Problems:

8- Given the angular wave function

$$\Psi(\theta,\phi)=\begin{cases}\sin\theta & 0<\theta<\pi/2\\ 0 & otherwise\end{cases}$$

Find its representation in terms of the spherical harmonics.

Determine the first three coefficients.

9- We define

$$L_+ = L_x + iL_y$$
$$L_- = L_x - iL_y$$

a-Show that in spherical coordinate they given by.

$$L_\pm = \hbar e^{\pm i\phi}\left[\pm\frac{\partial}{\partial\theta}+i\cot\theta\frac{\partial}{\partial\phi}\right]$$

b- show also that.

$$L_\pm Y_{lm}(\theta,\phi)=\hbar\left[l(l+1)-m(m\pm1)\right]^{1/2}Y_{lm\pm1}(\theta,\phi)$$

$$L_\pm L_\mp = L^2 - L_z^2 \pm \hbar L_z$$

10- show that

a- $\langle L_x\rangle = \langle L_y\rangle = 0$

b- $\langle L_x^2\rangle = \langle L_y^2\rangle = \frac{1}{2}\left[l(l+1)-m^2\right]\hbar^2$