾	0.06373 ⁽¹¹⁾	0.05376 ⁽⁸⁾	0.08061 ⁽¹³⁾	0.05822 ⁽⁹⁾	0.08052 ⁽¹²⁾
	D_{abs}	$\hat{\lambda}$	0.00617 ⁽¹⁾	0.00698 ⁽⁵⁾	0.00749 ⁽⁷⁾	0.00691 ⁽⁴⁾	0.0067 ⁽³⁾	0.00658 ⁽²⁾	0.00938 ⁽¹⁰⁾	0.00969 ⁽¹¹⁾	0.00816 ⁽⁸⁾	0.01221 ⁽¹³⁾	0.00884 ⁽⁹⁾	0.01217 ⁽¹²⁾
	D_{max}	$\hat{\lambda}$	0.00992 ⁽¹⁾	0.01123 ⁽⁵⁾	0.01205 ⁽⁷⁾	0.01111 ⁽⁴⁾	0.01076 ⁽³⁾	0.0106 ⁽²⁾	0.01502 ⁽¹⁰⁾	0.01549 ⁽¹¹⁾	0.01316 ⁽⁸⁾	0.01951 ⁽¹²⁾	0.01416 ⁽⁹⁾	0.01952 ⁽¹³⁾
	ASAE	0.00809 ^(2,5)	0.00825 ^(4,5)	0.00874 ⁽⁷⁾	0.00809 ^(2,5)	0.0086 ⁽⁶⁾	0.00791 ⁽¹⁾	0.00825 ^(4,5)	0.0099 ⁽¹¹⁾	0.00952 ⁽¹⁰⁾	0.00944 ⁽⁹⁾	0.01118 ⁽¹²⁾	0.00912 ⁽⁸⁾	0.01121 ⁽¹³⁾
400	$\sum$ Ranks	7.5 ⁽¹⁾	29.5 ⁽⁵⁾	42 ⁽⁷⁾	32.5 ⁽⁶⁾	26 ⁽⁴⁾	16 ⁽³⁾	14.5 ⁽²⁾	61 ⁽¹⁰⁾	65 ⁽¹¹⁾	49 ⁽⁸⁾	75 ^(12,5)	53 ⁽⁹⁾	75 ^(12,5)
	BIAS	$\hat{\lambda}$	0.14889 ⁽¹⁾	0.15069 ⁽²⁾	0.16834 ⁽⁷⁾	0.16723 ⁽⁵⁾	0.15277 ⁽³⁾	0.15283 ⁽⁴⁾	0.22575 ⁽¹¹⁾	0.21223 ⁽¹⁰⁾	0.19553 ⁽⁸⁾	0.25913 ⁽¹²⁾	0.20178 ⁽⁹⁾	0.26954 ⁽¹³⁾
	MSE	$\hat{\lambda}$	0.03503 ⁽¹⁾	0.03521 ⁽²⁾	0.04395 ⁽⁶⁾	0.04293 ⁽⁵⁾	0.03718 ⁽³⁾	0.03728 ⁽⁴⁾	0.08349 ⁽¹¹⁾	0.07484 ⁽¹⁰⁾	0.0601 ⁽⁸⁾	0.11011 ⁽¹²⁾	0.06514 ⁽⁹⁾	0.11504 ⁽¹³⁾
	MRE	$\hat{\lambda}$	0.03722 ⁽¹⁾	0.03767 ⁽²⁾	0.04208 ⁽⁷⁾	0.04195 ⁽⁶⁾	0.03819 ⁽³⁾	0.03821 ⁽⁴⁾	0.05644 ⁽¹¹⁾	0.05306 ⁽¹⁰⁾	0.04888 ⁽⁸⁾	0.06478 ⁽¹²⁾	0.05044 ⁽⁹⁾	0.06739 ⁽¹³⁾
	D_{abs}	$\hat{\lambda}$	0.00567 ⁽¹⁾	0.00574 ⁽²⁾	0.00642 ⁽⁷⁾	0.00638 ⁽⁵⁾	0.00582 ^(3,5)	0.00582 ^(3,5)	0.00859 ⁽¹¹⁾	0.00807 ⁽¹⁰⁾	0.00745 ⁽⁸⁾	0.00982 ⁽¹²⁾	0.00767 ⁽⁹⁾	0.01024 ⁽¹³⁾
400	D_{max}	$\hat{\lambda}$	0.00912 ⁽¹⁾	0.00922 ⁽²⁾	0.01031 ⁽⁷⁾	0.01023 ⁽⁶⁾	0.00935 ^(3,5)	0.00935 ^(3,5)	0.01376 ⁽¹¹⁾	0.01289 ⁽¹⁰⁾	0.01199 ⁽⁸⁾	0.01574 ⁽¹²⁾	0.01228 ⁽⁹⁾	0.01635 ⁽¹³⁾
	ASAE	0.00685 ^(4,5)	0.00685 ^(4,5)	0.0072 ⁽⁶⁾	0.00683 ⁽³⁾	0.00722 ⁽⁷⁾	0.00663 ⁽¹⁾	0.00673 ⁽²⁾	0.00856 ⁽¹¹⁾	0.00803 ⁽¹⁰⁾	0.00799 ⁽⁹⁾	0.00923 ⁽¹³⁾	0.0077 ⁽⁸⁾	0.0092 ⁽¹²⁾
	$\sum$ Ranks	9.5 ⁽¹⁾	14.5 ⁽²⁾	40 ⁽⁷⁾	33 ^(5,5)	33 ^(5,5)	17 ⁽³⁾	21 ⁽⁴⁾	66 ⁽¹¹⁾	60 ⁽¹⁰⁾	49 ⁽⁸⁾	73 ⁽¹²⁾	53 ⁽⁹⁾	77 ⁽¹³⁾

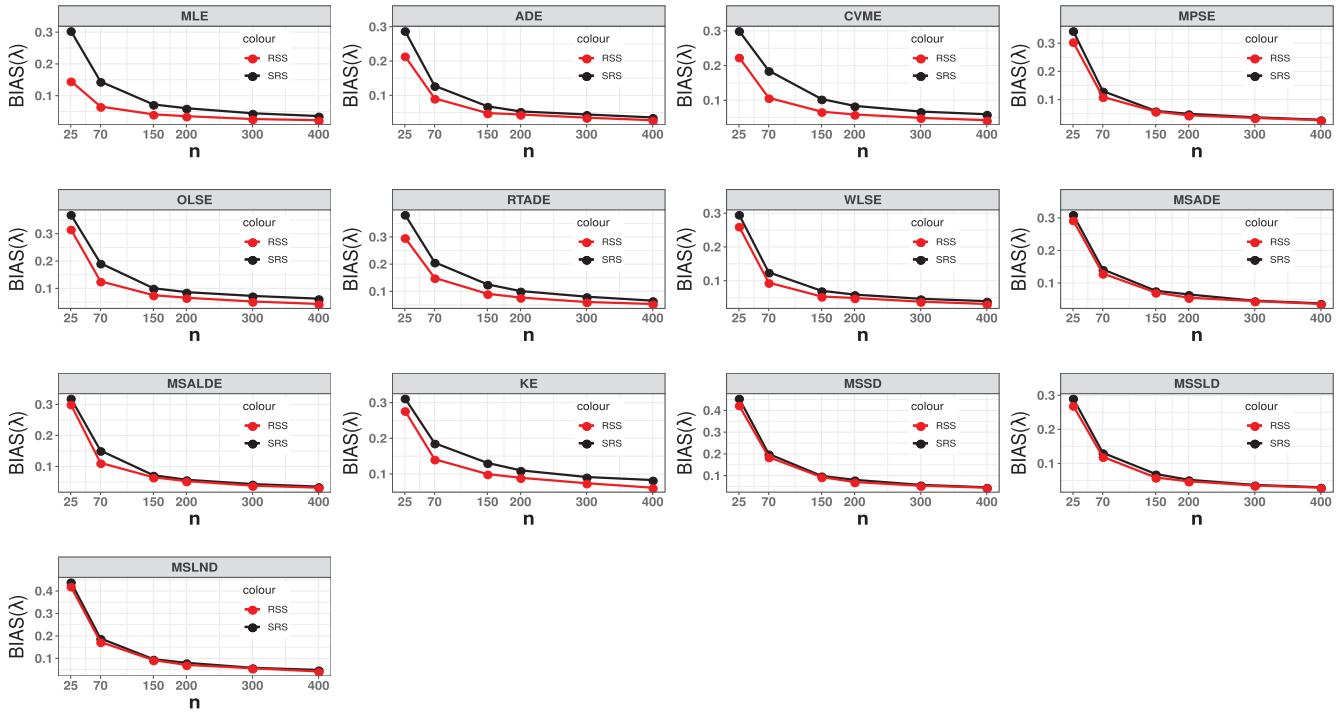


FIGURE 2 | A Graphical representation for the numerical values of BIAS presented in Tables 1 and 2.

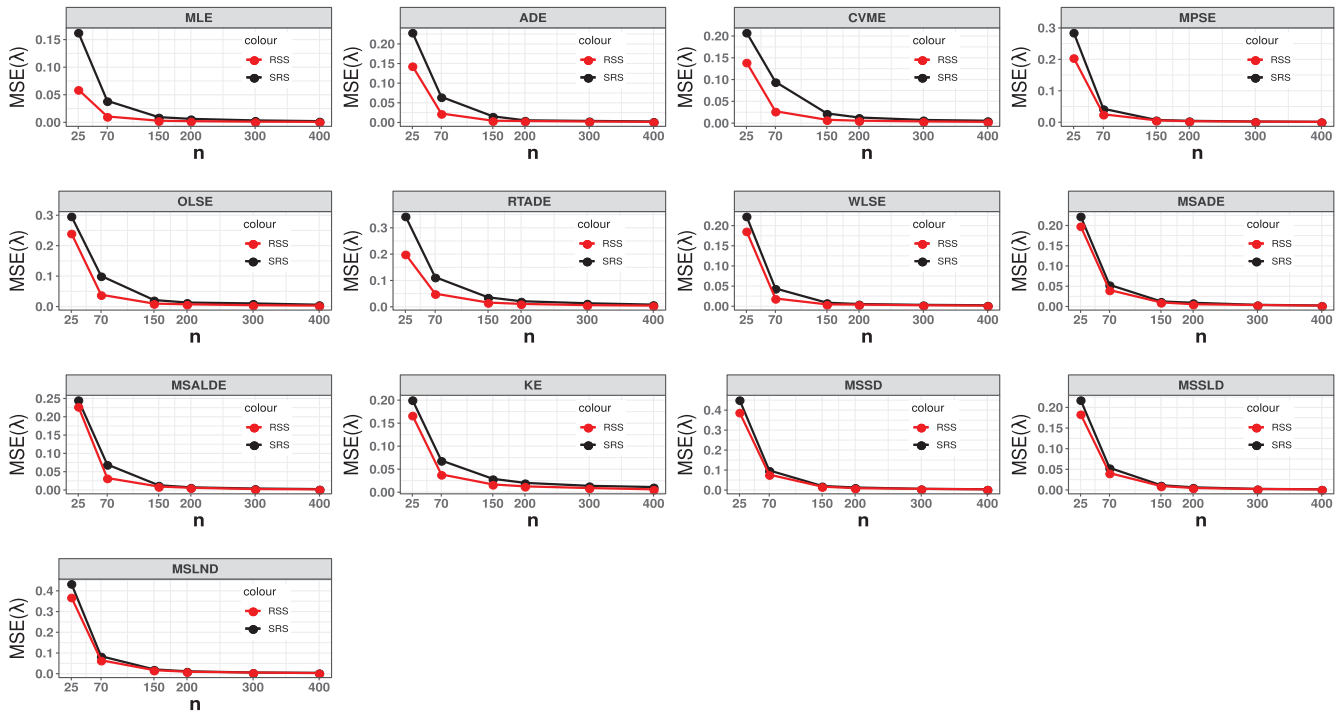


FIGURE 3 | A graphical representation for the numerical values of MSE presented in Tables 1 and 2.

