

Components of a Formal Hypothesis Test

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Null Hypothesis: H_0

- The **null hypothesis** (denoted by H_0) is a statement that the value of a population parameter (such as *proportion, mean, or standard deviation*) is **equal to** some claimed value.
- We test the null hypothesis directly.
- Either reject H_0 or fail to reject H_0 (in other words, accept H_0).

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Alternative Hypothesis: H_1

- The **alternative hypothesis** (denoted by H_1) is the statement that the parameter has a value that **somehow differs** from the null hypothesis.
- The symbolic form of the alternative hypothesis must use one of these symbols: \neq , $<$, $>$.
(not equal, less than, greater than)

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General rules:

- If the null hypothesis is rejected, the alternative hypothesis is accepted.
- If the null hypothesis is accepted, the alternative hypothesis is rejected.
- Acceptance or rejection of the null hypothesis is an **initial conclusion**.
- Always state the **final conclusion** expressed in terms of the **original claim**, not in terms of the null hypothesis or the alternative hypothesis.

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Type I Error

- A **Type I error** is the mistake of rejecting the null hypothesis when it is actually true.
- The symbol α (alpha) is used to represent the probability of a type I error.

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Type II Error

- A **Type II error** is the mistake of accepting the null hypothesis when it is actually false.
- The symbol β (beta) is used to represent the probability of a type II error.

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Type I and Type II Errors

		True State of Nature	
		The null hypothesis is true	The null hypothesis is false
Decision	We decide to reject the null hypothesis	Type I error (rejecting a true null hypothesis) $P(\text{type I error}) = \alpha$	Correct decision
	We fail to reject the null hypothesis	Correct decision	Type II error (failing to reject a false null hypothesis) $P(\text{type II error}) = \beta$

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Significance Level

The probability of the type I error (denoted by α) is also called the **significance level** of the test.

It characterizes the chances that the test fails (i.e., type I error occurs)

It must be a small number. Typical values used in practice: $\alpha = 0.1, 0.05, \text{ or } 0.01$ (in percents, 10%, 5%, or 1%).

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Testing hypothesis

Step 1: compute Test Statistic

The **test statistic** is a value used in making a decision about the null hypothesis.

The test statistic is computed by a specific formula depending on the type of the test.

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Section 8-3

Testing a Claim About a Proportion

Notation

n = number of trials

$\hat{p} = \frac{X}{n}$ (**sample** proportion)

p = population proportion (must be specified in the null hypothesis)

$q = 1 - p$

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Requirements for Testing Claims About a Population Proportion p

- 1) The sample observations are simple random samples.
- 2) The conditions for a **binomial distribution** are satisfied.
- 3) The conditions $np \geq 5$ and $nq \geq 5$ are both satisfied, **so the binomial distribution of sample proportions can be approximated by a normal distribution with $\mu = np$ and $\sigma = \sqrt{npq}$** . Note: p is the assumed proportion not the sample proportion.

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Test Statistic for Testing a Claim About a Proportion

$$Z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

Note: p is the value specified in the null hypothesis; $q = 1 - p$

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Example 1 again:

Claim: the XSORT method of gender selection **increases** the likelihood of having a baby girl.

Null hypothesis: $H_0 : p=0.5$

Alternative hypothesis: $H_1 : p>0.5$

Suppose 14 couples treated by XSORT gave birth to 13 girls and 1 boy.

Test the claim at a 5% significance level

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Compute the test statistic:

$$\hat{p} = 13/14 = 0.929$$

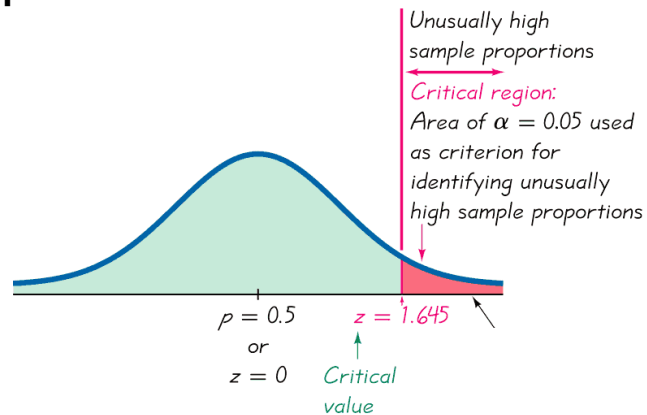
$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.929 - 0.5}{\sqrt{\frac{(0.5)(0.5)}{14}}} = 3.21$$

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Draw the diagram (the normal curve)

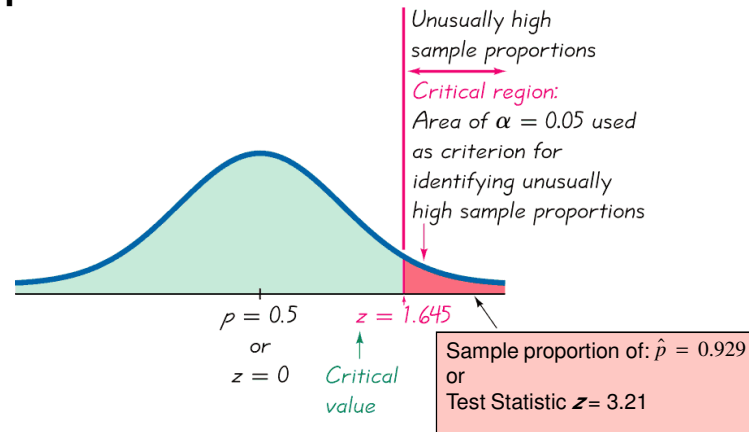
On the diagram, mark a region of extreme values that agree with the alternative hypothesis:



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Draw the diagram (the normal curve)

On the diagram, mark a region of extreme values that agree with the alternative hypothesis:



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Critical Region

The **critical region** (or **rejection region**) is the set of all values of the test statistic that cause us to reject the null hypothesis.

