Objectives

- To encourage "system" thinking
- Provide background to systems modeling concepts
- Opportunity for a practical appreciation for discrete event simulation
- Combine theory and practice

Outline of the course

- Introduction to Simulation.
- Hand Simulation.
- Review of basic Probability Theory.
- Random Number Generation
- Generation of Random Varieties.
- Analysis of Output.
- Elementary Queuing Models

Modeling and Simulation

Model

- It is a simplification of the reality
- A (usually miniature) representation of an actual system; an example for imitation or emulation
- A description of observed behavior, simplified by ignoring certain details.
- Models allow complex systems to be understood and their behavior predicted within the scope of the model
- A model can be **Analytical** (Queuing Theory) or by Simulation.

Discrete-Event Simulation

- Discrete Event System
  - a system whose ***state*** changes at discrete points in time due to the occurrence of asynchronous ***events***
- Example: M/M/1 queueing system
  - State
    - number of customers in system
  - Events
    - customer arrival
    - customer departure

Queuing System

- A system Characterized by three components
  - Arrival Process The distribution that determines how the tasks arrives in the system).
  - Service Process The distribution that determines the task processing time
  - Number of Servers Total number of servers available to process the tasks

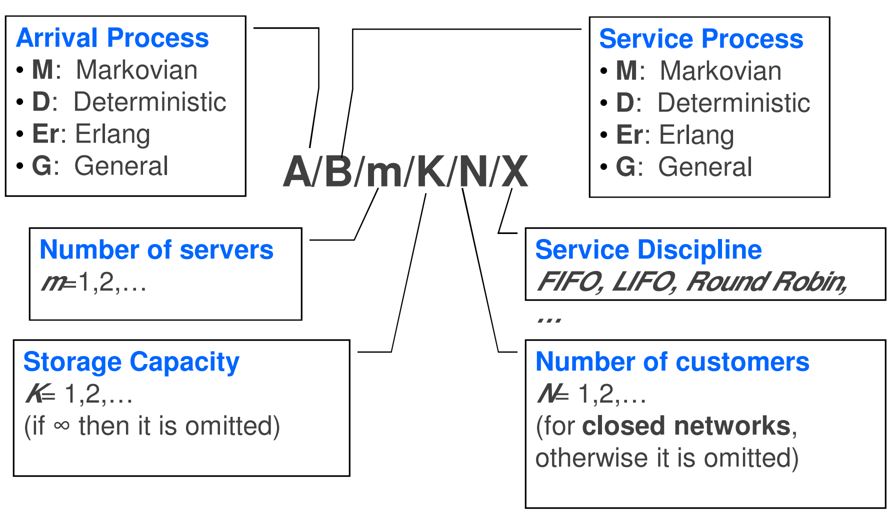
M/M/1 System

- Nomenclature: M stands for “Memoryless” (a property of the exponential distribution)
  - **M**/M/1 stands for Poisson arrival process (which is memoryless)
  - M/**M**/1 stands for exponentially distributed transmission times
  - M/M/1 stands for one server
- “M/M/1” is a special case of more general (Kendall) notation: X/Y/m/k
- Example of packet :
  - Arrival process is Poisson with rate l
  - Processing times are exponentially distributed with mean 1/m
  - One server
  - Independent interarrival times and processing times

Kendall Notation

- M/M/1 Queue Poisson arrivals (*exponential inter-arrival*), and exponential service, 1 server, infinite capacity and population, FCFS (FIFO). The most basic and important queuing model -Was the subject of our course
- M/M/m Queue ***Same, but m servers***
- M/M/m/k Queue system Same as M/M/m, but there is buffer space for at most k packets. Packets arriving at a full buffer are dropped, k is omitted when k = ∞
- M/D/1 Queue Poisson arrivals and CONSTANT service times, 1 server, infinite capacity and population, FIFO.
- G/G/3/20/1500/SPF General arrival and service distributions, 3 servers, 17 queues (20-3), 1500 total jobs, Shortest Packet First

Kendall Notation



Applications of Queuing Theory

- Telecommunications
- Computer Networks
- Predicting computer performance
- Health services (eg. control of hospital bed assignments)
- Airport traffic, airline ticket sales
- Layout of manufacturing systems.

Chapter 9 Output Analysis – Single System

Reading: Chapter 9

9.1 Introduction

9.2 Type of simulations with regard to output analysis

9.3 Statistical analysis for terminating simulations

9.4 Statistical analysis for steady-state parameters

9.5 Other measures of performance related to a single system

Why Output Data Analysis?