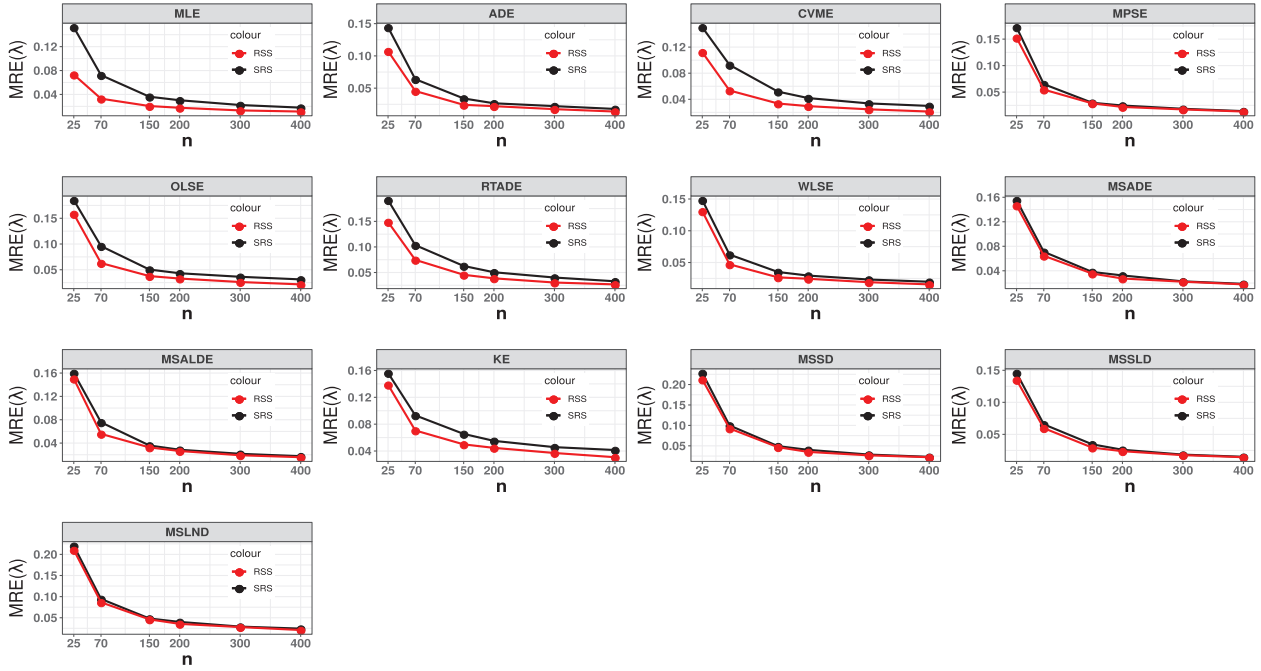


FIGURE 4 | A graphical representation for the numerical values of MRE presented in Tables 1 and 2.

The OLSEs $\hat{\lambda}_{11}$ can be produced by solving the following nonlinear equation:

$$\frac{\partial \tau}{\partial \lambda} = \sum_{i=1}^n \left[G(w_{(r_1)} | \lambda) - \frac{r_1}{n+1} \right] \delta(w_{(r_1)} | \lambda) = 0,$$

where $\delta(\cdot | \lambda)$, is given in (10).

After that, let $W_{(1:n)}, W_{(2:n)}, \dots, W_{(n:n)}$ be the sorted RSS taken from the RLD with sample size $n = tb$, where t denotes the set size and b the cycle numbers. The following function is minimized in order to give the OLSE $\hat{\lambda}_{11}$

$$\frac{\partial \tau^\circ}{\partial \lambda} = \sum_{i=1}^n \left[G(w_{(r_2:n)} | \lambda) - \frac{r_2}{n+1} \right] \delta(w_{(r_2:n)} | \lambda) = 0,$$

where $\delta(\cdot | \lambda)$, is given in (10).

5.2 | Weighted Least Squares Estimator

Let $W_{(1)}, W_{(2)}, \dots, W_{(n)}$ be the ordered SRS with sample size n . Following the function's minimization with regard to λ , the WLSE $\hat{\lambda}_{12}$ of λ is obtained.

$$\tau_1 = \sum_{r_1=1}^n \frac{(n+1)^2(n+2)}{r_1(n-r_1+1)} \left[G(w_{(r_1)} | \lambda) - \frac{r_1}{n+1} \right]^2.$$

The WLSE $\hat{\lambda}_{12}$ can be yielded by solving the following nonlinear equation:

$$\frac{\partial \tau_1}{\partial \lambda} = \sum_{r_1=1}^n \frac{(n+1)^2(n+2)}{r_1(n-r_1+1)} \left[G(w_{(r_1)} | \lambda) - \frac{r_1}{n+1} \right] \delta(w_{(r_1)} | \lambda) = 0,$$

where $\delta(\cdot | \lambda)$, is given in (10).

After that, given $W_{(1:n)}, W_{(2:n)}, \dots, W_{(n:n)}$ be the sorted RSS taken from the RLD with sample size $n = tb$, where t denotes the set size and b the cycle numbers. The following function is minimized in order to give the WLSE $\hat{\lambda}_{12}$

$$\frac{\partial \tau_1^\circ}{\partial \lambda} = \sum_{r_2=1}^n \frac{(n+1)^2(n+2)}{r_2(n-r_2+1)} \left[G(w_{(r_2:n)} | \lambda) - \frac{r_2}{n+1} \right] \delta(w_{(r_2:n)} | \lambda) = 0,$$

where $\delta(\cdot | \lambda)$, is given in (10).

5.3 | Kolmogorov Estimator

Let $W_{(1)}, W_{(2)}, \dots, W_{(n)}$ be the ordered SRS with sample size n . Following the function's minimization with regard to λ , the KE $\hat{\lambda}_{13}$ of λ is obtained.

$$\tau_2 = \text{Max}_{1 \leq r_1 \leq n} \sum_{r_1=1}^n \left[\frac{r_1}{n} - \left\{ 1 - \frac{e^{-\frac{w_{(r_1)}}{\lambda}}}{(\lambda-1)} \left(\lambda + \frac{w_{(r_1)}}{\lambda} - 1 \right) \right\} \right. \\ \left. \left\{ 1 - \frac{e^{-\frac{w_{(r_1)}}{\lambda}}}{(\lambda-1)} \left(\lambda + \frac{w_{(r_1)}}{\lambda} - 1 \right) \right\} - \frac{r_1-1}{n} \right]^2.$$

After that, given that $\hat{\lambda}_{13}$ be the sorted RSS taken from the RLD with sample size $n = tb$, where t denotes the set size and b the cycle numbers. The following function is minimized in order to give the KE $\hat{\lambda}_{13}$

$$\tau_2^\circ = \text{Max}_{1 \leq r_2 \leq n} \sum_{r_2=1}^n \left[\frac{r_2}{n} - \left\{ 1 - \frac{e^{-\frac{w_{(r_2:n)}}{\lambda}}}{(\lambda-1)} \left(\lambda + \frac{w_{(r_2:n)}}{\lambda} - 1 \right) \right\} \right. \\ \left. \left\{ 1 - \frac{e^{-\frac{w_{(r_2:n)}}{\lambda}}}{(\lambda-1)} \left(\lambda + \frac{w_{(r_2:n)}}{\lambda} - 1 \right) \right\} - \frac{r_2-1}{n} \right]^2.$$

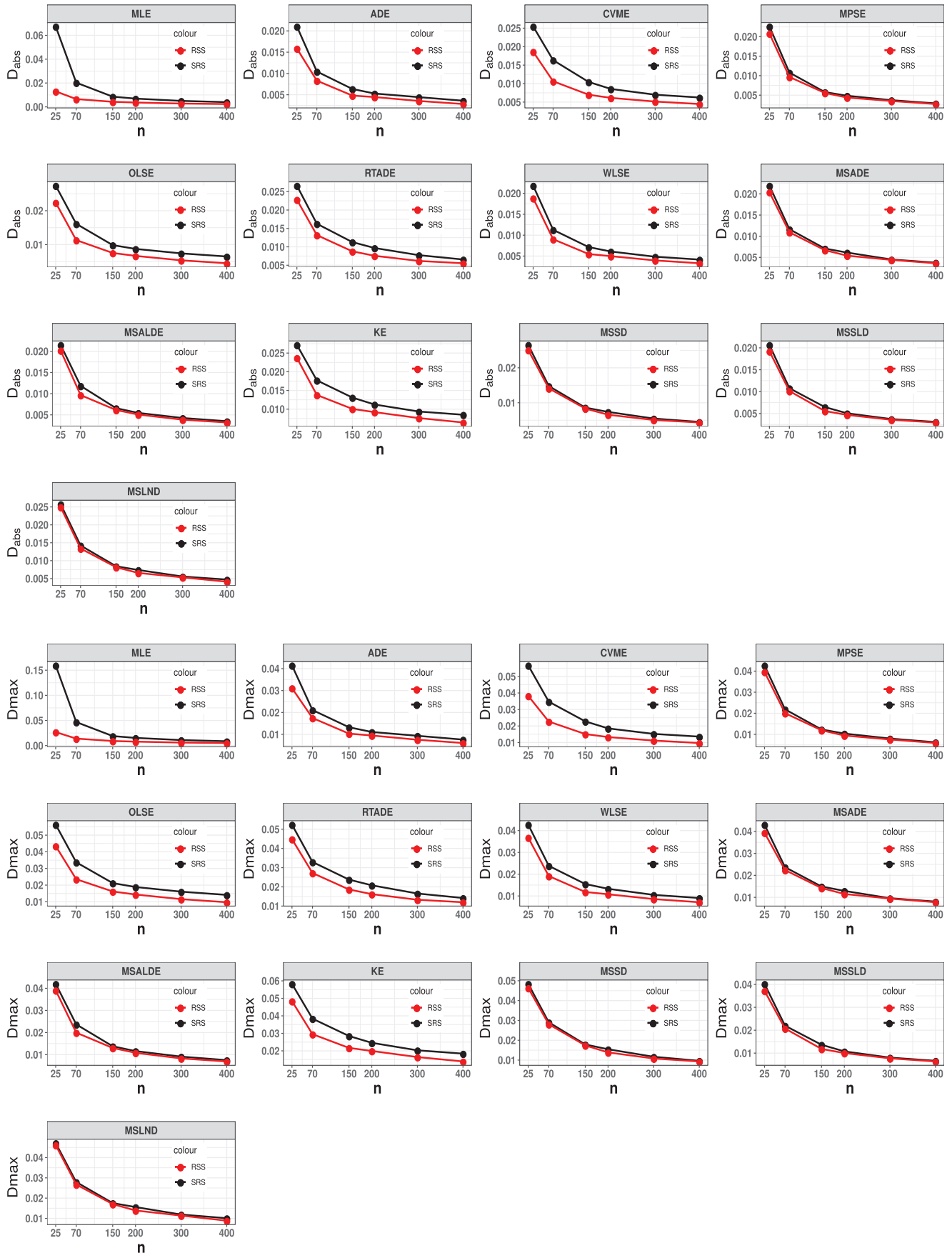


FIGURE 5 | A graphical representation for the numerical values of D_{abs} and D_{max} presented in Tables 1 and 2.

