# Problems

For a quantum particle in a harmonic oscillator potential $V(x)=\frac{1}{2}kx^2$ , with states $|n\rangle$, and assume the following two operators:

$a=\sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x}+\frac{i}{m\omega}\hat{p}_x\right)$ , $a^+=\sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x}-\frac{i}{m\omega}\hat{p}_x\right)$.

Prove the following:

1) $\left[a,a^+\right]=1$.

2) $\hat{x}=\sqrt{\frac{\hbar}{2m\omega}}\left(a+a^+\right)$.

3) $\hat{p}=-i\sqrt{\frac{m\omega\hbar}{2}}\left(a-a^+\right)$.

4) $H=\hbar\omega\left(a^+a+\frac{1}{2}\right)$.

5) $\left[H,a\right]=-\hbar\omega a$ .

6) $\left[H,a^+\right]=\hbar\omega a^+$ .

7) $a^+a|n\rangle=n|n\rangle$ .

8) $a|n\rangle = \sqrt{n}|n-1\rangle$.

9) $a^{+}|n\rangle = \sqrt{n+1}|n+1\rangle$.