The MidTerm Next Weak Until the end of Discrete Probability Distribution (Ch 5)

Chapter 6. Continuous Random Variables

Reminder:

Continuous random variable takes infinite values

Those values can be associated with <u>measurements</u> on a continuous scale (without gaps or interruptions)

Example: Uniform Distribution

A continuous random variable has a **uniform distribution** if its values are spread **evenly** over a certain range.

Example:

voltage output of an electric generator is between 123 V and 125 V, equally likely. The actual voltage level may be anywhere in this range.

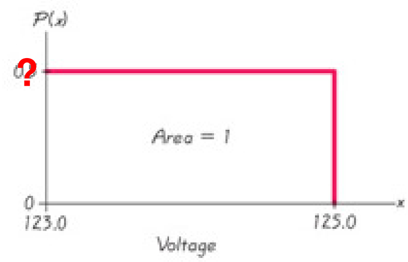


Figure 6-2 Uniform Distribution of Voltage Levels

Example: Uniform Distribution

A continuous random variable has a **uniform distribution** if its values are spread **evenly** over a certain range.

Example:

voltage output of an electric generator is between 123 V and 125 V, equally likely. The actual voltage level may be anywhere in this range.

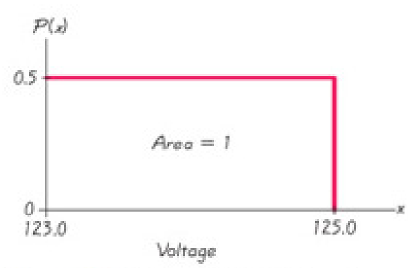


Figure 6-2 Uniform Distribution of Voltage Levels

Density Curve

A density curve is the graph of a continuous probability distribution. It must satisfy the following properties:

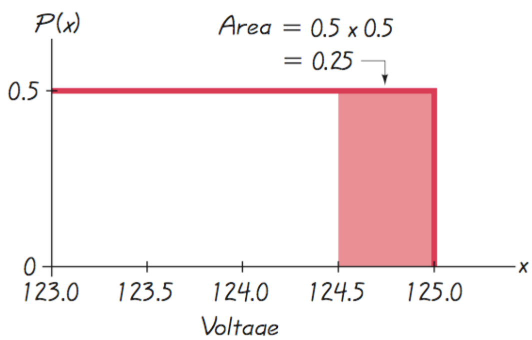
1. The total area under the curve must equal 1.
2. Every point on the curve must have a vertical height that is 0 or greater. (That is, the curve cannot fall below the *x*-axis.)

Area and Probability

Because the total area under the density curve is equal to 1, there is a correspondence between *area* and *probability*.

Using Area to Find Probability

Given the uniform distribution illustrated, find the probability that a randomly selected voltage level is greater than 124.5 volts.



Shaded area represents voltage levels greater than 124.5 volts. The area corresponds to probability: P = 0.25.

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Standard Normal Distribution

Standard normal distribution has three properties:

1. Its graph is bell-shaped.

2. It is symmetric about its center.

3. Its mean is equal to 0 ($\mu = 0$).

4. Its standard deviation is equal to 1 ($\sigma = 1$).

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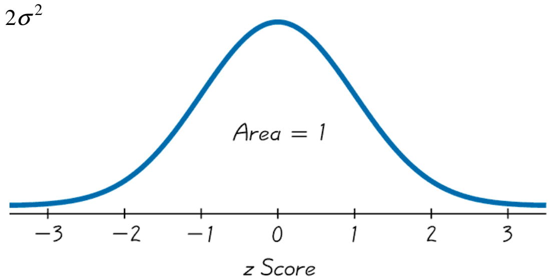
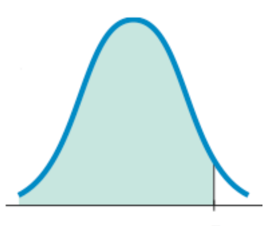
Standard Normal Distribution

The **standard normal distribution** is a bell-shaped probability distribution with $\mu = 0$ and $\sigma = 1$. The total area under its density curve is equal to 1.

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



Standard Normal Distribution: Areas and Probabilities



Probability that the standard normal random variable takes values less than z is given by the **area** under the curve from the left up to z (blue area in the figure)

Example – Thermometers

Thermometers are supposed to give readings of 0ºC at the freezing point of water.

Since these instruments are not perfect, some of them give readings below 0º (denoted by negative numbers) and some give readings above 0º (denoted by positive numbers).