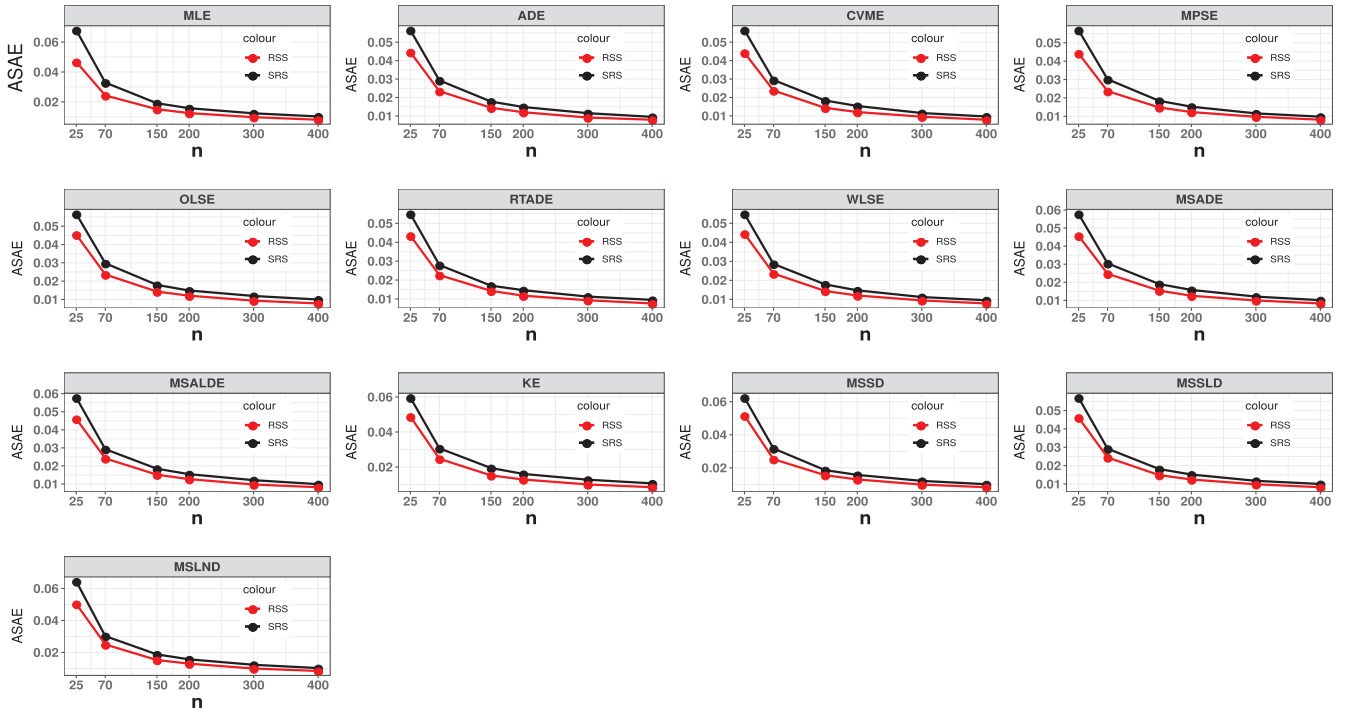


FIGURE 6 | A graphical representation for the numerical values of ASAE presented in Tables 1 and 2.

6 | Numerical Simulation

This section evaluates the effectiveness of various estimation techniques for a proposed model. SRS and RSS datasets will be generated and applied to determine the most suitable one. The simulation will proceed assuming faultless ranking as follows:

- Generate an SRS from the proposed model with sample sizes of $n = 30, 70, 150, 200, 300$, and 400 .
- To create an RSS from the proposed model, the perfect ranking is considered in this study with a fixed set size of $t = 5$ and varying cycle numbers $b = 6, 14, 30, 40, 60$, and 80 , we calculate the corresponding sample sizes $n = tb$, resulting in $n = bt = 30, 70, 150, 200, 300$, and 400 .
- Obtain our proposed model estimate of λ for the two methods of generation.
- Six metrics are used to assess the estimation methods, defined as follows
 1. The average of absolute bias (BIAS), computed by the formula: $|Bias(\hat{\lambda})| = \frac{1}{M} \sum_{i=1}^M |\hat{\lambda}_i - \lambda|$.
 2. The mean squared error (MSE), determined as follows: $MSE = \frac{1}{M} \sum_{i=1}^M (\hat{\lambda}_i - \lambda)^2$.
 3. The mean absolute relative error (MRE), evaluated using the expression: $MRE = \frac{1}{M} \sum_{i=1}^M |\hat{\lambda}_i - \lambda| / \lambda$.
 4. The average absolute difference, denoted as D_{abs} , calculated by: $D_{abs} = \frac{1}{nH} \sum_{i=1}^H \sum_{j=1}^n |G(w_{kj}; \lambda) - G(w_{kj}; \hat{\lambda})|$, where $G(w_{kj}; \lambda) = G(w_{kj})$ and w_{kj} represents values obtained at the k -th iteration sample and j -th component of this sample.

5. The maximum absolute difference, represented by D_{max} , obtained from:

$$D_{max} = \frac{1}{H} \sum_{i=1}^H \max_{j=1, \dots, n} |G(w_{ij}; \lambda) - G(w_{ij}; \hat{\lambda})|.$$

6. The average squared absolute error (ASAE), computed as: $ASAE = \frac{1}{H} \sum_{i=1}^H \frac{|w_{(i)} - \hat{w}_{(i)}|}{w_{(n)} - w_{(1)}}$, where $w_{(i)}$ denotes the ascending ordered observations.

- These metrics serve as impartial standards for evaluating the precision and reliability of the estimated parameters, providing insights into the effectiveness and suitability of the methods for the specific model.
- Through multiple iterations, a dependable evaluation of the techniques is obtained, ensuring consistent performance outcomes
- Evaluation metric results are presented in Tables 1–10, offering a detailed overview of the outcomes and facilitating comparison between estimation approaches.
- The numerical values in Tables 1 and 2 are provided in Figures 2–6 as a graphical representation of Bias, MSE, MRE, D_{abs} , D_{max} , and ASAE for different estimators as a function of sample size. In all cases, the metrics decrease as n increases, highlighting improved estimation accuracy with larger samples. The use of black and red lines effectively differentiates RSS and SRS methods, while the consistent layout across figures ensures comparability. Minor adjustments in subplot spacing and legend positioning could enhance readability, but overall, the visualizations clearly illustrate the performance trends of the estimators.

TABLE 11 | Numerical values for MSE of SRS divided by MSE of RSS for all estimates.

<i>n</i>	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	MSADE	MSALDE	KE	MSSDE	MSSLDE	MSLNDE
$\lambda = 2.0$													
30	2.74533	1.59389	1.49038	1.39326	1.23299	1.72030	1.19919	1.12242	1.07994	1.20023	1.15984	1.18602	1.17748
70	3.66479	2.87959	3.47581	1.70099	2.59629	2.25645	2.29031	1.31521	2.12767	1.78631	1.27405	1.31230	1.30094
150	3.20748	3.54338	2.91885	1.21725	2.19897	2.21867	1.88412	1.24826	1.36748	1.72275	1.19519	1.21218	1.22365
200	2.92558	1.48324	2.31720	1.32537	1.81344	1.91457	1.32850	1.65766	1.20564	1.59826	1.38162	1.34100	1.21967
300	2.97436	1.58898	1.86189	1.17241	2.28000	2.11076	1.45000	1.10165	1.52490	1.53786	1.00672	1.10256	1.06835
400	2.55294	1.78102	1.90311	1.16807	1.96417	1.81425	1.55346	1.17674	1.42614	1.81331	1.14244	1.07947	1.37542
$\lambda = 2.5$													
30	2.21542	1.85746	1.84153	1.31333	1.62091	1.85403	1.78050	1.07296	1.02466	1.68069	1.01410	1.10951	1.12489
70	1.93529	2.38895	2.33686	1.47887	2.21867	1.90779	1.99956	1.25467	1.17857	2.02987	1.20985	1.16807	1.17279
150	1.61665	2.37364	2.56810	1.24868	2.10242	1.98933	2.16973	1.25249	1.39408	2.01017	1.10514	1.43078	1.09166
200	1.45139	2.29553	2.61951	1.41229	2.16811	1.90454	2.50879	1.33966	1.18525	1.91450	1.07439	1.27802	1.11130
300	1.66292	2.30216	2.50548	1.28946	2.66512	1.68635	2.02418	1.29823	1.25263	1.83836	1.13770	1.27573	1.14438
400	1.56147	2.00684	2.35789	1.33849	1.97600	1.71891	2.08764	1.11693	1.09981	1.78592	1.10346	1.06304	1.27556
$\lambda = 3.0$													
30	1.99605	1.81985	2.02169	1.29945	1.97025	1.95686	1.85054	1.18888	1.14128	1.99917	1.08411	1.23437	1.03908
70	1.56573	2.00358	2.21095	1.43246	2.15439	2.03964	2.08268	1.26193	1.15785	2.00036	1.23065	1.46264	1.36222
150	1.77346	2.23312	2.25618	1.47323	2.44738	2.07356	2.32273	1.06435	1.23138	2.08332	1.27884	1.16428	1.23678
200	1.93710	2.17452	2.22518	1.41255	2.17729	2.21646	2.18379	1.27091	1.19823	1.83932	1.11015	1.06070	1.35409
300	1.95792	2.09337	2.52916	1.58030	2.40148	1.97687	2.26696	1.18650	1.15138	1.80462	1.18139	1.22146	1.23205
400	1.79643	2.17832	2.68685	1.37317	2.32397	1.98995	2.13183	1.23047	1.06442	2.38148	1.22168	1.37604	1.16732
$\lambda = 3.5$													
30	1.96802	2.13798	2.07823	1.33787	1.83034	1.90107	1.97866	1.20053	1.05818	1.86168	1.17992	1.32586	1.17858
70	1.88326	2.65484	2.34282	1.54820	2.38258	2.11856	2.39111	1.30133	1.32307	2.12662	1.23940	1.43754	1.30440
150	1.90613	2.23108	2.47920	1.43087	2.62522	2.26697	2.44593	1.25864	1.31071	2.38159	1.25523	1.41568	1.39156
200	2.13599	2.47594	2.43414	1.52728	2.36340	2.28436	2.75058	1.27216	1.50172	2.55374	1.19594	1.52803	1.15269
300	1.84840	2.61015	2.49822	1.46377	2.46151	2.13060	2.31978	1.40364	1.44992	2.22670	1.30135	1.38108	1.38201
400	2.30136	2.72975	2.96964	1.53852	2.82820	2.35036	2.48627	1.38599	1.34130	2.37571	1.26583	1.34540	1.33850
$\lambda = 4.0$													
30	2.07458	2.65172	2.46258	1.49306	2.34812	2.29256	2.48547	1.47540	1.13236	2.04720	1.22158	1.53391	1.20887
70	2.15151	2.65986	2.63086	1.52052	2.72794	2.29385	2.80870	1.44132	1.34953	2.23175	1.46424	1.45427	1.32525
150	2.27878	2.74535	3.08673	1.74069	2.45025	2.37210	2.53352	1.46554	1.39619	2.86976	1.44926	1.53978	1.16615
200	2.45192	2.64669	2.90985	1.74746	3.11851	2.15625	2.69067	1.25405	1.52143	2.69815	1.35870	1.41792	1.22588
300	2.21302	2.71726	2.73440	1.81445	3.10092	2.38764	3.08323	1.43081	1.54597	2.83051	1.19804	1.38686	1.24954
400	1.92150	2.84039	2.71741	1.68477	2.81691	2.35126	2.75376	1.47946	1.31293	2.46173	1.30579	1.46024	1.26938