Introduction

- Why output data analysis?
  - One realization (run) does not necessarily give the “correct” answer(s).
  - Variance exists in simulation results so we must be cautious about how we interpret results

  Output from our model $\{Y_1, Y_2, Y_3, \ldots\}$
  - $\{Y_1, Y_2, Y_3, \ldots\}$ may not be independent;
  - $\{Y_1, Y_2, Y_3, \ldots\}$ may have different distributions, depending on a number of different factors;
  - Estimators, confidence interval, and so on, must be constructed.

Simulation Replications

A single run of a simulation model is always a really, really bad idea.

- Replications
  - Run the simulation and sample from each run m times. Complete n runs:

$$
\begin{array}{l}
y_{11}, y_{12}, y_{13}, \ldots, y_{1m}; \\
y_{21}, y_{22}, y_{23}, \ldots, y_{2m}; \\
\vdots \\
y_{n1}, y_{n2}, y_{n3}, \ldots, y_{nm};
\end{array}
$$

(y_{ij} is called a ***realization*** of $\{Y_1, Y_2, Y_3, \ldots\}$)
Note that the observations along the rows are not IID, however observations down the columns

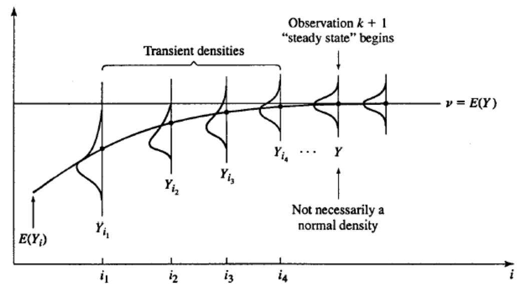
($\{y_{1i}, y_{2i}, y_{3i}, \ldots, y_{ni}\}$ from I.I.D. for Y_i)

- Estimators
  - $\hat{\mu}_j(m) = \frac{\sum_{i=1}^{n} y_{ji}}{m}$ may not be a unbiased estimator of the mean (i.e. across the rows)
  - $\hat{y}_i(n) = \frac{\sum_{j=1}^{n} y_{ji}}{n}$ is a unbiased estimator of **E** Y_i (i.e. down the columns)

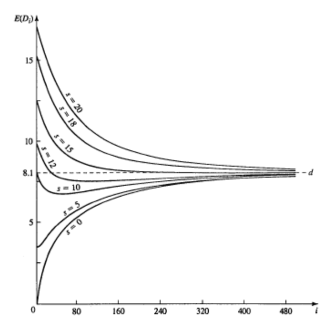
Transient vs. Steady-State

Definitions

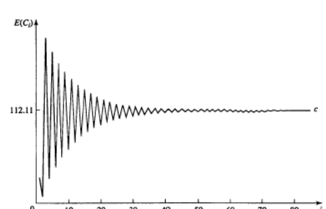
- ***Transient***: The period during which system state response is depends on the initial starting conditions.
- ***Steady-state***: System state after a long time – i.e. the system's state is independent of the initial starting conditions.



M/M/1 queue, *E*(delay in queue), different number of customers *s* present initially:

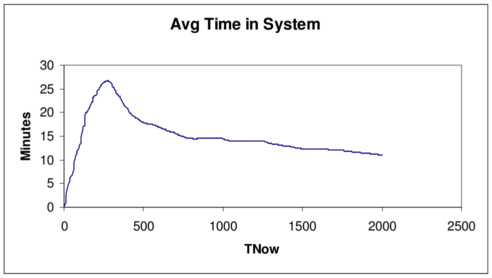


Inventory system, *E*(cost in month *i*):

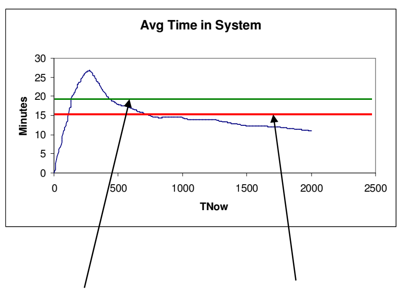


Transient Condition Example

Below, I have produced a plot of average time in system in an M/M/1 ($\rho = 0.90$) system. The observations are from a single run and are taken 5 minutes apart. The initial condition of the system was empty and idle.



- Typically (though not always) we are interested in the steady state performance of the system.
- If we include the transient, we get a different result than if we exclude it.
- There are two ways to get around this issue:
  - Run the model for a very long time (costly).
  - Chop off the transient state (tricky).



Model Nomenclature

We generally use different techniques to analyze the results of simulations, depending on the type of model we're running.

- **Types: Terminating simulation and Non-Terminating**

- **Terminating simulation:** There is a "natural" event that suggests a length for each run. Statistical analysis of terminating simulations is a ***lot*** easier.
  - **Examples**
    - A inventory planning model with a fixed horizon.
    - A contract to build four oil rigs at the Halifax shipyards.
- **Non-terminating**: No natural event to end the model.
  - **Examples**
    - 24/7 shopping/business
    - Networks and telecommunications