Example – Thermometers (continued)

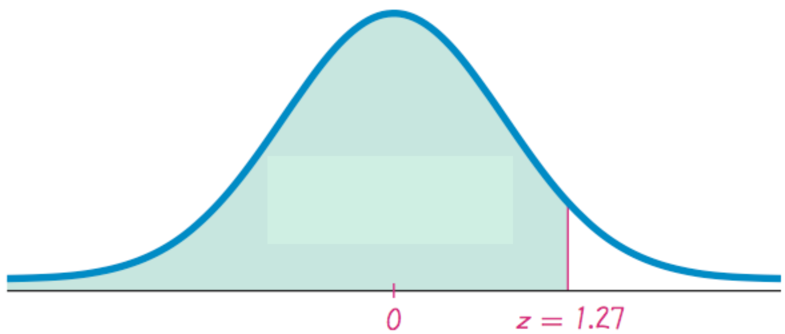
Assume that the mean reading is 0ºC and the standard deviation of the readings is 1ºC. Also assume that the readings are normally distributed.

Q: If one thermometer is randomly selected, find the probability that, at the freezing point of water, the reading is less than 1.27º.

Example – Thermometers (continued)

P(z< 1.27) =

area under the curve from the left up to 1.27



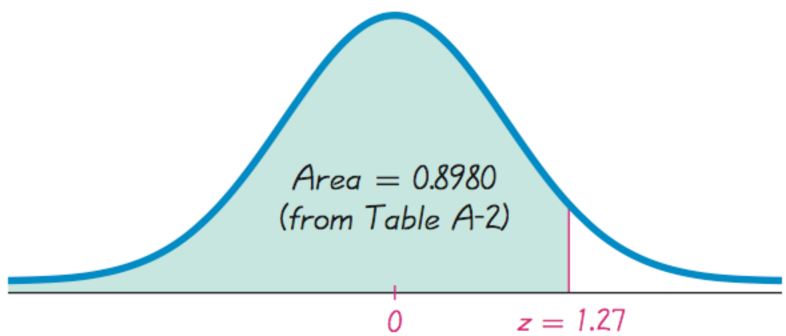
Look at Table A-2

TABLE A-2 (*continued*) Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292

Example – Thermometers (continued)

P (z< 1.27) = 0.8980



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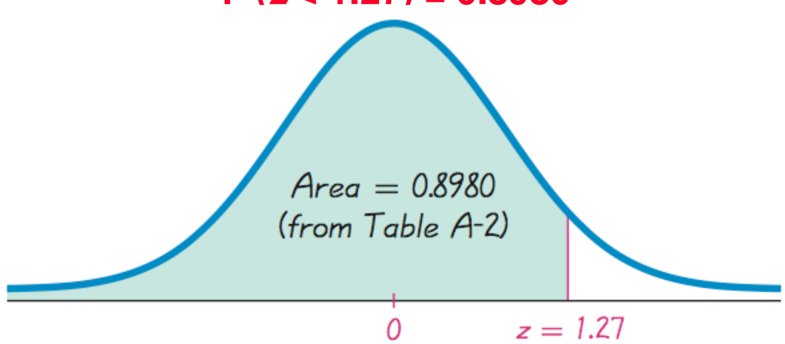
Example – Thermometers (continued)

P (z< 1.27) = 0.8980



The *probability* of randomly selecting a thermometer with a reading less than 1.27º is 0.8980.

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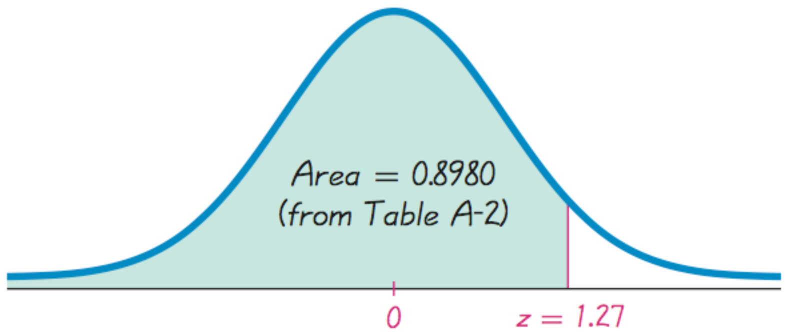
Example – Thermometers (continued)

P (z< 1.27) = 0.8980



Or 89.80% of randomly selected thermometers will have readings below 1.27º.

