Cairo University
Faculty of science
Mathematics Department
45 min.

MidTerm Exam (Spring 2017)
Simulation and Modeling
Comp 406

Answer the following questions

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1. Let the following table contains all the future arrivals A_i , Landings L_i and departures D_i events in a single runway aircraft (single-server queue)

Fight	A_i	L_i	D_i
F1	1	4	8
F2	3	7	11

i. Develop a conceptual model for the above problem

Customer (aircraft)
    Entities utilizing the system/resources
Server (runway)
    Resource that is serially reused; serves one customer at a time
Queue
    Buffer holding aircraft waiting to land

ii. Develop a specification for your conceptual model

- **Customers**
  Schedule of aircraft arrivals:
  Often, probability distribution defines time between successive customer arrivals (interarrival time)
  Assumes interarrival times *independent, and identically distributed*(iid)
  Customer attributes? e.g., priorities
- **Servers**
  How much *service time* is needed for each customer?
  May use probability distribution to specify customer service time (iid)
  How many servers?
- **Queue**
  Service discipline - who gets service next?
  - First-in-first-out (FIFO), Last-in-first-out (LIFO), random …
  - May depend on a property of the customer (e.g., priority, "smallest" first)

  Assume

2. Find the pseudo random numbers generated by the Von Neumann's Midsquare method starting with the seed 61.

Ans:

3721	**72**
5184	**18**
0324	**32**
1024	**2**
0004	**0**

3. **Let X be exponential random variable *exp(λ)* and $Y = \sqrt{X}$, X > 0**
Find $F_Y(Y)$

7.7 Example. Let X be exponential(λ) and $Y = \sqrt{X}$.

Solution. Since $0 < X < \infty$, we have $0 < \sqrt{X} < \infty$, then $0 < Y < \infty$. Now, for all $0 < y < \infty$ we have

$$F_Y(y) = \mathbb{P}(Y \le y) = \mathbb{P}(\sqrt{X} \le y) = \mathbb{P}(X \le y^2) = F_X(y^2) = 1 \quad e^{-\lambda y^2}.$$

Lastly, by differentiating, $f_Y(y) - 2\lambda y e^{-\lambda y^2}$.

4. **(7 points)** Suppose X_i for $i=1,2,3,4$ are independent random variables, with mean μ and standard deviation σ. Let $V = X_1 - X_2 + X_3$ and $W = X_1 - X_2 + X_3$. Find

a. The covariance $Cov(V,W)$

b. The Correlation $\rho_{V,W}$

c. How do you interpret the value of the correlation

3. (10 pts) Suppose X_i for $i = 1, 2, 3, 4$ are independent random variables with mean μ and standard deviation σ. Let $V = X_1 - X_2 + X_3$ and $W = X_2 - X_3 + X_4$.

(a) Find $\mathrm{Cov}(V, W)$.

Solution:

$$\mathrm{Cov}(V, W) = -\mathrm{Var}(X_2) - \mathrm{Var}(X_3) = -2\sigma^2$$

(b) Find $\rho_{V,W}$. How do you interpret the value of $\rho_{V,W}$, including its sign?

Solution:

$$\rho_{V,W} = \frac{-2\sigma^2}{\sqrt{3\sigma^2}\sqrt{3\sigma^2}} = -\frac{2}{3}$$

Strongly negatively correlated (typical fluctuations occur in opposite directions).

5.