CHAPTER

1

MDEN471

Mechanical Vibration

Course Instructors

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Introduction to Vibration

Syllabus

1. Introduction and Fundamentals of Vibrations
2. Free vibrations of SDOF systems
3. Harmonically Excited Vibration of SDOF systems
4. Vibration Under General Forcing Conditions
5. Systems of two degrees of freedom
6. Vibration control.
7. Vibration measurement.

Introduction to Vibration

Course Grade Distribution (100 Points)

1. Assignments : 20 Points
   - Laboratory Reports:
   - Tutorial Assignments
   - Project

2. Quizzes: 20 Points
   - Laboratory
   - Tutorial
   - Lecture

3. Mid term exam: 20 Points

4. Final Exam: 40 Points

Introduction to Vibration

Classroom rules

- أي محاولة للغش في أي من إمتحانات أعمال السنة أو إمتحان منتصف العام، أو نقل تقارير ستقابل بعقوبة تصل إلى الحصول على صفر في أعمال السنة.
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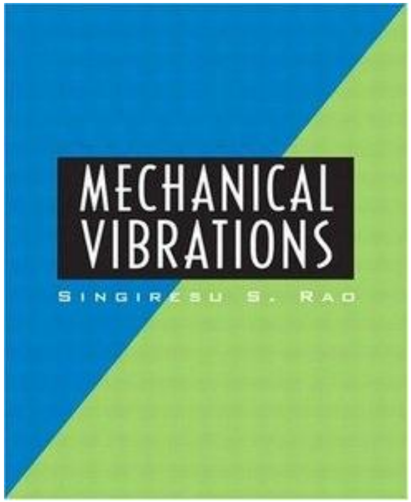
Introduction to Vibration

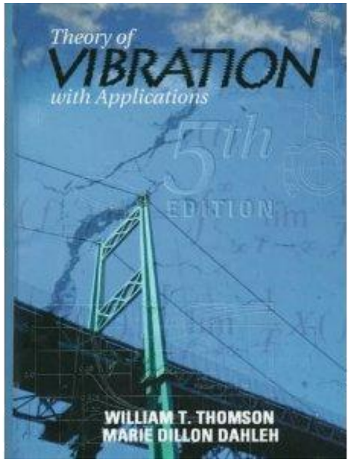
Course Material

- Lecture Slides



- S.S. Rao, Mechanical Vibration, Pearson

- W. T. Thomson, Theory of Vibration with Applications (5th Edition)



Course Website

http://www.facebook.com/groups/MDPN471.2013/

Website includes:

- PowerPoint slides
- Grades and scores
- Problem sets
- Handouts
- Other material
- **Announcements**

Why Study Vibration ??

Introduction to Vibration

Earthquake-induced vibration

00:44

Introduction to Vibration

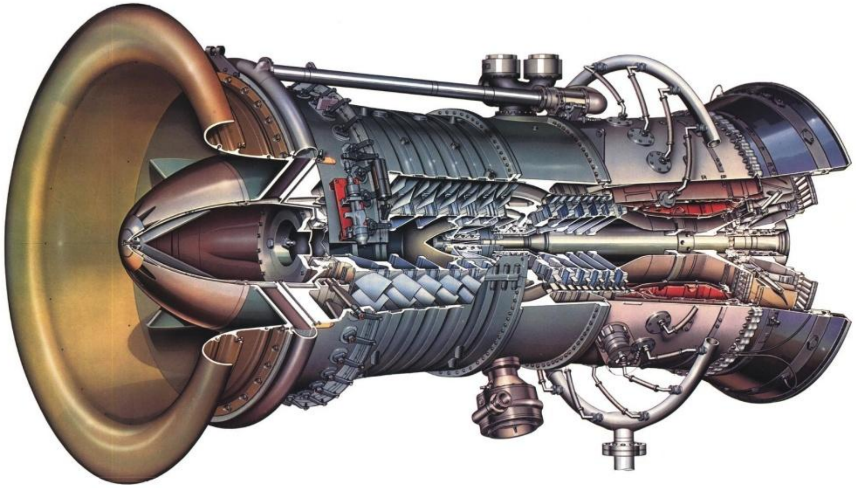
Wind-induced vibration

Tacoma Narrows bridge during wind-induced vibration. The bridge opened on July 1, 1940, and collapsed on November 7, 1940.

