Case Study

The Cakes' shop Problem

An owner of a bakery shop would like to determine how many 10-inch birthday cakes he should produce each day in order to maximize his profit. His present method of determining the quantity to bake is based on his best guess. For example if he estimates the daily demand will be five cakes, then five cakes will be produced for the day's operation. Since it is more economical to process all the cakes in one batch, all five cakes would be produced early in the morning.

The production costs are $2.00 per cake. And the profit for each cake is $2.5. However, If over estimates the daily demand, some cakes will be left over at the end of the day. The policy is to sell all leftover cakes to a local store that specializes in day-old items. He is currently receiving $1.50 per cake for the surplus cakes, thus incurring a loss of $0.50 per cake.

Unfortunately, daily decision of how many cakes to produce is difficult because the daily demand is uncertain. The owner has experienced some days when there was no demand, and yet one day he experienced a demand of eight cakes of the birthday cakes.

Ideally, he wanted a recommendation on the day production quantity for the birthday cakes that would maximize his profit over the long run.

What he needs is a way to carry out his trial-and-error procedure without actually performing the experiments.

Actually, computer simulation can be used to experiment with production sizes in just this manner. In the simulation technique we will need to develop a mathematical model and a computer program that describe or recreate the bakery's shop operation for the sale of white birthday cakes. In the simulation model we will choose a trial production quantity, simulate a number of daily demand, and then compute the resulting total profit. Then by choosing other production quantities, we can continue this trial-and –error procedure until we find what appears to be best profit maximizing production quantity. While this trial-and –error procedure will not guarantee an optimal solution, the simulated "best" solution should be close to the optimal solution and thus a very good production-quantity decision.

Model development:

Let

x = number of 10-inch birthday cakes produced

d =daily demand for 10-inch birthday cakes

z =daily profit associated with producing x 10-ich birthday cakes

When writing a mathematical model to describe how profits are related to the production quantity x and d , we need only consider two separate case:

1) The production quantity is less than or equal to the daily demand
2) The production quantity is greater than the daily demand.

Case 1: The production quantity is less than or equal to demand

If $x \leq d$, $z = 2.5x$

Case 1: The production quantity is greater than the demand

If $x > d$ $z = 2.5d + (x-d)(-0.5)$

$Z = 3.00d - 0.5x$

To develop a general model that are appropriate for similar problems, we must introduce some additional notation.

Let

p = selling price for each cake

c = cost of each unit

s = day-old price

Case 1: The production quantity is less than or equal to demand

If x<= d, $z = (p-c)\ x$

Case 1: The production quantity is greater than the demand

If x > d $z = (p-c)\ d + (x-d)\ (s-c)$

$Z = (p-s)\ d - (s-c)\ x$

<u>Generating daily demand</u>

Since we want the model to be a good representation of the real situation, it is important that the generated daily demand be a good representation of the acual daily demands that exist for the birthday cakes.

Assume that we have available data showing the daily demand during the past month (10 days of operation) This history of daily demand is shown in the table below

Where $relative\_frequency = \frac{frequency\_of\_observation}{total\_number\_of\_observations}$

Daily demand d	Frequency of Days observed	Relative frequency
0	1	0.05
1	2	0.1
2	1	0.05
3	2	0.1
4	3	0.15
5	6	0.3
6	3	0.15
7	1	0.05
8	1	0.05
	Total	1

If we believe that this relative frequency distribution is representative of the future pattern of daily demand, we can use it as the basis for generating hypothetical daily demand in our simulation model. This is a critical part of any simulation study and needs serious consideration by both the analyst and the decision maker. If the demand distribution is not an accurate representation of the actual demand the simulation results will be of little value. In general decision models based on inaccurate or non representative inputs will not provide useful output information for the decision maker. We must now develop a method of simulating daily demand d=0, d=1,d=2,…etc. Our method should be in the long run generate a demand of o units approximately 5% of the time, a demand of 1 unit approximately 10% of the time, and so on. First we explain that will a simple method, where you need nothing more than a paper and a pencil.

Manual Simulation:

Take a sheet of paper and cut it into twenty equal pieces. Following the historical daily demand frequency in the table, write the number zero on one piece. On two of the remaining pieces write the number one, which stands for the demand of one unit. Then on one piece write the number two, on two pieces write the number three… and so on.

Check the numbers you have written carefully, because this "deck" of twenty pieces will be used in our simulation of the process. Note that your deck has 5% 0's. 10%1's, 5% 2'S, and so on. And represents the historical relative frequency distribution of the above table Now shuffle your deck of twenty pieces of paper so as to thoroughly mix up the numbers. Since the slips are all the same size, if we select one slip of paper at random, we will have simulated drawing a specific daily demand from the distribution shown in the table.

Let us now see how we can use this "deck" in the simulation of the shop operation. The first step is the selection of the production quantity, Assume (x=3). The second step in the simulation process is to select a "hypothetical" daily production quantity. Now use the deck of twenty slips of paper to generate a demand by selecting one slip of paper at random. Suppose the first slip drawn has a 5 written on it. We shall then use a demand of 5 cakes for the first simulated day of bakery shop operation. This level of demand corresponds to an underproduction of 2 cakes. Since $x<d$, we can computer our first day's profit using the expression $2.5x=25(3)=\$7.5$. i.e. Total profit of \$7.5.

We generate a hypothetical demand for the second day of our simulation by returning the previously selected slip of paper to the deck, reshuffling all twenty pieces of paper, and then drawing another slip at random. And compute the profit z of the second day then add it to the profit of the first day to upgrade the total profit.

And so on. Here is an example of 10-day simulation results for a production quantity of x=3

Day	Generated demand	Daily profit	Total profit
1	5	7.5	7.5
2	1	1.5	9
3	6	7.5	16.5
4	3	7.5	24
5	4	7.5	31.5
6	4	7.5	39
7	3	7.5	46.5
8	0	-1.5	45
9	5	7.5	52.5
10	6	7.5	60

Now Consider changing the production rate. Using the same set of hypothetical daily demand for x=1,2,3,4,5,6,7,8.

Here is an example of a Ten-Day Simulation Results for the x's

Production Size	Ten Day Simulated profit $
1	25
2	44
3	60
4	79
5	90
6	93
7	91
8	89

From the table it is clear that the best production quantity that maximizes the profit is at x=6. The results are based on only 10-day simulation.

A much longer simulation period is required to develop confidence in our best production quantity conclusion