- The MSE ratio for SRS to RSS is provided in Table 11, aiding in evaluating the relative performance of the sampling methods
- Partial and total ranks of the estimates are presented in Tables 12 and 13 for SRS and RSS, offering a comprehensive view of their performance.

After careful examination of the simulation outcomes and rankings, we have the following results:

1. Consistency in model estimates is observed for both SRS and RSS datasets, indicating convergence to true parameter values as sample size increases.

2. All metrics consistently decrease as sample size increases, indicating improved precision with larger samples.
3. The MPS and the ML methods appear to be particularly effective in assessing the quality of estimates for SRS and RSS, respectively.
4. RSS datasets demonstrate higher efficiency compared with SRS datasets, suggesting RSS as a more efficient sampling method with lower MSE and other measures.

In Table 14, we check the impact of increasing the set size ($t = 2, 5$) on the simulation results for all estimation methods. We need to have sample sizes equal to 20, 60, and 130, then for $t = 2$, we will take cycle numbers equal to 10, 30, and 65, for $t = 5$, we will take

TABLE 12 | Partial and overall ranks for all estimation methods of our proposed model by SRS.

True value	<i>n</i>	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	MSADE	MSALDE	KE	MSSDE	MSSLDE	MSLNDE
$\lambda = 2.0$	30	9.0	1.5	4.0	7.0	11.0	10.0	3.0	5.0	6.0	8.0	13.0	1.5	12.0
	70	7.5	1.5	9.0	3.5	11.5	10.0	1.5	5.0	6.0	11.5	13.0	3.5	7.5
	150	7.0	2.0	11.0	1.0	10.0	12.0	4.0	6.0	5.0	13.0	9.0	3.0	8.0
	200	6.0	2.0	10.0	1.0	11.0	12.0	4.0	7.0	5.0	13.0	9.0	3.0	8.0
	300	7.0	3.0	9.0	1.0	11.5	11.5	5.0	6.0	4.0	13.0	8.0	2.0	10.0
	400	6.0	3.0	9.0	1.0	11.5	11.5	5.0	7.0	4.0	13.0	8.0	2.0	10.0
$\lambda = 2.5$	30	7.0	5.5	8.0	5.5	10.0	4.0	3.0	2.0	11.0	9.0	12.0	1.0	13.0
	70	1.0	6.0	9.0	2.5	11.0	5.0	4.0	8.0	7.0	10.0	13.0	2.5	12.0
	150	1.0	3.0	10.0	2.0	9.0	6.0	5.0	7.0	8.0	11.0	13.0	4.0	12.0
	200	1.0	4.0	9.0	2.0	10.0	6.5	6.5	8.0	5.0	13.0	11.0	3.0	12.0
	300	1.0	5.0	10.0	2.0	12.0	6.0	3.0	8.0	7.0	13.0	9.0	4.0	11.0
	400	1.0	4.0	12.0	2.0	9.0	7.0	5.0	8.0	6.0	13.0	11.0	3.0	10.0
$\lambda = 3.0$	30	6.0	1.0	10.0	3.0	8.0	5.0	4.0	7.0	11.0	9.0	13.0	2.0	12.0
	70	1.0	3.0	10.5	2.0	9.0	4.0	6.0	7.0	8.0	10.5	12.0	5.0	13.0
	150	1.0	5.0	9.0	2.0	10.0	4.0	7.0	6.0	8.0	11.0	13.0	3.0	12.0
	200	1.0	3.5	10.0	2.0	9.0	5.0	6.5	6.5	8.0	11.0	12.0	3.5	13.0
	300	1.0	3.0	10.0	2.0	9.0	5.0	7.0	8.0	6.0	12.0	13.0	4.0	11.0
	400	2.0	5.0	10.5	1.0	9.0	3.0	7.0	8.0	4.0	13.0	12.0	6.0	10.5
$\lambda = 3.5$	30	3.5	5.0	11.0	6.0	7.0	3.5	8.0	2.0	10.0	9.0	13.0	1.0	12.0
	70	1.0	7.0	10.0	2.0	9.0	3.0	5.0	6.0	8.0	11.0	13.0	4.0	12.0
	150	1.0	5.0	10.0	2.0	9.0	3.0	7.0	6.0	8.0	11.5	11.5	4.0	13.0
	200	1.0	4.0	10.0	2.0	9.0	3.0	6.0	7.0	8.0	13.0	11.0	5.0	12.0
	300	1.0	6.0	9.0	2.0	10.0	3.0	4.0	8.0	7.0	12.5	12.5	5.0	11.0
	400	1.0	7.0	10.0	2.0	9.0	3.0	5.5	8.0	5.5	13.0	11.0	4.0	12.0
$\lambda = 4.0$	30	1.0	6.0	11.0	3.0	9.0	4.0	8.0	2.0	7.0	10.0	12.0	5.0	13.0
	70	1.0	6.0	10.0	2.0	9.0	3.0	7.0	5.0	8.0	11.0	12.0	4.0	13.0
	150	1.0	4.0	10.0	2.0	8.0	3.0	5.0	9.0	7.0	13.0	12.0	6.0	11.0
	200	1.0	5.0	9.0	2.0	10.0	3.0	6.0	7.0	8.0	11.0	13.0	4.0	12.0
	300	1.0	5.0	10.0	2.0	9.0	3.0	7.0	6.0	8.0	13.0	11.0	4.0	12.0
	400	1.0	5.0	8.0	2.0	9.5	3.0	7.0	9.5	6.0	13.0	11.0	4.0	12.0
$\sum$ ranks		81.0	126.0	288.0	71.5	289.0	165.0	162.0	195.0	209.5	348.0	347.0	106.0	342.0
Overall rank		2	4	9	1	10	6	5	7	8	13	12	3	11

cycle numbers equal to 4, 12, and 26. As expected, performance improves with increasing sample size, and the results also show that the effect of increasing t from 2 to 5 can lead to variations in estimator rankings.

7 | Real Data Analysis

To show how useful our estimation methods are in practice, we carefully picked a real-world dataset and analyzed it thoroughly in this section. We aim to demonstrate their practical applications by conducting an in-depth data analysis. This analysis is a real-world example of how these methods can be used, highlighting their effectiveness and value for research and decision-making. To demonstrate the effectiveness of our proposed model, we analyze a real-world dataset. The RSS technique was chosen for data collection due to its ability to improve estimation accuracy, especially in situations where measurement costs

are high or there are difficulties in directly measuring the units. This prior ranking helps in selecting more representative samples compared with SRS, resulting in enhanced estimation efficiency and reduced variance in the results.

In the real data applications presented in this section, the RSS technique was adopted due to its superior efficiency over SRS, especially in situations where actual measurement is costly or time-consuming. Still, the ranking of sampling units can be easily achieved. In our datasets, ranking was feasible using auxiliary information or visual inspection, such as size, weight, or operational time variables that correlate with the underlying measurement of interest. These features made the RSS method both practical and appropriate for the data collection process in the analyzed applications.

First, this dataset consists of uncensored data published by Maguire et al. [56]. It records the time intervals, in days, between

TABLE 13 | Partial and overall ranks for all estimation methods of our proposed model by RSS.