Introduction to Vibration

Turbomachinery

Jet Engines



Power Plant Turbine

Introduction to Vibration

Other Examples

Wind Turbine

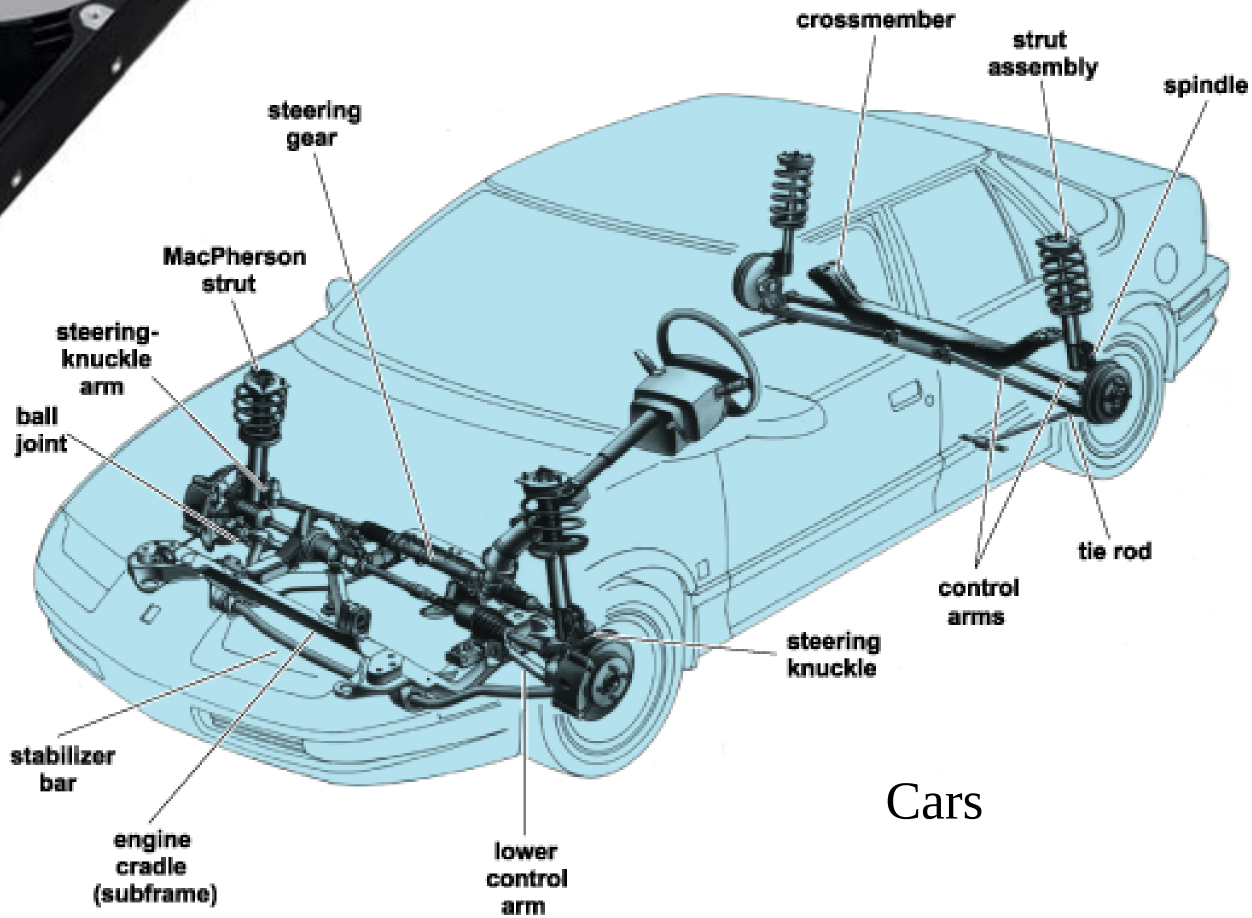
Washing Machines

Compressors and
Air Conditioners

Introduction to Vibration

Other Examples

Hard Drives



Cars

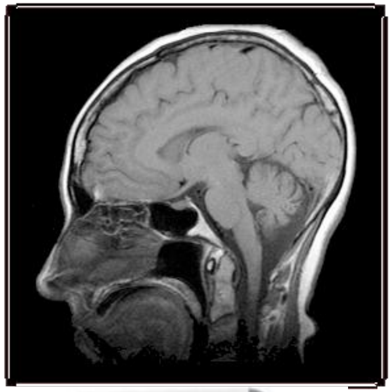
Is all Vibration Bad??

Desirable Vibrations

Musical Instruments

Time Keeping

Medical Diagnostics Equipment



Basic Concepts of Vibrations

Vibration :Any motion that repeats itself after an interval of time. [Ex. swinging of a pendulum]

A vibratory system includes:

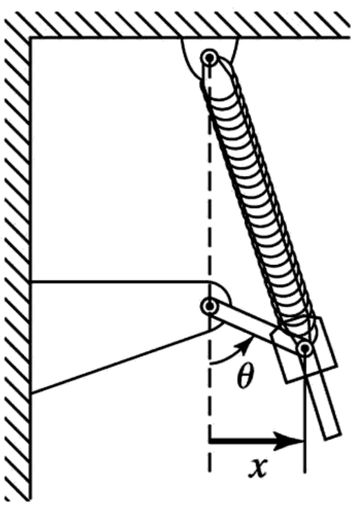
- means for storing potential energy (springs or elasticity)
- means for storing kinetic energy (mass or inertia)
- means for gradually losing energy (damper).

