

Some Examples

❖ **Are the following simple events or not?**

❖ **Throwing Three coins**

-> **All coins show head**

Yes

-> **Exactly one coin shows head**

No

-> **At least one coin show head**

No

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Some Examples

❖ **What is the complement of the following**

1- **A: Every one failed exam**

Ans: someone passed the exam

2- **At most 5 students got an A in the exam**

Ans: At least 6 students got an A in the exam

❖ **Two fair 6-sided dice are rolled. What is the probability that their sum is 7 or exactly one die is 2**

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$= 6/36 + 10/36 - 2/36 = 7/18$$

Some Examples

❖ Suppose that two cards are drawn from a standard 52 card deck without replacement. Find the probability

1- both cards are two

$$\text{Ans: } 4/52 * 3/51 = 1/221$$

2- both cards are hearts

$$\text{Ans: } 13/52 * 12/51 = 156/2652$$

❖ The first card is an ace and the second card is 2

$$\text{Ans: } 4/52 * 4/51 = 4/663$$

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Some Examples

❖ Suppose that two cards are drawn from a standard 52 card deck with replacement. Find the probability

1- both cards are two

$$\text{Ans: } 4/52 * 4/52 = 1/169$$

2- both cards are hearts

$$\text{Ans: } 13/52 * 13/52 = 1/16$$

❖ The first card is an ace and the second card is 2

$$\text{Ans: } 4/52 * 4/52 = 1/169$$

Some Examples

❖ A card is drawn from a standard 52-cards. Find the probability that the card is

1- A king given that the card is heart

Ans: (# of hearts that are kings)/(# of hearts)= 1/13

2- A heart given that the card is a king

Ans: (# of kings that are hearts)/(#of kings)= 1/4

❖ The first card is 1 and the second card is 2

Ans: $4/52 * 4/52 = 1/169$

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Random Variables Chapter 5

- ❖ **Random variable**
a variable (typically represented by X) that takes a numerical value by chance.
- ❖ For each outcome of a procedure, X takes a certain value, but for different outcomes that value may be different.

Examples:

- ❖ **Number of boys in a randomly selected family with three children.**
Possible values: $x=0,1,2,3$
- ❖ **The weight of a randomly selected person from a population.**
Possible values: positive numbers, $x>0$

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Discrete and Continuous Random Variables

- ❖ **Discrete random variable**
either a finite number of values or countable number of values (resulting from a counting process)
- ❖ **Continuous random variable**
infinitely many values, and those values can be associated with measurements on a continuous scale without gaps or interruptions

Probability Distributions

- ❖ **Probability distribution**
a description that gives the probability for each value of the random variable;
- ❖ Three ways to describe a distribution
 - ❖ Formula
 - ❖ Table
 - ❖ Graph

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Method 1: Using the Binomial Probability Formula

$$P(x) = \frac{n!}{(n-x)!x!} \cdot p^x \cdot q^{n-x}$$

for $x = 0, 1, 2, \dots, n$

Tables

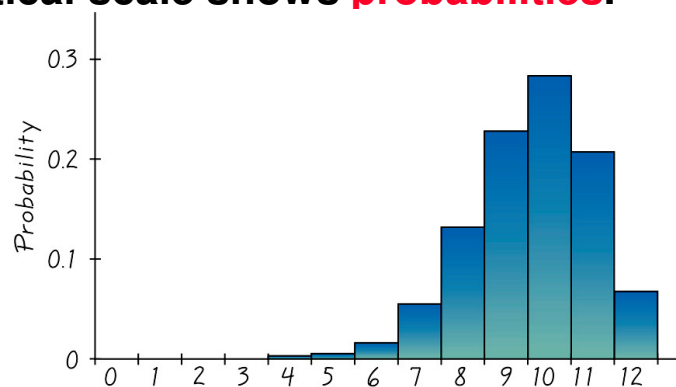
Values: Probabilities:

x	$P(x)$
0	$1/8$
1	$3/8$
2	$3/8$
3	$1/8$

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Graphs

The **probability histogram** is very similar to a relative frequency histogram, but the vertical scale shows **probabilities**.



Probability Histogram for Number of Mexican-American Jurors Among 12

Requirements for Probability Distribution

$$\sum P(x) = 1$$

where x assumes all possible values.

$$0 \leq P(x) \leq 1$$

for every individual value of x .