True value	<i>n</i>	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	MSADE	MSALDE	KE	MSSD	MSSLD	MSLND
$\lambda = 2.0$	30	1.0	2.0	3.0	8.0	11.0	7.0	4.0	6.0	9.0	10.0	13.0	5.0	12.0
	70	1.5	1.5	6.0	4.0	8.0	10.0	3.0	9.0	5.0	11.0	13.0	7.0	12.0
	150	1.0	2.0	6.0	4.0	8.0	10.0	3.0	9.0	7.0	13.0	12.0	5.0	11.0
	200	1.0	3.0	8.0	2.0	9.0	10.0	4.0	7.0	6.0	13.0	11.0	5.0	12.0
	300	1.0	2.0	7.5	3.0	9.0	11.0	5.0	7.5	6.0	13.0	10.0	4.0	12.0
	400	1.0	3.0	8.0	2.0	10.0	11.0	5.0	7.0	6.0	13.0	12.0	4.0	9.0
$\lambda = 2.5$	30	1.0	2.0	5.0	9.0	7.0	3.5	3.5	10.0	11.0	8.0	12.0	6.0	13.0
	70	1.0	2.0	6.0	5.0	8.0	4.0	3.0	11.0	10.0	9.0	13.0	7.0	12.0
	150	1.0	2.0	6.0	5.0	8.0	4.0	3.0	10.0	11.0	9.0	13.0	7.0	12.0
	200	1.0	3.0	6.0	4.0	7.0	5.0	2.0	9.0	10.0	11.0	12.0	8.0	13.0
	300	1.0	2.0	8.0	4.0	6.0	5.0	3.0	11.0	9.0	10.0	13.0	7.0	12.0
	400	1.0	3.0	6.0	4.0	7.0	5.0	2.0	9.0	10.0	11.0	13.0	8.0	12.0
$\lambda = 3.0$	30	1.0	3.0	7.0	9.0	5.0	2.0	4.0	10.0	11.0	6.0	13.0	8.0	12.0
	70	1.0	4.0	6.0	7.0	5.0	2.0	3.0	10.0	11.0	9.0	13.0	8.0	12.0
	150	1.0	4.0	7.0	6.0	5.0	2.0	3.0	11.0	10.0	9.0	13.0	8.0	12.0
	200	1.0	3.0	7.0	5.0	6.0	2.0	4.0	10.0	11.0	9.0	13.0	8.0	12.0
	300	1.0	2.0	7.0	5.0	6.0	4.0	3.0	11.0	10.0	9.0	13.0	8.0	12.0
	400	1.0	3.0	6.0	5.0	7.0	2.0	4.0	11.0	10.0	9.0	13.0	8.0	12.0
$\lambda = 3.5$	30	1.0	2.0	6.0	9.0	5.0	3.0	4.0	10.0	11.0	7.0	13.0	8.0	12.0
	70	1.0	2.0	6.0	7.0	5.0	4.0	3.0	10.0	11.0	9.0	13.0	8.0	12.0
	150	1.0	4.0	6.0	7.0	5.0	2.0	3.0	10.0	11.0	9.0	12.0	8.0	13.0
	200	1.0	3.0	6.5	6.5	5.0	2.0	4.0	11.0	10.0	8.0	12.0	9.0	13.0
	300	1.0	3.0	5.0	6.0	7.0	4.0	2.0	11.0	10.0	8.0	13.0	9.0	12.0
	400	1.0	2.0	6.0	7.0	5.0	3.0	4.0	11.0	9.0	10.0	12.0	8.0	13.0
$\lambda = 4.0$	30	1.0	2.0	6.0	9.0	5.0	3.0	4.0	8.0	11.0	7.0	12.0	10.0	13.0
	70	1.0	2.5	6.0	7.0	5.0	4.0	2.5	10.0	11.0	8.0	12.0	9.0	13.0
	150	1.0	2.0	5.0	6.5	6.5	3.0	4.0	11.0	10.0	8.0	12.0	9.0	13.0
	200	1.0	2.0	5.0	7.0	6.0	4.0	3.0	11.0	10.0	8.0	12.0	9.0	13.0
	300	1.0	5.0	7.0	6.0	4.0	3.0	2.0	10.0	11.0	8.0	12.5	9.0	12.5
	400	1.0	2.0	7.0	5.5	5.5	3.0	4.0	11.0	10.0	8.0	12.0	9.0	13.0
$\sum$ ranks		30.5	78.0	187.0	174.5	196.0	137.5	101.0	292.5	288.0	280.0	372.5	226.0	366.5
Overall rank		1	2	6	5	7	4	3	11	10	9	13	8	12

109 consecutive coal-mining disasters that occurred in Great Britain between 1875 and 1951. The data are presented in sorted order below: 1, 4, 4, 7, 11, 13, 15, 15, 17, 18, 19, 19, 20, 20, 22, 23, 28, 29, 31, 32, 36, 37, 47, 48, 49, 50, 54, 54, 55, 59, 59, 61, 61, 66, 72, 72, 75, 78, 78, 81, 93, 96, 99, 108, 113, 114, 120, 120, 120, 123, 124, 129, 131, 137, 145, 151, 156, 171, 176, 182, 188, 189, 195, 203, 208, 215, 217, 217, 217, 224, 228, 233, 255, 271, 275, 275, 275, 286, 291, 312, 312, 312, 315, 326, 326, 329, 330, 336, 338, 345, 348, 354, 361, 364, 369, 378, 390, 457, 467, 498, 517, 566, 644, 745, 871, 1312, 1357, 1613, 1630.

Figure 7 shows several graphs and statistics that help us understand the data better. These include total time on test plots (TTT), the estimated risk of failure (hazard rate), density plots, and violin and box plots summarizing the data’s distribution. We can identify important features and patterns in the data by looking at all these elements together.

We checked how well the data fit a particular model using a statistical test called the Kolmogorov-Smirnov test. To fit the RLD model to the data, we estimated its parameters using a method called ML as 232.3423 with a standard error (StEr) of 22.35159. The KS test resulted in a KS distance (KSD) of 0.078652 and a *p*-value of KS (PVKS) of 0.5101. The results so far suggest that the RLD is a good option for analyzing real data. Figure 8 provides more visual evidence to support this fitting. The figure shows several plots, including a P-P plot, the estimated CDF, and a histogram with the estimated PDF. When we look at all these elements together, it seems like the RLD is a good fit for the data set because the plots closely resemble the expected patterns for the RLD.

Following the theoretical analysis, we explored two sampling methods (SRS and RSS) on the actual data. Tables 15 and 16 summarize the results for each method using the RLD. The tables

TABLE 14 | Numerical results of simulation for all measures when ($\lambda = 3.25$) under RSS for $t = 2$ and $t = 5$.

n		Estimate	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	MSADE	MSALDE	KE	MSSE	MSSLDE	MSLNDE
$t = 2.0$															
20	BIAS	...	0.6317	0.68833	0.77169	0.81964	0.80372	1.88765	0.98603	0.74983	0.92384	0.63558	1.1213	0.72088	1.01592
	MSE	...	0.65047	0.76559	0.89395	1.09161	1.04867	4.35423	1.59591	0.92388	1.34685	0.94592	1.93422	0.8682	1.64057
	MRE	...	0.19437	0.21179	0.23744	0.2522	0.2473	0.58082	0.30339	0.23072	0.28426	0.19556	0.34501	0.22181	0.31259
	D_{abs}	...	0.02316	0.02554	0.02869	0.03014	0.02983	0.06939	0.03694	0.02727	0.03377	0.02346	0.04071	0.02643	0.03702
	D_{max}	...	0.04128	0.0442	0.0497	0.05143	0.05078	0.11006	0.06046	0.04765	0.05751	0.03931	0.0683	0.04619	0.06235
60	ASAE	...	0.05374	0.0528	0.05445	0.05436	0.05449	0.13608	0.07982	0.05805	0.057	0.07643	0.06848	0.05473	0.06722
	BIAS	...	0.42709	0.46137	0.50181	0.46606	0.47926	2.33224	0.94404	0.52936	0.52738	0.55682	0.6817	0.46971	0.69241
	MSE	...	0.27898	0.3302	0.36517	0.34345	0.36522	5.7508	1.5148	0.45181	0.4627	0.81987	0.72773	0.36007	0.75993
	MRE	...	0.13141	0.14196	0.1544	0.1434	0.14746	0.71761	0.29047	0.16288	0.16227	0.17133	0.20975	0.14453	0.21305
	D_{abs}	...	0.01539	0.01692	0.0181	0.01695	0.01751	0.08424	0.03517	0.01925	0.01933	0.0206	0.02498	0.01703	0.02528
130	D_{max}	...	0.02863	0.03051	0.03366	0.03058	0.03157	0.13529	0.05832	0.03483	0.03424	0.03443	0.04347	0.03093	0.04416
	ASAE	...	0.02512	0.025	0.02662	0.02516	0.02552	0.12255	0.04817	0.02794	0.02724	0.04376	0.0326	0.02718	0.03258
	BIAS	...	0.28114	0.30972	0.36297	0.31176	0.35491	2.57827	0.8776	0.35684	0.34777	0.49119	0.44983	0.33211	0.47694
	MSE	...	0.128	0.15254	0.20721	0.1608	0.19455	6.81283	1.25673	0.21514	0.203	0.72765	0.31642	0.18523	0.35534
	MRE	...	0.08651	0.0953	0.11168	0.09593	0.1092	0.79331	0.27003	0.1098	0.10701	0.15113	0.13841	0.10219	0.14675
20	D_{abs}	...	0.01	0.01115	0.01307	0.01124	0.01276	0.09292	0.03267	0.01287	0.01247	0.01819	0.01643	0.012	0.01739
	D_{max}	...	0.01889	0.0206	0.02411	0.02052	0.02357	0.1481	0.05468	0.02368	0.02289	0.03027	0.02933	0.02193	0.03115
	ASAE	...	0.01539	0.0151	0.01593	0.01509	0.0162	0.11178	0.038	0.01797	0.01751	0.02985	0.01971	0.01661	0.01975
$t = 5.0$															
20	BIAS	...	0.56543	0.5984	0.62952	0.74413	0.59996	1.17926	0.7338	0.73658	0.81733	0.77288	1.0848	0.66639	0.97735
	MSE	...	0.47917	0.57626	0.5786	0.86394	0.59718	1.92324	0.82997	0.92841	1.14231	1.14443	1.74523	0.73619	1.50583
	MRE	...	0.17398	0.18412	0.1937	0.22896	0.1846	0.36285	0.22578	0.22664	0.25149	0.23781	0.33378	0.20504	0.30072
	D_{abs}	...	0.02044	0.02187	0.02306	0.02759	0.02193	0.04371	0.02733	0.02731	0.02998	0.02792	0.0395	0.0243	0.0355
	D_{max}	...	0.03755	0.03879	0.0411	0.04689	0.03837	0.07153	0.04733	0.04659	0.05104	0.04804	0.06647	0.04267	0.06027
60	ASAE	...	0.04892	0.04866	0.04851	0.04762	0.04762	0.06413	0.05651	0.051	0.05017	0.06546	0.06104	0.04831	0.06168
	BIAS	...	0.32403	0.35094	0.38886	0.41749	0.40184	1.18324	0.71932	0.49918	0.49876	0.66027	0.62374	0.43271	0.6411
	MSE	...	0.16474	0.20283	0.24115	0.27033	0.24799	1.61539	0.72679	0.39045	0.40801	0.83577	0.59649	0.29763	0.63064
	MRE	...	0.0997	0.10798	0.11965	0.12846	0.12364	0.36407	0.22133	0.15359	0.15347	0.20316	0.19192	0.13314	0.19726
	D_{abs}	...	0.01161	0.01263	0.01402	0.01527	0.01451	0.04386	0.02693	0.01822	0.01812	0.02385	0.02288	0.01579	0.02346
130	D_{max}	...	0.02173	0.02336	0.02592	0.02727	0.02671	0.07312	0.0492	0.03262	0.03249	0.04177	0.04003	0.0284	0.04114
	ASAE	...	0.02311	0.02306	0.02412	0.02254	0.02459	0.04947	0.03615	0.02703	0.02615	0.03967	0.03023	0.02519	0.03081
	BIAS	...	0.23304	0.25203	0.27324	0.26899	0.26957	1.18008	0.86567	0.33662	0.3519	0.63726	0.40626	0.29046	0.41516
	MSE	...	0.08433	0.0998	0.11344	0.11601	0.11165	1.48664	0.90326	0.18414	0.1943	0.75291	0.26239	0.13186	0.2672
	MRE	...	0.0717	0.07755	0.08407	0.08276	0.08295	0.3631	0.26636	0.10357	0.10828	0.19608	0.125	0.08937	0.12774
20	D_{abs}	...	0.00828	0.00896	0.00972	0.00967	0.00962	0.04415	0.03352	0.01212	0.01267	0.02321	0.0148	0.01041	0.01512
	D_{max}	...	0.01563	0.01685	0.01831	0.01777	0.01796	0.0734	0.06086	0.02238	0.02324	0.04045	0.02668	0.01924	0.02718
	ASAE	...	0.01426	0.0139	0.01479	0.01413	0.01475	0.04523	0.03397	0.01664	0.0166	0.03027	0.01866	0.01509	0.01841