Using Table A-2

1. **It is designed only for the *standard* normal distribution, which has a <u>mean of 0</u> and a <u>standard deviation of 1</u>.**
2. **It is on two pages, with one page for *negative* *z*-scores and the other page for *positive* *z*-scores.**
3. **Each value in the body of the table is a *cumulative area from the left* up to a vertical boundary above a specific *z*-score.**

Using Table A-2

4. When working with a graph, avoid confusion between *z*-scores and areas.

   ***z* Score**
   Distance along horizontal scale of the standard normal distribution; refer to the <u>leftmost column and top row</u> of Table A-2.

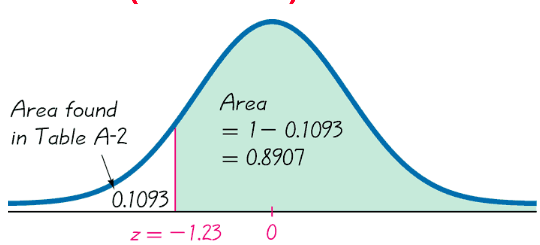
   Area
   Region under the curve; refer to the values in the <u>body</u> of Table A-2.

5. The part of the z-score denoting hundredths is found across the top.

Example - Thermometers Again

If thermometers have an average (mean) reading of 0 degrees and a standard deviation of 1 degree for freezing water, and if one thermometer is randomly selected, find the probability that it reads (at the freezing point of water) <u>above –1.23 degrees</u>.

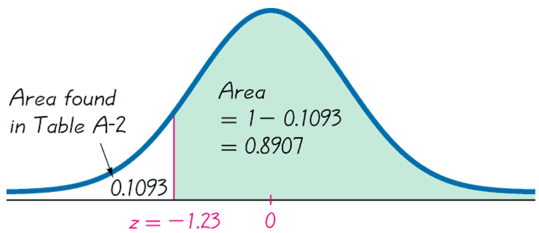
$$P(z > -1.23) = 0.8907$$



Probability of randomly selecting a thermometer with a reading above –1.23º is 0.8907.

Example - cont

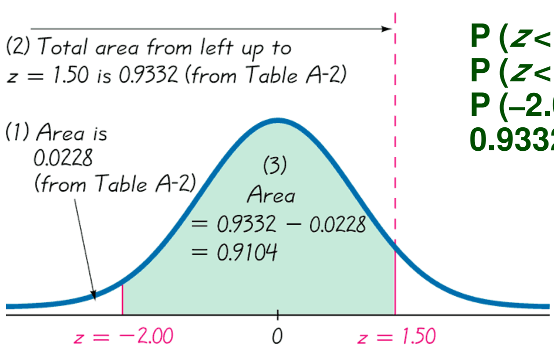
P (z> –1.23) = 0.8907



89.07% of the thermometers have readings above –1.23 degrees.

Example - Thermometers III

A thermometer is randomly selected. Find the probability that it reads (at the freezing point of water) <u>between</u> –2.00 and 1.50 degrees.

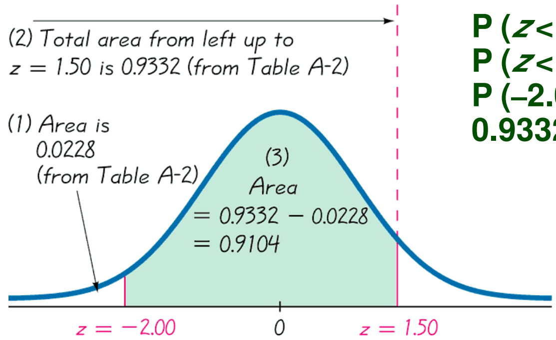


P (z< –2.00) = 0.0228
P (z< 1.50) = 0.9332
P (–2.00 < z< 1.50) =
0.9332 – 0.0228 = 0.9104

The probability that the chosen thermometer has a reading between – 2.00 and 1.50 degrees is 0.9104.

Example - continued

A thermometer is randomly selected. Find the probability that it reads (at the freezing point of water) between –2.00 and 1.50 degrees.



$P(z < -2.00) = 0.0228$
$P(z < 1.50) = 0.9332$
$P(-2.00 < z < 1.50) =$
$0.9332 - 0.0228 = 0.9104$

If many thermometers are selected and tested at the freezing point of water, then 91.04% of them will read between –2.00 and 1.50 degrees.