The vibration of a system involves the transfer of its potential energy to kinetic energy and vice versa. If damping exists, energy is dissipated in each cycle.

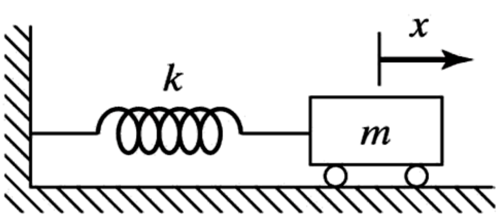
Basic Concepts of Vibrations

Degree of freedom of the system: The minimum number of independent coordinates required to determine completely the positions of all parts of a system at any instant of time.

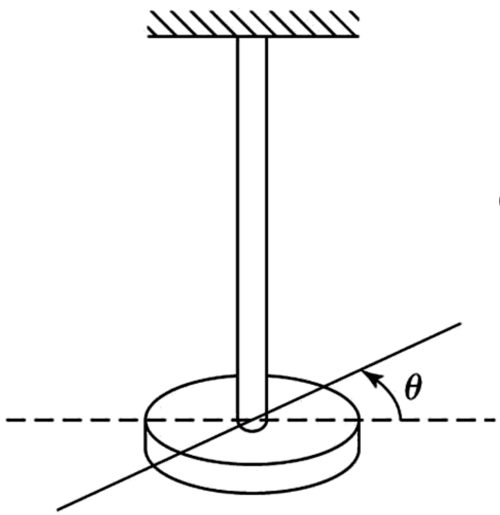
Example of SDOF systems



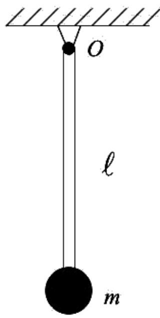
(a) Slider-crank-spring mechanism



(b) Spring-mass system

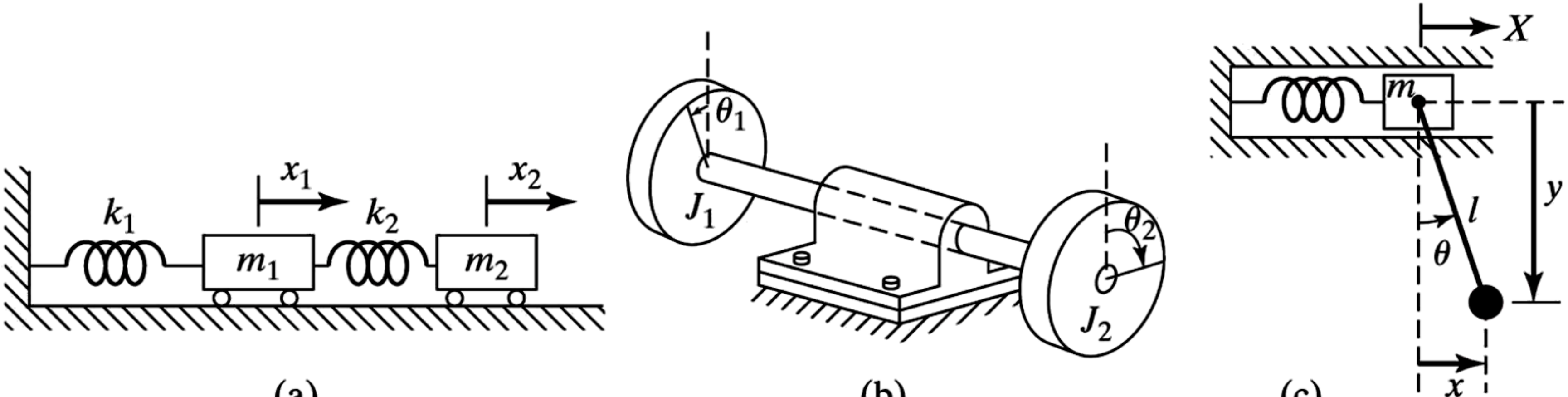


(c) Torsional system



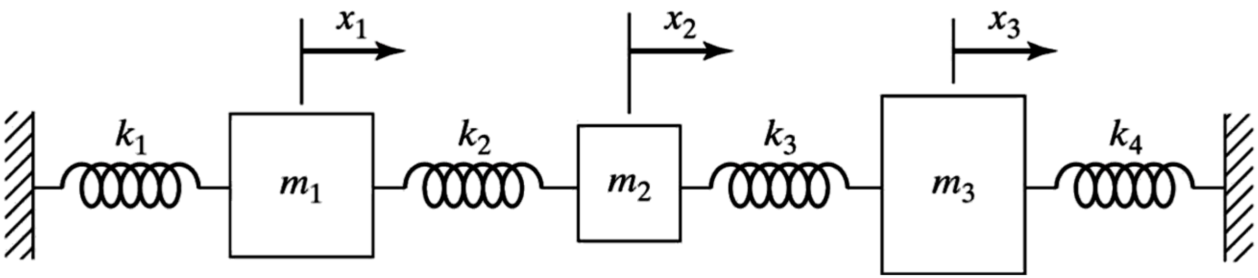
Introduction to Vibration

Examples of two and three DOF systems:

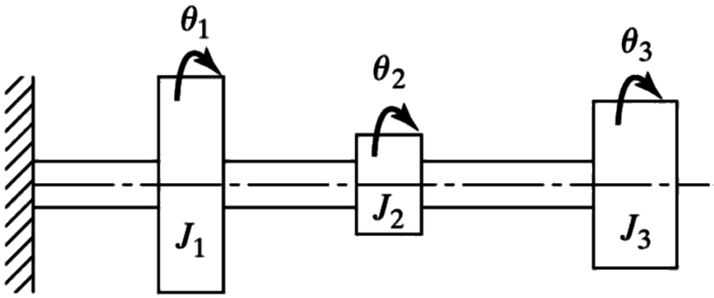


(a) (b) (c)

2-DOF

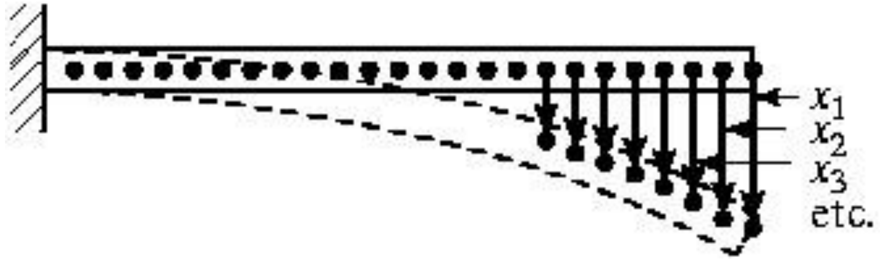


3-DOF



Basic Concepts of Vibrations

- Example of Infinite-number-of-degrees-of-freedom system:



- *Infinite* number of degrees of freedom system are termed *continuous* or *distributed* systems

- *Finite* number of degrees of freedom are termed *discrete* or *lumped* parameter systems

- More accurate results obtained by increasing number of degrees of freedom

Introduction to Vibration

Classification of Vibration

- Free
- Forced

  – Undamped

  – Damped

    » Linear

    » Nonlinear

      ❑ Deterministic

      ❑ Random

Classification of Vibration

- **Free Vibration:**
  A system is left to vibrate on its own after an initial disturbance and no external force acts on the system. [E.g. simple pendulum]

- **Forced Vibration:**
  A system that is subjected to a repeating external force. [E.g. oscillation arises from diesel engines]

Introduction to Vibration

Classification of Vibration

- Undamped Vibration:

  When ***no*** energy is lost or dissipated in friction or other resistance during oscillations

- Damped Vibration:

  When ***any*** energy is lost or dissipated in friction or other resistance during oscillations

- Linear Vibration:

  When ***all*** basic components of a vibratory system, i.e. the spring, the mass and the damper behave linearly

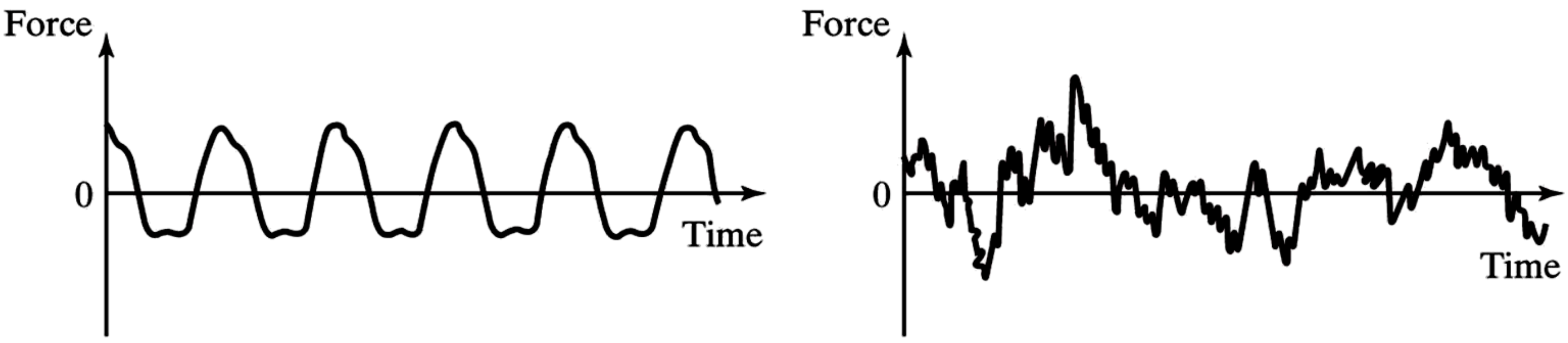
- Nonlinear Vibration:

  If ***any*** of the components behave nonlinearly

Classification of Vibration

- **Deterministic Vibration:**
  If the value or magnitude of the excitation (force or motion) acting on a vibratory system is known at any given time