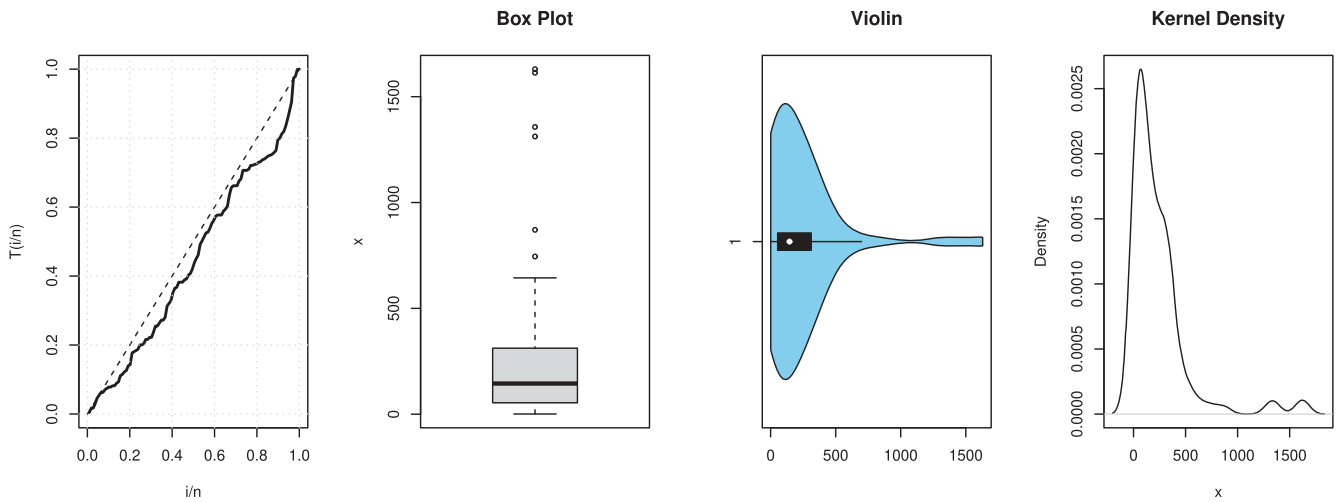


FIGURE 7 | Some plots for coal mining disasters data.

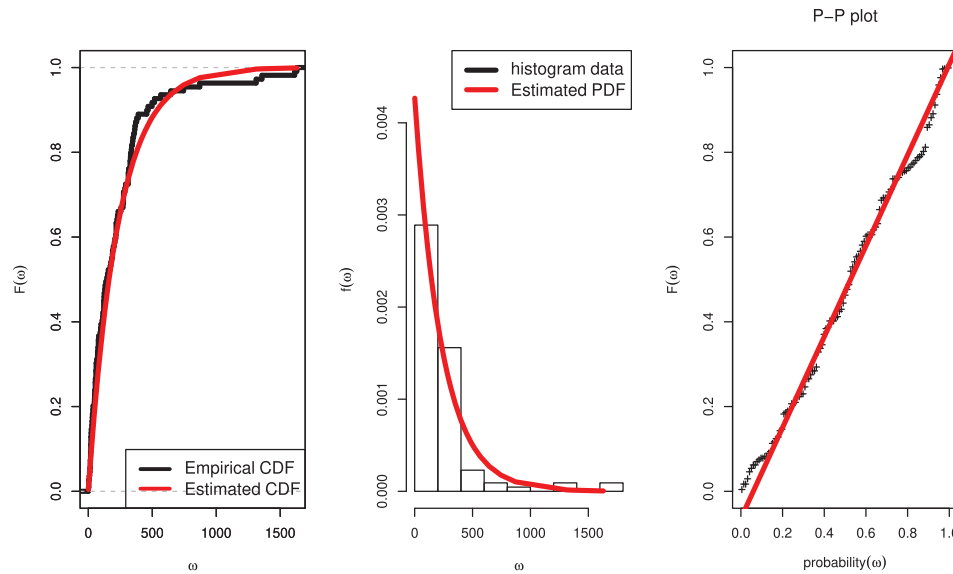


FIGURE 8 | Fitting of the RLD by estimated CDF, PDF, and PP-plot: Coal Mining Disasters data.

show estimates obtained with different sample sizes across different cycles using various estimation techniques. An R software package was used to generate the data for both methods (SRS and RSS).

These tables provide a comprehensive comparison of the sampling methods and estimation techniques. We compared them using goodness-of-fit tests to see whether RSS performs better than SRS for different estimation methods. We used three tests: Anderson-Darling (A^*), Cramer-von Mises (C^*), KSD, and PVKS. These tests assess how well the data fits the model. By comparing the test results for SRS and RSS, we can see which sampling method better captures the actual distribution of the data.

Figure 9 suggests that our estimates are not just the highest values (maximum) but also the only ones (unique). The graph of the derivative, which shows how the function changes, is constantly decreasing. Additionally, the curve representing the function

itself touches the x-axis at only one point. Since the derivative is related to the slope of the function, a constantly decreasing derivative with a single point touching the x-axis indicates a unique maximum value. Therefore, we can be confident that these estimates are the only maximum points and are thus unique.

The estimates that performed better generally had two characteristics:

Higher p -values: A p -value greater than 5% indicates that we cannot reject the null hypothesis, which in this case suggests a good fit between the data and the model. **Lower goodness-of-fit values:** Lower values of goodness-of-fit statistics typically indicate a better fit. Tables 15 and 16 compares the performance of SRS and RSS methods by showing their goodness-of-fit values and PVKS associated with the KS test. By analyzing this table, we can determine which design (SRS or RSS) and estimation techniques achieve a better fit for the data according to the chosen model.

TABLE 15 | Estimates, StEr, KSD, PVKS, C*, and A* for parameter based on SRS, and RSS: Data I size 15 (cycles is 3).

	SRS					RSS				
	Estimates	KSD	PVKS	C*	A*	Estimates	KSD	PVKS	C*	A*
ML	427.2921	0.1709	0.7120	0.0428	0.2800	197.2297	0.1402	0.9296	0.0365	0.2569
AD	392.7130	0.1403	0.8905	0.0435	0.2845	233.4314	0.1401	0.9257	0.0363	0.2561
CVM	359.3592	0.1168	0.9718	0.0442	0.2893	228.3943	0.1137	0.9739	0.0364	0.2562
MPS	475.5757	0.2091	0.4662	0.0419	0.2745	217.7302	0.1290	0.9642	0.0364	0.2564
KE	232.3423	0.2707	0.1841	0.0481	0.3143	232.0213	0.1402	0.9296	0.0363	0.2561
OLS	362.3542	0.1187	0.9672	0.0442	0.2888	207.7921	0.1123	0.9762	0.0364	0.2566
WLS	371.6797	0.1247	0.9510	0.0439	0.2875	209.4087	0.1218	0.9792	0.0364	0.2566
RTAD	428.5627	0.1720	0.7050	0.0428	0.2799	285.6177	0.1687	0.7588	0.0363	0.2554
MSAD	349.4993	0.1227	0.9468	0.0445	0.2908	231.2713	0.1204	0.9573	0.0364	0.2561
MSALD	519.5029	0.2399	0.3030	0.0413	0.2701	231.2713	0.1397	0.9316	0.0364	0.2561
MSLND	204.0256	0.3150	0.0804	0.0493	0.3221	235.6096	0.1429	0.9195	0.0363	0.2560
MSSqD	220.9067	0.2882	0.1348	0.0486	0.3173	235.7912	0.1430	0.9190	0.0363	0.2560
MSSQLD	462.5065	0.1993	0.5267	0.0422	0.2760	207.0887	0.1246	0.9741	0.0364	0.2566

TABLE 16 | Estimates, StEr, KSD, PVKS, C*, and A* for parameter based on SRS, and RSS: Data I size 50 (cycles is 5).

	SRS					RSS				
	Estimate	KSD	PVKS	C*	A*	Estimate	KSD	PVKS	C*	A*
MLE	254.4875	0.1436	0.7525	0.1173	0.9119	198.7854	0.1275	0.7904	0.0674	0.5477
AD	213.9523	0.0888	0.9083	0.1241	0.9561	217.1856	0.0739	0.9261	0.0676	0.5471
CVM	199.6295	0.0812	0.9762	0.1269	0.9741	219.6361	0.0746	0.9787	0.0676	0.5473
MPS	268.4643	0.1596	0.1566	0.1152	0.8986	196.7649	0.1231	0.3570	0.0682	0.5457
KE	232.3423	0.1153	0.5192	0.1208	0.9349	231.3139	0.1127	0.6399	0.0679	0.5465
OLS	200.0549	0.0811	0.9756	0.1268	0.9736	214.6103	0.0805	0.9807	0.0675	0.5475
WLS	206.9866	0.0785	0.9618	0.1254	0.9647	202.0913	0.0712	0.9645	0.0673	0.5481
RTAD	224.6505	0.1046	0.7064	0.1222	0.9435	231.7297	0.0913	0.7394	0.0681	0.5460
MSAD	201.8724	0.0804	0.9503	0.1264	0.9712	201.8726	0.0712	0.9644	0.0666	0.5511
MSALD	229.9486	0.1120	0.5568	0.1212	0.9376	193.7819	0.1036	0.5631	0.0702	0.5438
MSLND	199.7599	0.0812	0.8966	0.1269	0.9740	225.9892	0.0801	0.9047	0.0665	0.5516
MSSqD	200.4860	0.0809	0.8989	0.1267	0.9730	225.3993	0.0791	0.9148	0.0665	0.5516
MSSQLD	289.7472	0.1817	0.5737	0.1124	0.8798	204.0256	0.1185	0.5840	0.0693	0.5443

Second, this part introduces the Boag data set. These data (‘0.3, 4, 7.4, 15.5, 23.4, 46, 46, 51, 65, 68, 83, 88, 96, 110, 111, 112, 132, 162’) show the ages, in months, of 18 patients who passed away due to causes other than cancer. This data comes from the research by Boag [57], who explored using the Lognormal distribution to model such data.

