Cairo University
Faculty of science
Mathematics Department
120 min.

Final Exam (Fall 2015)
Simulation and Modeling
Comp 406

The Exam is in <u>seven</u> Pages

Answer the following questions. Full credit is 60.

Question (1) (15 points) Short Answers:

A. (10 points) Write T (for true) or F (for false).

(i) (…..) Any random number generator is able to reproduce the same sequence of random numbers if required

(ii) (…..) M/M/1 is a queuing model with Poisson arrivals, exponential service times and one server.

(iii) (…..) Pseudo random generators are fast and easy, therefore, it is recommended in all simulation applications.

(iv) (…..) In random-variate generation, convolution is a special case of composition.

(v) (…..) The Box-Muller method uses two uniform random numbers to generate two independent standard normal random variates.

(vi) (…..) The Box-Muller algorithm generates normal random variates via the inverse transformation.

(vii) (…..) Composition is applicable for generating either continuous or discrete random variables.

(viii) (…..) The expected number of iterations for the acceptance/rejection method depends on the choice of the majorizing function.

(ix) (…..) Thinning is a way to generate realizations of a specified Poisson process.

(x) (…..) A Poisson process has Poisson times between events.

B. (3 points) Validation may be considered as the most important step in the simulation study, Explain why?

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C. (2 points) Give an inverse transformation algorithm for generating $U(3,10)$; that is the continuous uniform distribution on the interval $(3, 10)$

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Question (2) (15 points) Write in Details:

A. (7 points) Consider the problem of minimizing the waiting time in a pizza delivery operation.

(i) Discuss whether the simulation model should be static or dynamic, deterministic or stochastic, continuous or discrete. Always give a reason to your choice.

(ii) Suggest a simulation model. Explain the conceptual and the specification models and discuss what variables that should be effective in the simulation.

B. **(8 points)** Suppose that X and Y are jointly continuous random variables with

$$f(x) = \begin{cases} y - x & 0 < x < 1, and, 1 < y < 2 \\ 0 & Otherwise \end{cases}$$

a. Compute $f_X(x)$ and $f_Y(y)$
b. Are X and Y independent?
c. Compute $F_X(x)$ and $F_Y(y)$
d. Compute the covariance $Cov(X,Y)$ and the correlation $Cor(X,Y)$

Question (3) (15 points) Answer the following questions:

A. **(5 points)** Outline a procedure to generate 10 random numbers normally distributed with mean of 27 and standard deviation 3.

B. **(3 points)** An insurance company insures 6000 people, each of whom has a 1/2000 chance of an accident in one year. Use the Poisson approximation to find the probability that the number of accidents in one year will be at least 2 and at most 5.

C. **(3 points)** Determine if the following mixed LCG has a full period or not, why ?

$Z_i = (13\ Z_{i-1} + 12) \pmod{16}$,

D. **(4 points)** Use the composition method to generate a random variate with the density:

$$f(x) = \begin{cases} 3x^2 & -1 < x \le 1 \\ 0 & Otherwise \end{cases}$$

Question (4) (15 points) Answer the following questions

A. **(5 points)** Give the inverse transform algorithm for generating the random variate *x* with the density function

$$f(x) = \frac{x-1}{50}, 1 \le x \le 11$$

B. **(4 points)** A baker is trying to determine how many Dozens of donuts to bake each day. The probability distribution of the number of donuts is as follows:

No. of customers per day:	8	10	12	14
Probability	0.35	0.30	0.25	0.10

How can you use a pseudo random number generator to simulate this distribution.

A. **(6 points)** Consider the Von Neumann's Midsquare Method for generating random numbers between 0.01 and 0.99. Using 12 as your seed :

(i) List the generated numbers

(ii) Use the numbers generated in (i) to generate random numbers exponentially distributed with $\lambda = 3.8$

(iii) Test the numbers in (i) for uniformity using Chi square test, given that the corresponding value from the table is 14.07

Best Wishes
Areeg Abdalla