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Mean, Variance and Standard Deviation of a Probability Distribution

$$\mu = \sum [x \cdot P(x)] \quad \text{Mean}$$

$$\sigma^2 = \sum [(x - \mu)^2 \cdot P(x)] \quad \text{Variance}$$

$$\sigma^2 = \sum [x^2 \cdot P(x)] - \mu^2 \quad \text{Variance (shortcut)}$$

$$\sigma = \sqrt{\sum [x^2 \cdot P(x)] - \mu^2} \quad \text{Standard Deviation}$$

Example

- Find the mean and Standard deviation of the following discrete probability distribution

x	P(x)		
0	0		
1	.15		
2	.45		
3	.20		
4	.20		

$$\sigma^2 = \Sigma [x^2 \cdot P(x)] - \mu^2$$

$$\bullet \mu = \Sigma [x \cdot P(x)]$$

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Example

- Find the mean and Standard deviation of the following discrete probability distribution

x	P(x)	xP(x)	
0	0	0	
1	.15	.15	
2	.45	.9	
3	.20	.6	
4	.20	.4	

Example

- Find the mean and Standard deviation of the following discrete probability distribution

x	P(x)	xP(x)	x ² P(x)
0	0	0	0
1	.15	.15	.15
2	.45	.9	1.8
3	.20	.6	1.8
4	.20	.4	1.6

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Example

- Find the mean and Standard deviation of the following discrete probability distribution

x	P(x)	xP(x)	x ² P(x)
0	0	0	0
1	.15	.15	.15
2	.45	.9	1.8
3	.20	.6	1.8
4	.20	.4	1.6

$$\mu = \Sigma [x \cdot P(x)] = 2.05$$

$$\sigma^2 = \Sigma [x^2 \cdot P(x)] - \mu^2 = 1.07$$

Roundoff Rule for μ , σ , and σ^2

Round results by carrying one more decimal place than the number of decimal places used for the random variable x .

If the values of x are integers, round μ , σ , and σ^2 to one (better two) decimal place.

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Identifying *Unusual* Results Range Rule of Thumb

According to the **range rule of thumb**, most values should lie within 2 standard deviations of the mean.

We can therefore identify “unusual” values by determining if they lie outside these limits:

$$\text{Maximum usual value} = \mu + 2\sigma$$

$$\text{Minimum usual value} = \mu - 2\sigma$$

Identifying *Unusual* Results By Probabilities

Using Probabilities to Determine When Results Are Unusual:

- ❖ **Unusually high**: a particular value x is **unusually high** if $P(x \text{ or more}) \leq 0.05$.
- ❖ **Unusually low**: a particular value x is **unusually low** if $P(x \text{ or fewer}) \leq 0.05$.

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Bernoulli Trials

Any simple trial with two possible outcomes.
p and q

EX: Tossing a coin, repeat, with counting #
of success p “ the number of heads”,

Then # of failure $q = (1 - p)$, “ # of tails”

$P(\text{HHT}) = p \cdot p \cdot q$

$P(\text{TTT}) = q \cdot q \cdot q$

If we have k as the number of successes and
n-k failures

Then the probability is $p^k q^{n-k}$

Binomial Random Variable

If we have $X : S \rightarrow \{0,1,2,3\}$

Where X is the number of successes

$$X(sss) = 3$$

$$X(sfs) = X(ssf) = 2$$

X now is a random variable.

X is named Binomial random variable resulted from n Bernoulli trials denoted:

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Binomial Random Variable

Now the probability that $X = k$ $0 \leq k \leq n$
that is all strings with k success and $n - k$
fails, there are $\binom{n}{k}$ different ways

$$P(X = k) = \binom{n}{k} p^k q^{n-k}$$

Binomial Probability Distribution

A **binomial probability distribution** results from a procedure that meets all the following requirements:

1. The procedure has a **fixed number of trials**.
2. The trials must be **independent**. (The outcome of any individual trial doesn't affect the probabilities in the other trials.)
3. Each trial must have all outcomes classified into **two categories** (commonly referred to as **success** and **failure**).
4. The probability of a success remains the same in all trials.

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Notation for Binomial Probability Distributions

S and **F** (success and failure) denote the two possible categories of all outcomes; p and q denote the probabilities of **S** and **F**, respectively:

$$P(S) = p \quad (p = \text{probability of success})$$

$$P(F) = 1 - p = q \quad (q = \text{probability of failure})$$

Notation (continued)

- n denotes the fixed number of trials.
- x denotes a specific number of successes in n trials, so x can be any whole number between 0 and n , inclusive.
- p denotes the probability of **success** in **one** of the n trials.
- q denotes the probability of **failure** in **one** of the n trials.
- $P(x)$ denotes the probability of getting exactly x successes among the n trials.

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Methods for Finding Probabilities

We will now discuss two methods for finding the probabilities corresponding to the random variable x in a binomial distribution.

Method 1: Using the Binomial Probability Formula

$$P(x) = \frac{n!}{(n-x)!x!} \cdot p^x \cdot q^{n-x}$$

for $x = 0, 1, 2, \dots, n$

where

n = number of trials

x = number of successes among n trials

p = probability of success in any one trial

q = probability of failure in any one trial ($q = 1 - p$)

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Rationale for the Binomial Probability Formula

$$P(x) = \frac{n!}{(n-x)!x!} \cdot p^x \cdot q^{n-x}$$



Number of
outcomes with
exactly x
successes
among n trials

Interpretation of Results

It is especially important to interpret results. The **range rule of thumb** suggests that values are unusual if they lie outside of these limits:

$$\text{Maximum usual values} = \mu + 2\sigma$$

$$\text{Minimum usual values} = \mu - 2\sigma$$