Figure 10 presents various visualizations and statistical measures aimed at enhancing our comprehension of the Boag dataset. These encompass plots illustrating TTT, the estimated risk of failure (hazard rate), density representations, and violin and box plots offering a concise summary of the distribution of the Boag dataset. We can discern crucial characteristics and trends within the Boag data through a comprehensive examination of these components.

We assessed the suitability of a specific model for the data through a statistical evaluation known as the KS test. Using the ML method, we determined the parameters of the RLD to be 66.74664 with a StEr of 15.96987. The KS test yielded a KSD of 0.21497 and a PVKS of 0.3763. These findings thus far indicate that the RLD is a viable choice for analyzing the actual dataset. Additional visual support for this fit is provided in Figure 11. This figure exhibits various plots, such as a P-P plot, the estimated CDF, and a histogram with the estimated PDF. On examining all these elements collectively, it appears that the RLD adequately captures the characteristics of the dataset, as the plots closely resemble the anticipated patterns for the RLD.

After conducting a theoretical analysis, we examined the performance of two sampling techniques (SRS and RSS) on the

Boag data. Table 17 presents a summary of the outcomes for each method employing the RLD. This table illustrates the estimates for diverse estimation methodologies and compares the performance of the SRS and RSS methods. An R software package facilitated Boag data generation for both the SRS and RSS methods.

Figure 12 indicates that our estimations are not only the maximum values but also singular ones. The derivative graph, depicting the function's rate of change, consistently declines. Furthermore, the curve representing the function intersects the x-axis at a solitary point. Given the relationship between the derivative and the function's slope, a continuously decreasing derivative coupled with a solitary intersection with the x-axis signifies a solitary maximum value. Consequently, we can assert with confidence that these estimations represent the sole maximum points, ensuring their uniqueness.

Third, the final data set, sourced from Pham [58], represents failure times associated with time-dependent dielectric breakdown in metal-oxide-semiconductor integrated circuits. The experiment was performed at three distinct high temperatures. Our study exclusively examines data collected at a temperature of

170°C. These data: 164, 164, 218, 230, 263, 467, 538, 639, 669, 917, 1148, 1678, 1678, 1678, 1678.

Figure 13 displays various visualizations and statistical metrics designed to improve our understanding of the failure times dataset. These include charts depicting TTT, the predicted failure risk (hazard rate), density representations, and violin and box plots that provide a succinct summary of the failure times dataset distribution. A thorough analysis of these components allows us to identify essential traits and trends in the failure times data.

We assessed the suitability of a specific model for the data through a statistical evaluation known as the KS test. Using the ML method, we determined the parameters of the RLD to be 807.7138 with a StEr of 208.836. The KS test yielded a KSD of 0.1835 and a PVKS of 0.693. These findings thus far indicate that the RLD is a viable choice for analyzing the actual dataset. Additional visual support for this fit is provided in Figure 14. This figure exhibits various plots, such as a P-P plot, the estimated CDF, and a histogram with the estimated PDF. On examining all these elements collectively, it appears that the RLD adequately captures the characteristics of the dataset, as the plots closely resemble the anticipated patterns for the RLD.

Figure 15 demonstrates that our estimations represent not only the maximum values but also unique ones. The derivative graph, illustrating the function's rate of change, continually decreases. Moreover, the curve depicting the function intersects the x-axis at a single point. The correlation between the derivative and the function's slope indicates that a consistently declining derivative with a single intersection at the x-axis denotes a unique maximum value. Thus, we may confidently declare that these estimations reflect the only maximum points, guaranteeing their uniqueness.

After conducting a theoretical analysis, we examined the performance of two sampling techniques (SRS and RSS) on the failure times data. Table 18 presents a summary of the outcomes for each method employing the RLD. This table illustrates the estimations derived from 10 sample sizes with cycles 2, utilizing

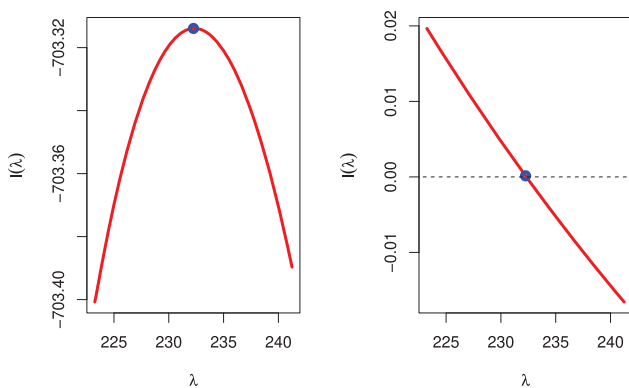


FIGURE 9 | Profile likelihood of the RLD parameter: Coal Mining Disasters data.

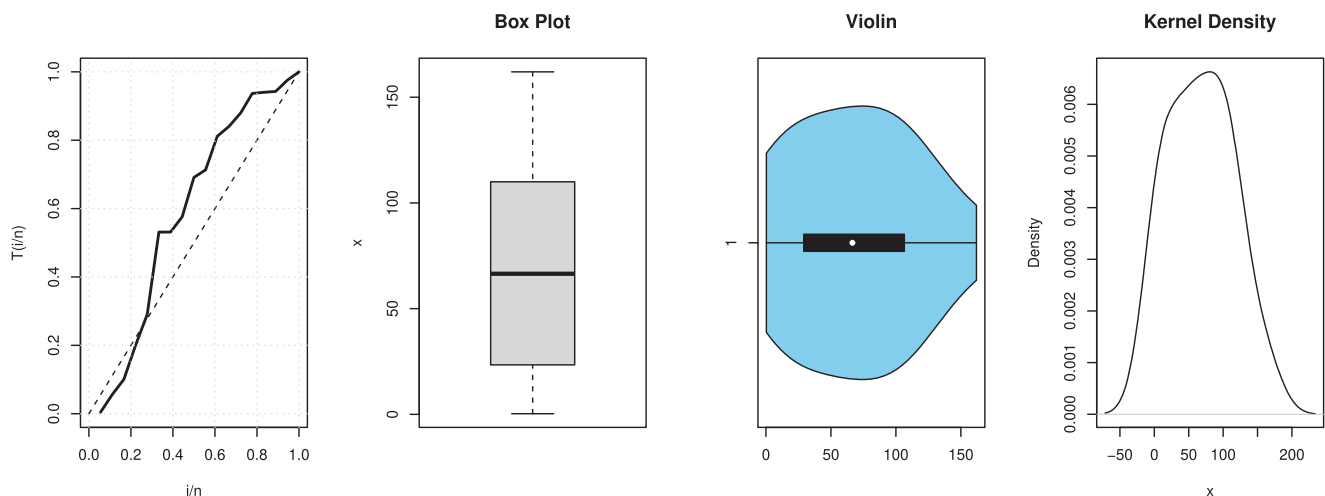


FIGURE 10 | Some plots for Boag data.

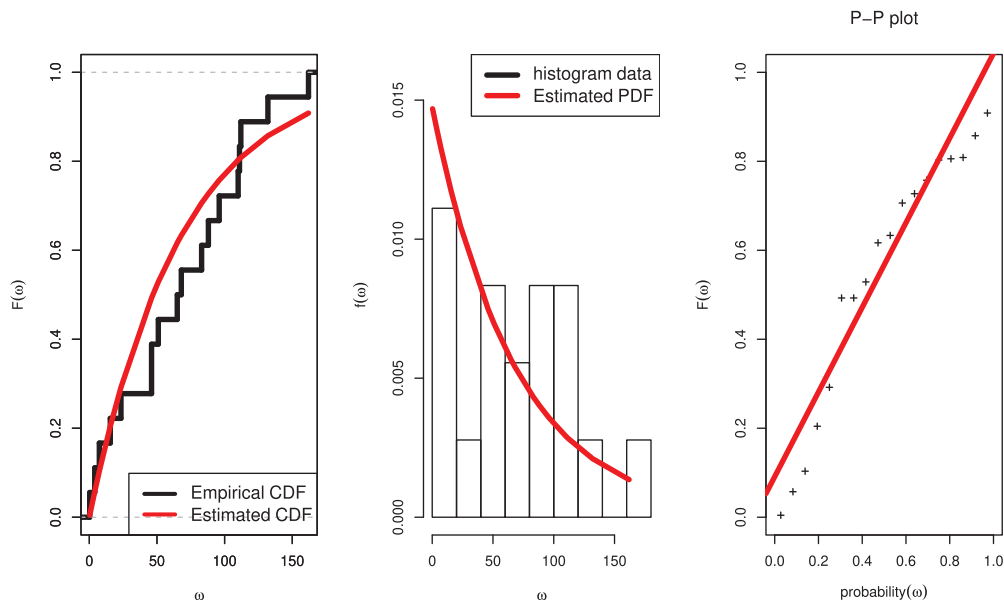


FIGURE 11 | Fitting of the RLD model by estimated CDF, PDF, and PP-plot: Boag data.

TABLE 17 | Estimates, StEr, KSD, PVKS, C*, and A* for parameter based on SRS, and RSS: Data II.

	SRS					RSS				
	Estimates	KSD	PVKS	C*	A*	Estimates	KSD	PVKS	C*	A*
ML	75.6636	0.2511	0.3007	0.1679	1.0112	72.4690	0.2314	0.3981	0.1430	0.8208
AD	91.0075	0.1934	0.6289	0.1730	1.0400	84.4110	0.1764	0.7392	0.1445	0.8287
CVM	94.7098	0.1852	0.6825	0.1741	1.0462	89.2736	0.1756	0.7441	0.1450	0.8315
MPS	80.8521	0.2298	0.4064	0.1697	1.0216	75.6636	0.2159	0.4864	0.1434	0.8230
KE	66.7466	0.2927	0.1528	0.1644	0.9917	3.8976	0.2756	0.1731	0.0868	0.5402
OLS	96.0330	0.1900	0.6513	0.1745	1.0483	86.5588	0.1711	0.7719	0.1447	0.8300
WLS	87.9276	0.2038	0.5617	0.1720	1.0346	79.8939	0.1963	0.6099	0.1439	0.8258
RTAD	86.1123	0.2102	0.5215	0.1715	1.0314	92.6501	0.1808	0.7107	0.1454	0.8335
MSAD	99.5985	0.2026	0.5691	0.1755	1.0540	59.9155	0.2010	0.6141	0.1411	0.8112
MSALD	83.1681	0.2210	0.4566	0.1705	1.0260	60.9451	0.2192	0.5581	0.1413	0.8121
MSLND	86.1123	0.2102	0.5215	0.1715	1.0314	88.3480	0.1741	0.7535	0.1449	0.8310
MSSqD	86.1058	0.2102	0.5214	0.1715	1.0314	87.5528	0.1728	0.7616	0.1448	0.8305
MSSQLD	75.4289	0.2521	0.2962	0.1678	1.0107	84.5935	0.1756	0.7442	0.1445	0.8288

diverse estimation methodologies. An R software package facilitated the generation of failure times data for both the SRS and RSS methods.

