Problems:

1- Prove that

$$-\frac{\hbar^2}{2m_1}\nabla_{r_1}^2-\frac{\hbar^2}{2m_2}\nabla_{r_2}^2=-\frac{\hbar^2}{2M}\nabla_R^2-\frac{\hbar^2}{2\mu}\nabla_r^2$$

Where,

$$M=m_1+m_2$$
$$\vec{r}=\vec{r}_1-\vec{r}_2$$
$$\vec{R}=\frac{m_1\vec{r}_1+m_2\vec{r}_2}{m_1+m_2}$$

2- Show that the Schrodinger equation

$$\left[-\frac{\hbar^2}{2m_1}\nabla_{r_1}^2-\frac{\hbar^2}{2m_2}\nabla_{r_2}^2+V(r_1-r_2)\right]\Psi(r_1,r_2,t)=i\hbar\frac{\partial}{\partial t}\Psi(r_1,r_2,t)$$

Can be separated to the following two equations

$$\left[-\frac{\hbar^2}{2M}\nabla_R^2\right]\Phi(R)=E_{CM}\Phi(R)$$
$$\left[-\frac{\hbar^2}{2\mu}\nabla_r^2+V(r)\right]\psi(r)=E\psi(r)$$

Where $\Psi(r_1,r_2,t)=\Phi(R)\psi(r)\exp[-i(E_{CM}+E)t/\hbar]$

3- Consider the motion of a free particle of mass m in three dimension, solve this problem quantum mechanically and show that the wave function and the energy of that particle are given by

$$\psi_k(r)=\frac{1}{(2\pi)^{2/3}}\exp[i\vec{k}\cdot\vec{r}]$$
$$E=\frac{(\hbar k)^2}{2m}$$

4- a particle of mass m in a three dimensional Box of impenetrable walls of sides a, b, and c. The potential inside the box is zero while at the walls it is infinite. Solve this problem quantum mechanically and determine the wave function and the energy of the particle.

5- consider a particle of mass m move in a potential given by

$$V(r) = \frac{1}{2}kr^2$$

Solve this problem quantum mechanically and determine the wave function and the energy.