- **Nondeterministic or random Vibration:**
  When the value of the excitation at a given time cannot be predicted



(a) A deterministic (periodic) excitation (b) A random excitation

Introduction to Vibration

Spring Elements

- *Linear* spring is a type of mechanical link that is generally assumed to have negligible mass and damping

- *Spring force* is given by:

$$F = kx$$

F = spring force,
k = spring stiffness or spring constant, and
x = deformation (displacement of one end with respect to the other)

- *Work done (U)* in deforming a spring or the strain (potential) energy is given by:

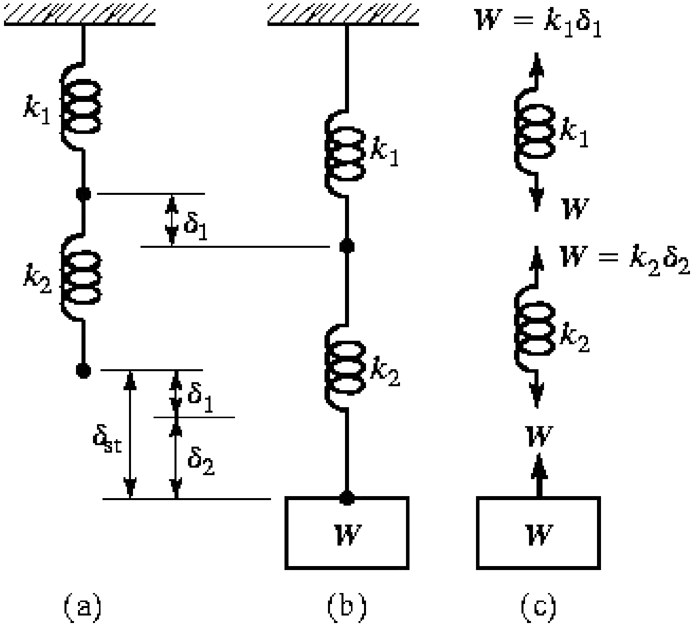
$$U = \frac{1}{2}kx^2$$

Spring Elements

- Combination of Springs:

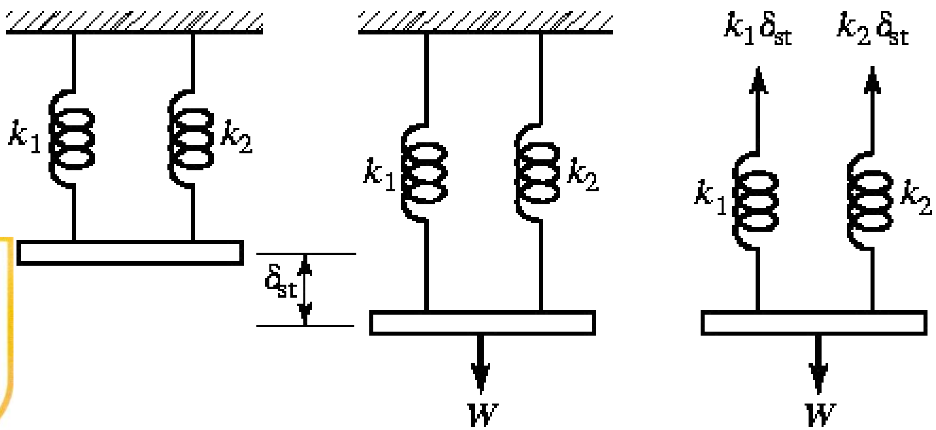
  2) Springs in series – if we have *n* spring constants k_1, k_2, …, k_n in *series*, then the equivalent spring constant k_{eq} is:

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \ldots + \frac{1}{k_n}$$



- Combination of Springs:

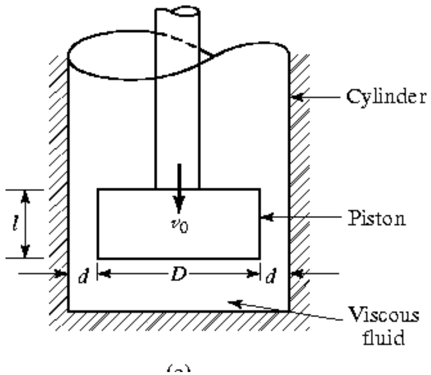
  1) Springs in parallel – if we have *n* spring constants k_1, k_2, …, k_n in *parallel*, then the equivalent spring constant k_{eq} is:



$$k_{eq} = k_1 + k_2 + \ldots + k_n$$

Damping Elements

- **Viscous Damping:**
  Damping force is proportional to the velocity of the vibrating body in a fluid medium such as air, water, gas, and oil.