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Notation

$P(a < z < b)$

denotes the probability that the z score is between a and b.

$P(z > a)$

denotes the probability that the z score is greater than a.

$P(z < a)$

denotes the probability that the z score is less than a.

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Special Cases in Table A-2

- If z-score is above 3.49, then the area=0.9999
- If z-score is below -3.49, then the area=0.0001
- Some special values are marked by stars:

  z-score=1.645 corresponds to the area=0.95

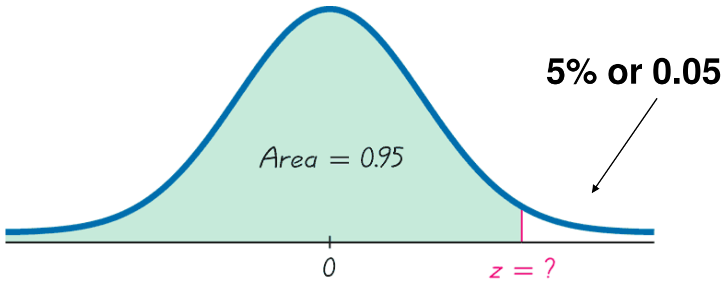
  z-score=2.575 corresponds to the area=0.995

Finding a z Score When Given a Probability Using Table A-2

1. Draw a bell-shaped curve and identify the region under the curve that corresponds to the given probability. If that region is NOT a cumulative region from the left, work instead with a known region that is a cumulative region from the left.

2. Using the cumulative area from the left, locate the closest probability in the body of Table A-2 and identify the corresponding z score.

Finding *z* Scores When Given Probabilities



(*z* score will be positive)

Finding the z-score separating 95% bottom values from 5% top values.

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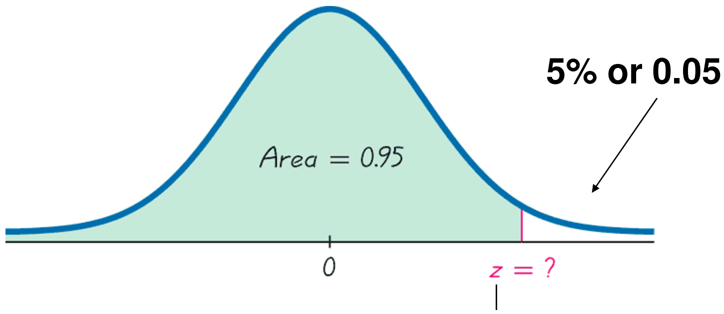
Finding *z* Scores When Given Probabilities

TABLE A-2 (*continued*) Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495 *	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633

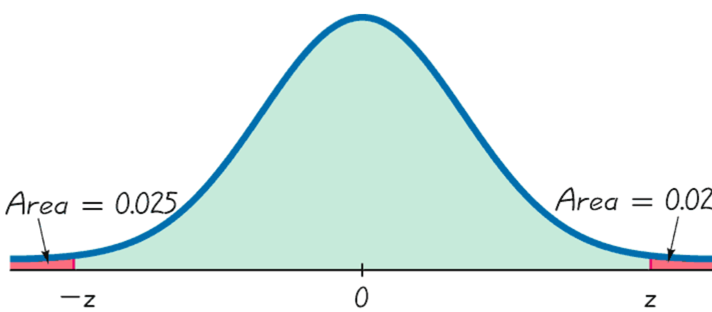
Finding z Scores When Given Probabilities



(z score will be positive)

Finding the z-score separating 95% bottom values from 5% top values.

Finding z Scores When Given Probabilities - cont



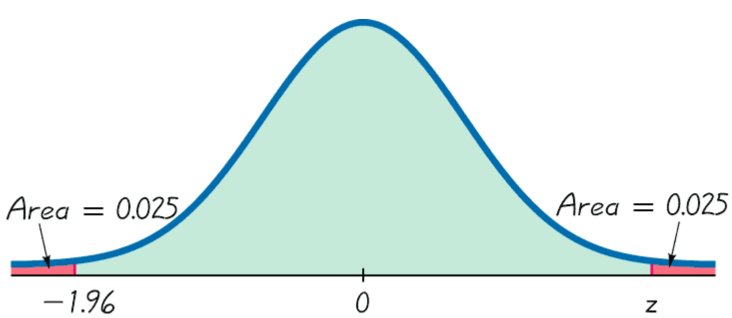
(One z score will be negative and the other positive)

