Problems

1) In the case of quantum particle in a potential step with E > U .Using $A+B=C$, $\mathrm{K}_1(A-B)=K_2C$ show that

a)The transmission probability is given by

$$T=\frac{4K_1K_2}{(K_1+K_2)^2}=\frac{4\sqrt{1-\left(\frac{U}{E}\right)}}{\left(1+\sqrt{1-\left(\frac{U}{E}\right)}\right)^2}$$

b) The Reflection probability is given by

$$T=\frac{(K_1-K_2)^2}{(K_1+K_2)^2}=\frac{\left(1-\sqrt{1-\left(\frac{U}{E}\right)}\right)^2}{\left(1+\sqrt{1-\left(\frac{U}{E}\right)}\right)^2}$$

2- In the case of quantum particle in a potential barrier with E > U. Using

$$A+B=C+D,$$
$$K_1(A-B)=K_2(C-D)$$
$$Ce^{iK_2L}+De^{-iK_2L}=Fe^{iK_1L}$$
$$K_2\left(Ce^{iK_2L}-De^{-iK_2L}\right)=K_1Fe^{iK_1L}$$

Show that

a)The transmission probability is given by

$$T=\frac{\dfrac{4K_1^2K_2^2}{\left(K_1^2-K_2^2\right)^2}}{\sin^2(K_2L)+\dfrac{4K_1^2K_2^2}{\left(K_1^2-K_2^2\right)^2}}=\frac{4(E/U)[(E/U)-1]}{\sin^2[\sqrt{2m(E-U)}L/\hbar]+4(E/U)[(E/U)-1]}$$

The Reflection probability is given by

$$R=\frac{\sin^2(K_2L)}{\sin^2(K_2L)+\dfrac{4K_1^2K_2^2}{\left(K_1^2-K_2^2\right)^2}}=\frac{\sin^2[\sqrt{2m(E-U)}L/\hbar]}{\sin^2[\sqrt{2m(E-U)}L/\hbar]+4(E/U)[(E/U)-1]}$$

3- In the case of quantum particle in a potential barrier with E < U. Using

$$
\begin{aligned}
&A+B=C+D \\
&iK(A-B)=\alpha(C-D) \\
&Ce^{\alpha L}+De^{-\alpha L}=Fe^{iKL} \\
&\alpha\left(Ce^{\alpha L}-De^{-\alpha L}\right)=iKFe^{iKL}
\end{aligned}
$$

Show that

a) The transmission probability is given by

$$T=\frac{4(E/U)[1-(E/U)]}{\sinh^2[\sqrt{2m(U-E)}L/\hbar]+4(E/U)[1-(E/U)]}$$

b) The Reflection probability is given by

$$R=\frac{\sinh^2[\sqrt{2m(U-E)}L/\hbar]}{\sinh^2[\sqrt{2m(U-E)}L/\hbar]+4(E/U)[1-(E/U)]}$$