For example, see the red-shaded region in the previous figure.

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Critical Value

A **critical value** is a value that separates the critical region (where we reject the null hypothesis) from the values of the test statistic that do not lead to rejection of the null hypothesis.

See the previous figure where the critical value is $z = 1.645$. It corresponds to a significance level of $\alpha = 0.05$.

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Significance Level

The **significance level** (denoted by α) is the probability that the test statistic will fall in the critical region (when the null hypothesis is actually true).

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Conclusion of the test

Since the test statistic ($z=3.21$) falls in the critical region ($z>1.645$), we **reject the null hypothesis**($p=.5$).

Final conclusion: the original claim is accepted, the XSORT method of gender selection indeed increases the likelihood of having a baby girl.

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Types of Hypothesis Tests: Two-tailed, Left-tailed, Right-tailed

The **tails** in a distribution are the extreme regions bounded by critical values.

Determinations of P values and critical values are affected by whether a critical region is in **two tails**, the **left tail**, or the **right tail**.

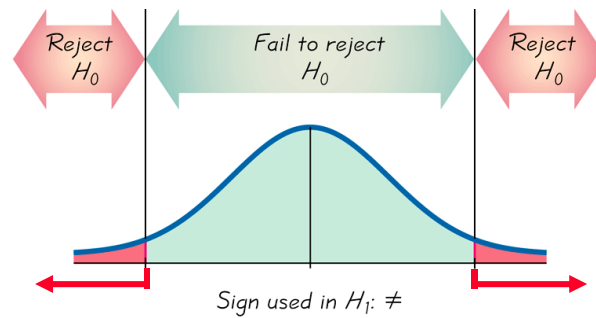
It therefore becomes important to correctly characterize a hypothesis test as **two-tailed**, **left-tailed**, or **right-tailed**.

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Two-tailed Test

$H_0: p=0.5$ α is divided equally between
 $H_1: p \neq 0.5$ the two tails of the critical
region

Means less than or greater than



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Critical values for a two-tailed test

A two-tailed test requires
two critical values:

$$z_{\alpha/2} \quad \text{and} \quad -z_{\alpha/2}$$

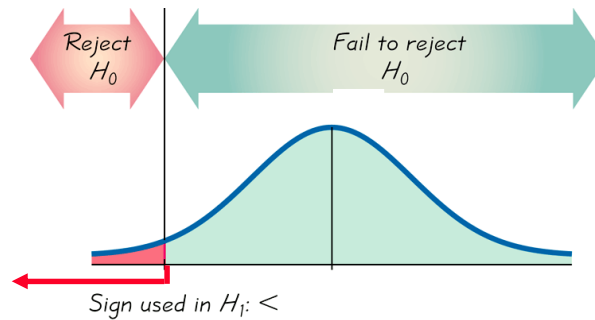
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Left-tailed Test

$H_0: p=0.5$ α is in the left tail

$H_1: p < 0.5$

Points Left



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Critical value for a left-tailed test

A left-tailed test requires
one (negative) critical value:

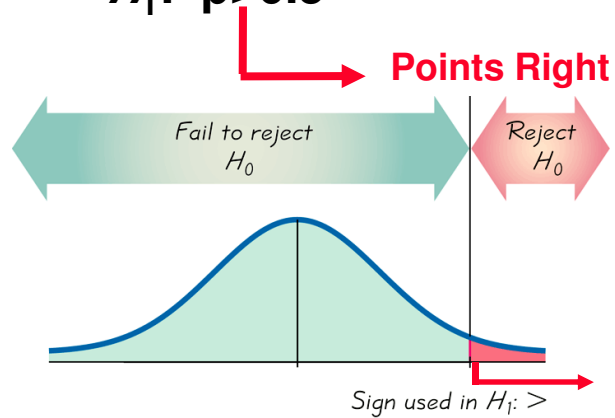
$$-z_{\alpha}$$

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Right-tailed Test

$H_0: p=0.5$ α is in the right tail

$H_1: p>0.5$



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Critical value for a right-tailed test

A right-tailed test requires
one (positive) critical value:

$$z_{\alpha}$$

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Traditional method:

If the test statistic falls within the critical region, **reject H_0** .

If the test statistic does not fall within the critical region, **fail to reject H_0** .

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Do we prove a claim?

- **A statistical test cannot prove a hypothesis or a claim.**
- **Our conclusion can be only stated like this: the available evidence is not strong enough to warrant rejection of a hypothesis or a claim (such as not enough evidence to convict a suspect).**

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