Finding the Bottom 2.5% and Upper 2.5%

TABLE A-2 Standard Normal (z) Distribution: Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.50 and lower	.0001									
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	* .0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294

Finding *z* Scores When Given Probabilities - cont



(One *z* score will be negative and the other positive)

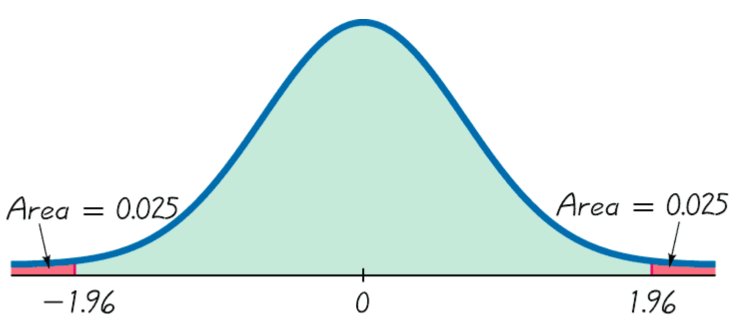
Finding the Bottom 2.5% and Upper 2.5%

1-.025=.975

TABLE A-2 (*continued*) Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495 *	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767

Finding *z* Scores When Given Probabilities - cont



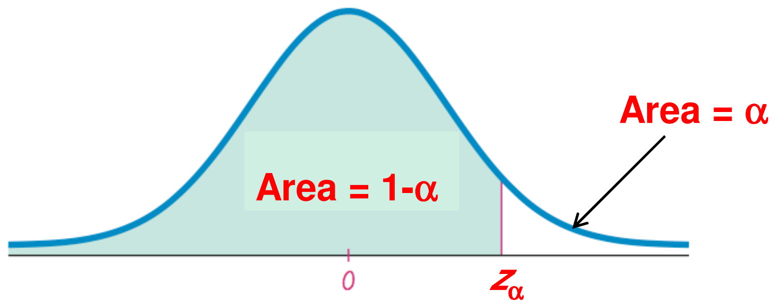
(One *z* score will be <u>negative</u> and the other <u>positive</u>)

Finding the Bottom 2.5% and Upper 2.5%

Notation

We use z_α to represent the z-score separating the top α from the bottom $1-\alpha$.

Examples: $z_{0.025}$ **= 1.96**, $z_{0.05}$ **= 1.645**



Normal distributions that are not standard

All normal distributions have bell-shaped density curves.

A normal distribution is standard if its mean μ is 0 and its standard deviation σ is 1.

A normal distribution is not standard if its mean μ is not 0, or its standard deviation σ is not 1, or both.

We can use a simple conversion that allows us to standardize any normal distribution so that Table A-2 can be used.

Conversion Formula

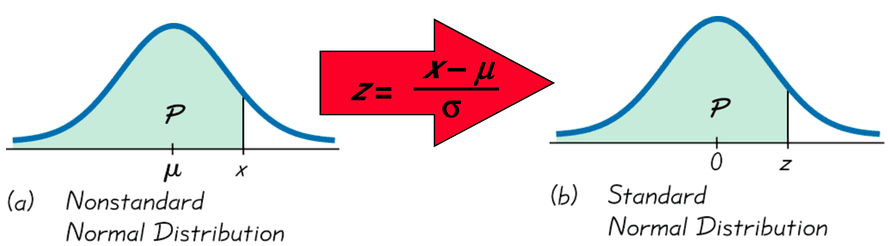
Let x be a score for a normal distribution with mean μ and standard deviation σ

We convert it to a z score by this formula:

$$z = \frac{x - \mu}{\sigma}$$

(round z scores to 2 decimal places)

Converting to a Standard Normal Distribution

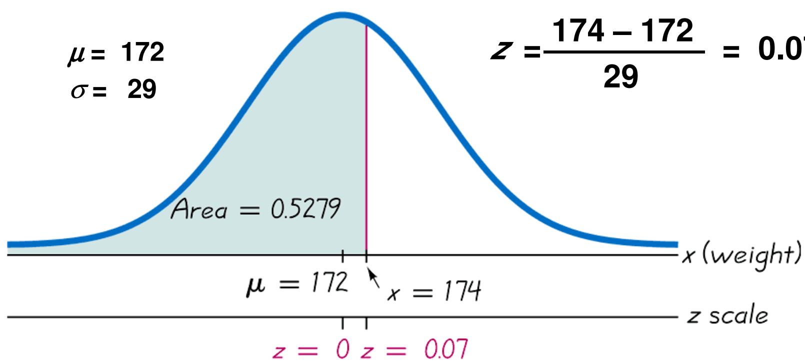


Example – Weights of Passengers

Weights of taxi passengers have a normal distribution with mean 172 lb and standard deviation 29 lb. If one passenger is randomly selected, what is the probability he/she weighs less than 174 pounds?