Overall, the RSS method seems to be more efficient than the SRS method for fitting this data to the model. This conclusion is based on three observations from Tables 15–18:

- The RSS method consistently achieves lower goodness-of-fit values compared with SRS, even when using the same number of data points. Lower goodness-of-fit values indicate a better fit.
- The KSPV tends to be higher for the RSS method. In general, a higher p -value suggests we cannot reject the null

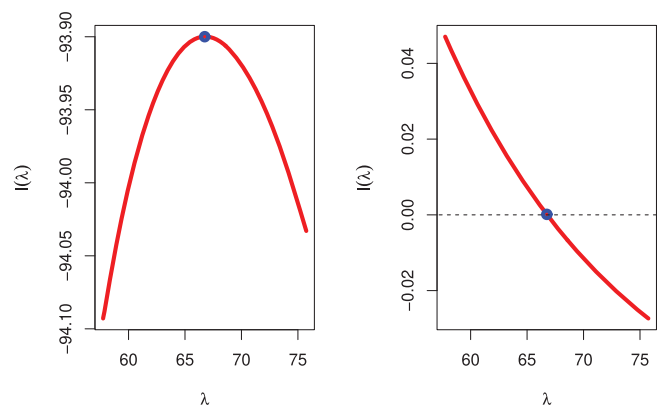


FIGURE 12 | Profile likelihood of the RLD parameter: Boag data.

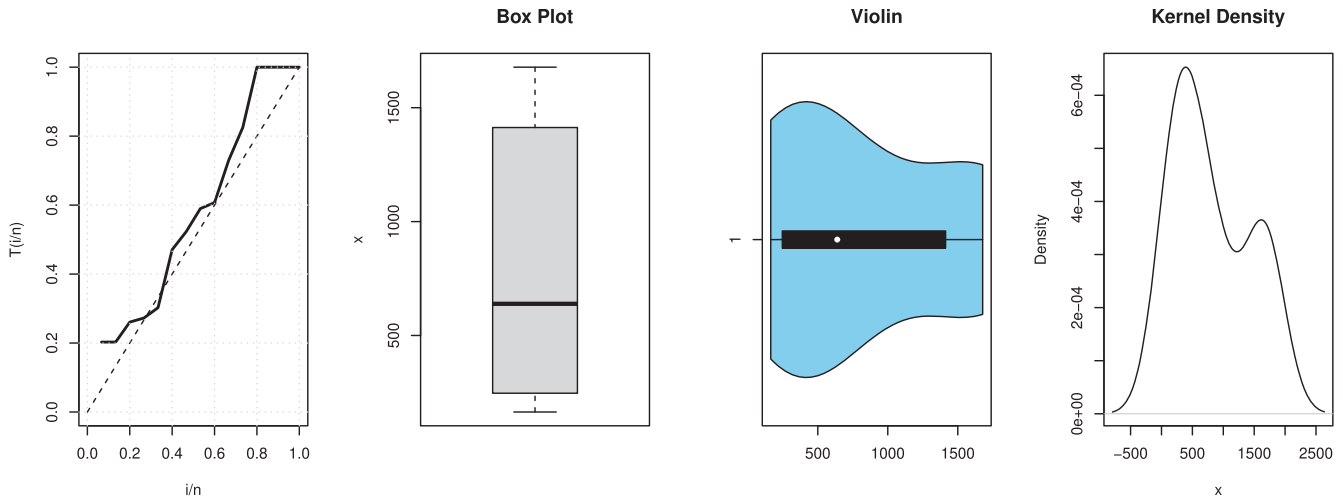


FIGURE 13 | Some plots for failure times data.

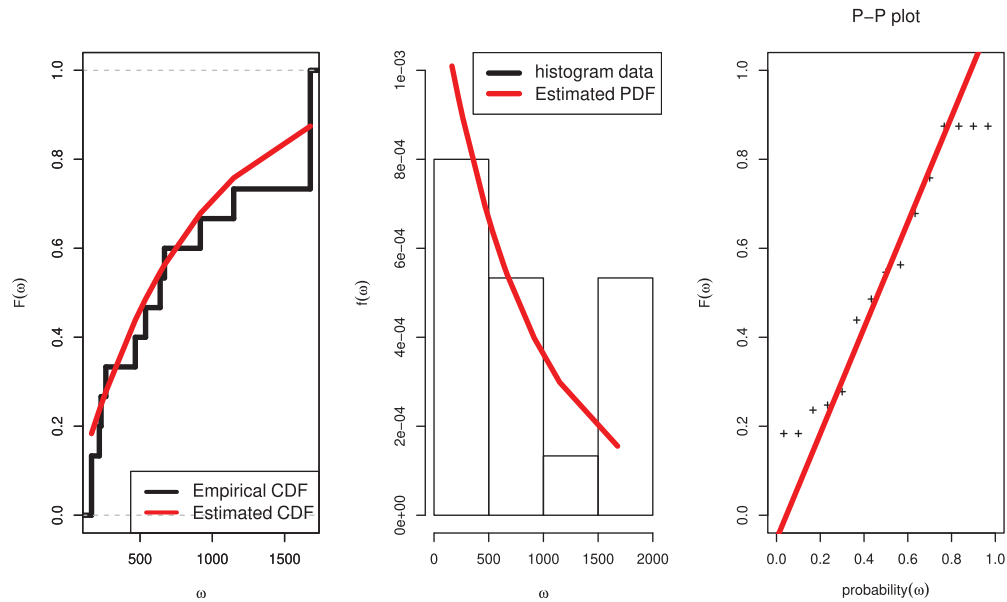


FIGURE 14 | Fitting of the RLD model by estimated CDF, PDF, and PP-plot: failure times data.

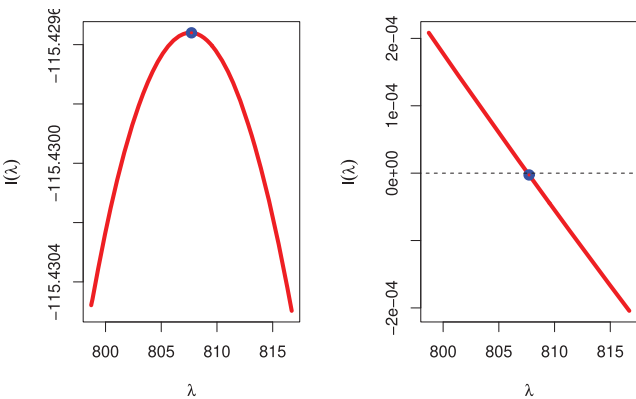


FIGURE 15 | Profile likelihood of the RLD parameter: failure times data.

hypothesis, which, in this case, means the data aligns well with the model.

- These results consistently favor the RSS method across different estimates, highlighting its efficiency in fitting the data and obtaining reliable results.

8 | Concluding Remarks

For any scientific investigation, data collection is essential. Complex data collection methods, such as RSS, use the ranking information of the sample unit to provide representative sample data. The latter often yields significantly superior results when comparing identical procedures based on SRS with statistical ones based on RSS. Selecting a suitable statistical distribution is

TABLE 18 | Estimates, StEr, KSD, PVKS, C*, and A* for parameter based on SRS, and RSS: Data III.

	SRS					RSS				
	Estimates	KSD	PVKS	C*	A*	Estimates	KSD	PVKS	C*	A*
MLE	1039.3005	0.2007	0.8153	0.1752	1.1703	807.7141	0.2004	0.8439	0.0841	0.6136
AD	1266.1545	0.2660	0.4788	0.1745	1.1659	1017.2146	0.2224	0.4848	0.0843	0.6145
CVM	1271.0450	0.2674	0.4722	0.1743	1.1643	1110.2910	0.2393	0.4737	0.0843	0.6145
KS	1119.4409	0.2237	0.6991	0.1748	1.1679	914.9883	0.2150	0.7557	0.0842	0.6139
MSAD	807.5788	0.2783	0.4209	0.1752	1.1703	807.595	0.2447	0.4385	0.0838	0.6124
RTAD	1292.6576	0.2733	0.4438	0.1749	1.1681	908.4414	0.2489	0.5654	0.0844	0.6146
MSSD	1192.324	0.2451	0.5853	0.1749	1.1681	908.047	0.2388	0.6592	0.0843	0.6142
MSALD	1186.5663	0.2434	0.5940	0.1739	1.1620	1253.8495	0.2311	0.6288	0.0843	0.6142
MSAD	813.3931	0.2757	0.4329	0.1757	1.1731	698.7792	0.2309	0.5950	0.0838	0.6125
MSALD	966.8902	0.2234	0.7007	0.1751	1.1696	839.5806	0.2064	0.8787	0.0840	0.6132
MSSqD	964.5239	0.2241	0.6967	0.1751	1.1694	844.3133	0.2163	0.6995	0.0840	0.6132
MSSQLD	1495.5326	0.3259	0.2387	0.1743	1.1644	1103.9807	0.2882	0.3773	0.0846	0.6154

essential for data analysis and modeling in order to get more precise results. An excellent choice for simulating leptokurtic, positively skewed, overdispersed data is the one-parameter RLD. This study uses RSS and SRS techniques to investigate 13 traditional methods for estimating the RLD parameter. The effectiveness of the generated estimators is compared using a Monte Carlo simulation study. In order to assess the accuracy and reliability of the estimated parameters, several accuracy metrics are used as objective benchmarks. These metrics offer information on the effectiveness and appropriateness of the techniques for the proposed distribution. Based on the partial and total rankings metrics, we assess the quality of estimates for SRS and RSS and find that the ML and MPS techniques appear to be extremely helpful for both sampling procedures. Given that RSS data sets outperform SRS data sets in terms of efficiency, analysis suggests that RSS is a more effective sampling strategy. The results of the two real data sets likewise support the superiority of the RSS design over the SRS concept.

Author Contributions

Diaa S. Metwally: conceptualization; methodology; software; writing – original draft; writing – review and editing; formal analysis. **Amal S. Hassan:** conceptualization; writing – original draft; methodology; writing – review and editing; software; formal analysis. **Mohammed Elgarhy:** conceptualization; writing – original draft; writing – review and editing; methodology; software; formal analysis. **Ehab M. Almetwally:** conceptualization; writing – original draft; writing – review and editing; methodology; software; formal analysis. **Abdoulie Faal:** conceptualization; writing – original draft; methodology; writing – review and editing. **Ahmed M. Gemeay:** conceptualization; writing – original draft; methodology; writing – review and editing; software; formal analysis.

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Conflicts of Interest

The authors declare no conflicts of interest.

Data Availability Statement

The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available due to privacy or ethical restrictions.

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