

Problems

Q.1 Prove the following:

- 1- The operator $I = \sum_i |i\rangle\langle i|$ is the identity operator.
- 2- The eigen values of the Hermitian operators are real and the eigen functions are orthogonal.
- 3- Ehrenfest's theorem

Q.2 Suppose that $|u_1\rangle, |u_2\rangle, |u_3\rangle$ is an orthonormal basis. In this basis let,

$$|\Psi\rangle = 2i|u_1\rangle - 3|u_2\rangle + i|u_3\rangle$$

$$|\Phi\rangle = 3|u_1\rangle - 2|u_2\rangle + 4|u_3\rangle$$

a) Find $\langle\Psi|$ and $\langle\Phi|$

b) Compute the inner product $\langle\Psi|\Phi\rangle$ and show that $\langle\Psi|\Phi\rangle^* = \langle\Phi|\Psi\rangle$

Q.3 Let $|\Psi\rangle = 3i|\Phi_1\rangle + 2|\Phi_2\rangle - 4i|\Phi_3\rangle$ where $|\Phi_i\rangle$ are an orthonormal basis. Normalize $|\Psi\rangle$.

Q.4 Suppose that in some orthonormal basis $\{|u_1\rangle, |u_2\rangle, |u_3\rangle\}$ an operator

A acts as follows:

$$A|u_1\rangle = 2|u_1\rangle$$

$$A|u_2\rangle = 3|u_1\rangle - i|u_3\rangle$$

$$A|u_3\rangle = -|u_2\rangle$$

Write the matrix representation of the operator.