Problems:

1- use the wave function $\psi(r) = A.e^{-\alpha r}$ to prove that

a- The value of the constant $A = \sqrt{\frac{1}{4\pi}\frac{(2\alpha)^3}{2}}$

b- $\langle \nabla^2 \rangle = -\alpha^2$

c- $\left\langle \frac{1}{r} \right\rangle = \alpha$

2- Consider as a variation approximation to the ground state of the hydrogen atom the wave function $\psi(r) = A.e^{-\alpha r}$. Calculate the corresponding energy $E(\alpha)$, then optimize with respect to the parameter α. Compare with the exact solution.

3- Using $E_{g.s} = -79.0\,\text{eV}$ for the ground-state energy of helium,

Calculate the ionization energy (the energy required to remove just one electron).