$\mu = 172$
$\sigma = 29$

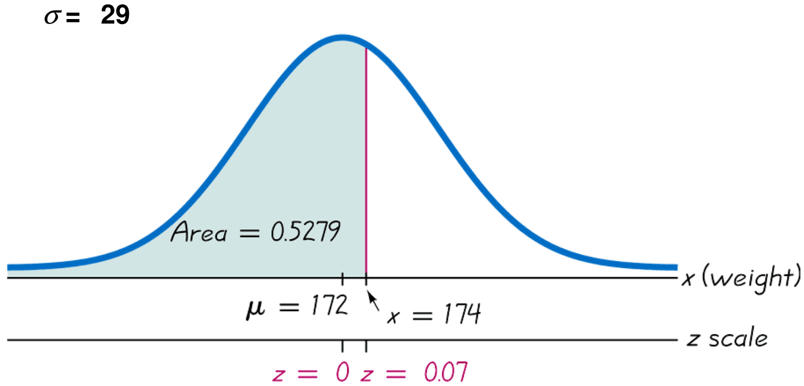
$$z = \frac{174 - 172}{29} = 0.07$$



Example - continued

$\mu = 172$
$\sigma = 29$

P (x < 174 lb.) = P(z < 0.07) = 0.5279



Finding x Scores When Given Probabilities

1. Use Table A-2 to find the z score corresponding to the given probability (the area to the left).

2. Use the values for μ, σ, and the z score found in step 1, to find x:

$$x = \mu + (z \cdot \sigma)$$

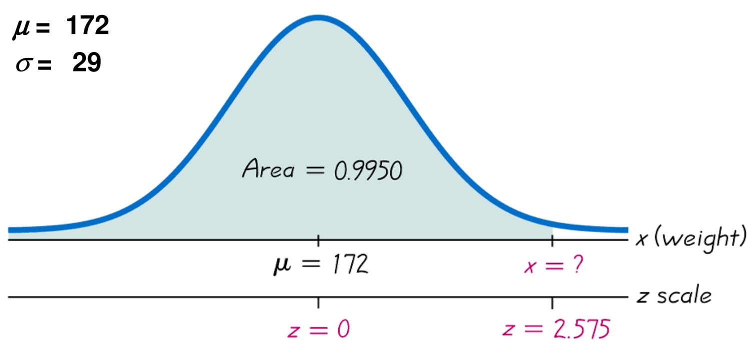
(If z is located to the left of the mean, be sure that it is a negative number.)

Example – Lightest and Heaviest

Weights of taxi passengers have a normal distribution with mean 172 lb and standard deviation 29 lb. Determine what weight separates the lightest 99.5% from the heaviest 0.5%?

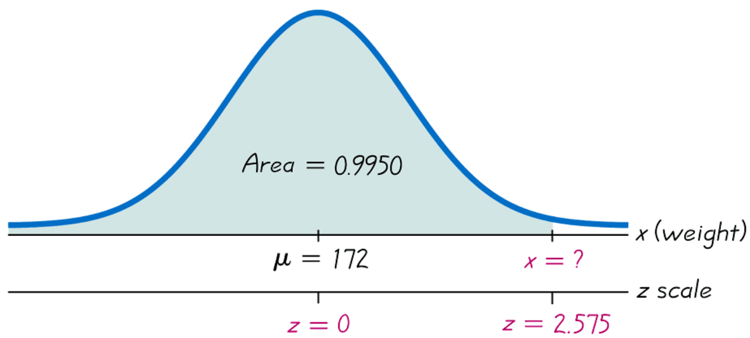
μ = 172

σ = 29



Example –
Lightest and Heaviest - cont

$x = \mu + (z \bullet \sigma)$
$x = 172 + (2.575 \bullet 29)$
$x = 246.675$ (247 rounded)



Example –
Lightest and Heaviest - cont

The weight of 247 pounds separates the lightest 99.5% from the heaviest 0.5%