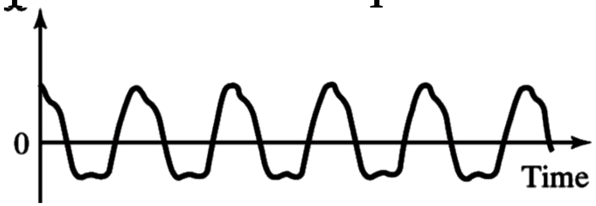
$$F = cv$$

- **Coulomb or Dry Friction Damping:**
  Damping force is constant in magnitude but opposite in direction to that of the motion of the vibrating body between dry surfaces

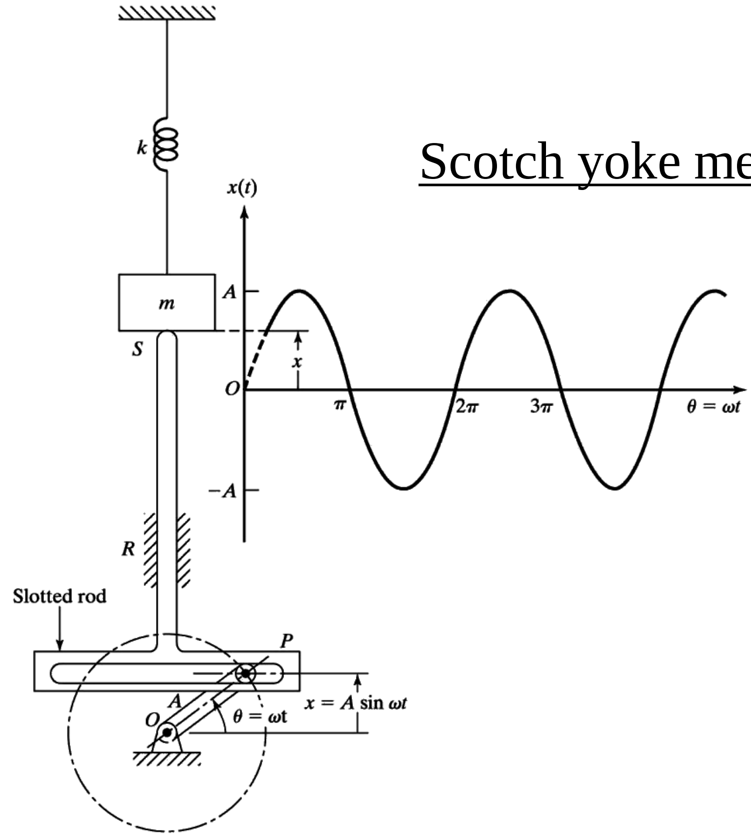
- **Material or Solid or Hysteretic Damping:**
  Energy is absorbed or dissipated by material during deformation due to friction between internal planes

Harmonic Motion

- Periodic Motion: motion repeated after equal intervals of time



- Harmonic Motion: simplest type of periodic motion



Scotch yoke mechanism

Introduction to Vibration

Harmonic Motion - vectorial

- Displacement (x): *(on horizontal axis)*

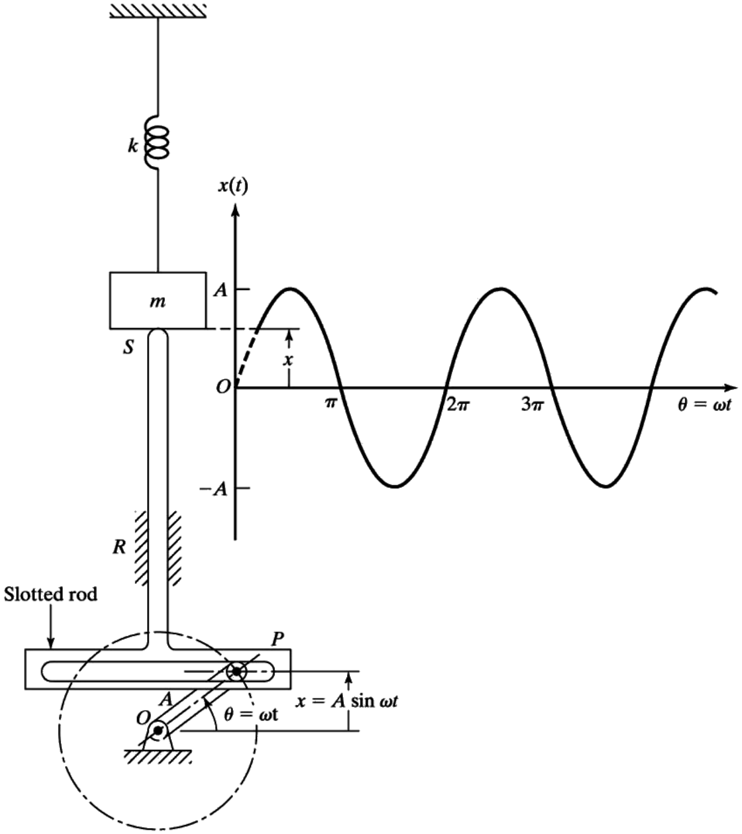
$$x = A\sin\theta = A\sin\omega t$$

- Velocity:

$$\frac{dx}{dt} = \omega A\cos\omega t$$

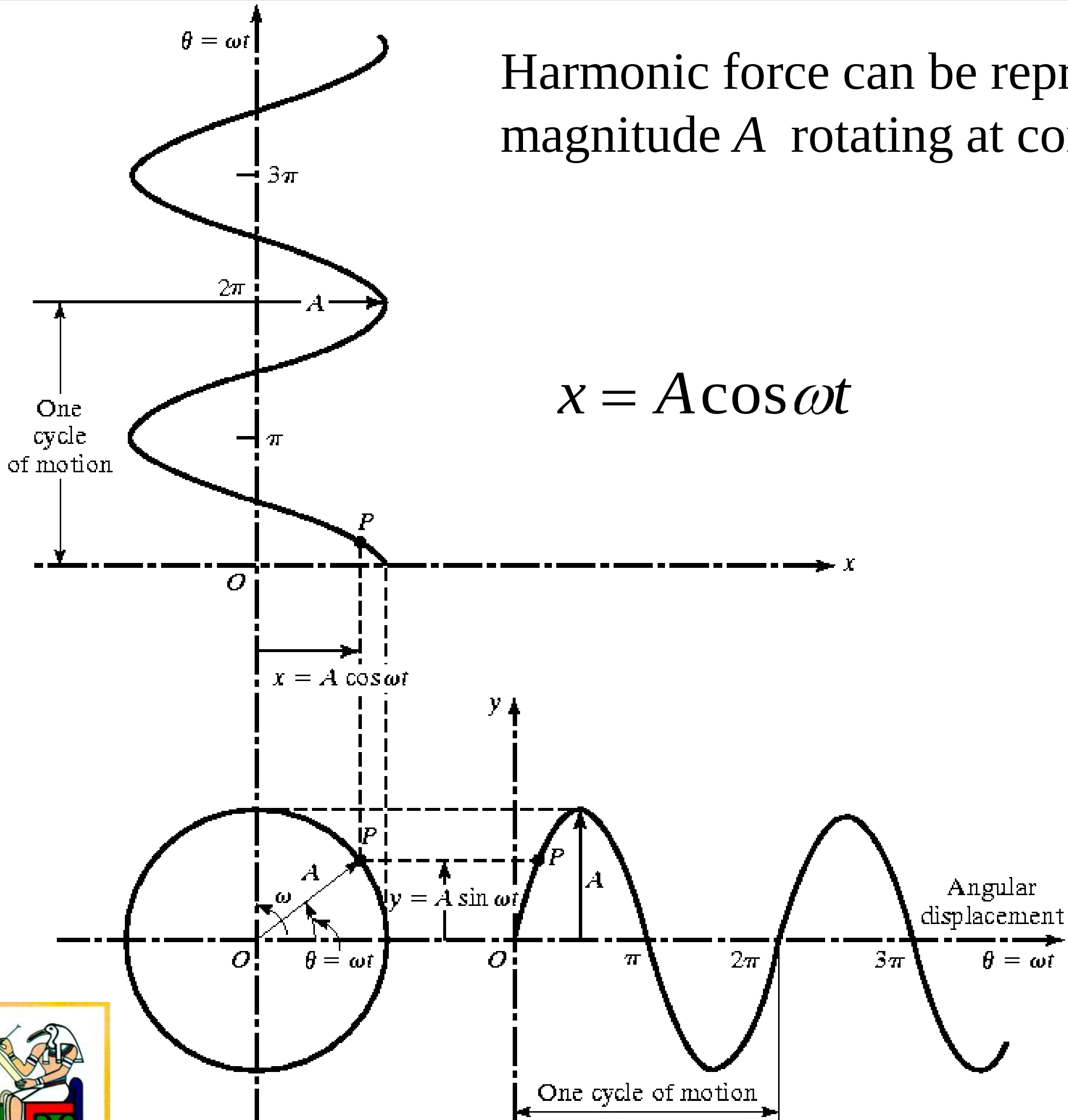
- Acceleration:

$$\frac{d^2x}{dt^2} = -\omega^2 A\sin\omega t = -\omega^2 x$$



Introduction to Vibration

Harmonic Motion - vectorial



Harmonic force can be represented by vector *OP* of magnitude *A* rotating at constant velocity ω.

$$x = A\cos\omega t$$

$$\overrightarrow{OP}\ (x,y)$$

Rotating Vector

$$y = A\sin\omega t$$

Harmonic Motion - complex

- Complex number representation of harmonic motion:

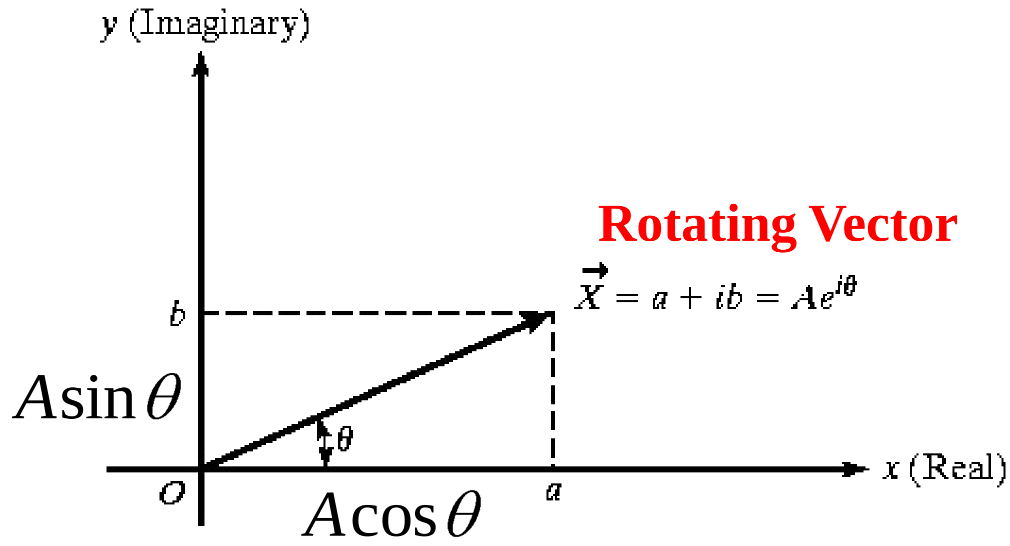
$$\vec{X} = a + ib$$

where $i = \sqrt{(-1)}$ and a and b denote the real and imaginary x and y components of X, respectively.

$$\vec{X} = A\cos\theta + iA\sin\theta$$

where $$A = \sqrt{(a^2 + b^2)}$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$



$$\vec{X} = A\ (\cos\theta + i\sin\theta) = Ae^{i\theta}$$

Harmonic Motion - complex

Displacement = $\mathrm{Re}[Ae^{i\omega t}] = A\cos\omega t$

Velocity = $\mathrm{Re}[i\omega Ae^{i\omega t}] = -\omega A\sin\omega t$

$$= \omega A\cos\ \ \omega t + \pi / 2$$

Acceleration = $\mathrm{Re}[-\omega^2 Ae^{i\omega t}]$

$$= -\omega^2 A\cos\omega t$$

$$= \omega^2 A\cos\ \ \omega t + \pi$$

Harmonic Motion

- Definitions of Terminology:
  - **Amplitude (*A*)** is the maximum displacement of a vibrating body from its equilibrium position
  - **Period of oscillation (*T*)** is time taken to complete one cycle of motion

$$T = \frac{2\pi}{\omega}$$

  - **Frequency of oscillation (*f*)** is the no. of cycles per unit time

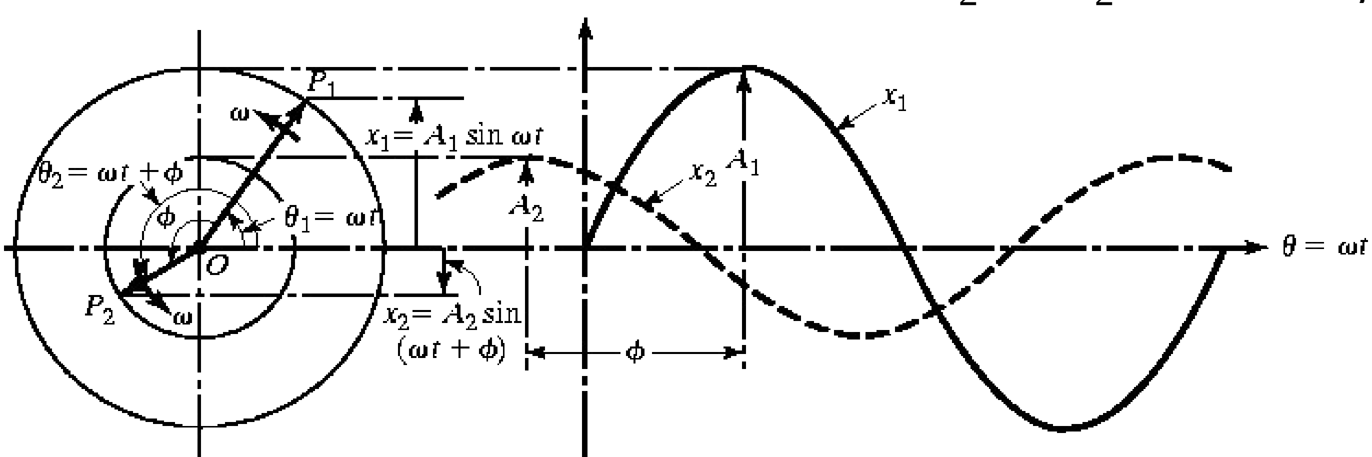
$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

- Definitions of Terminology:
  - **Natural frequency** is the frequency which a system oscillates without external forces
  - **Phase angle (ϕ)** is the angular difference between two synchronous harmonic motions

    Same frequency

$$x_1 = A_1 \sin \omega t$$

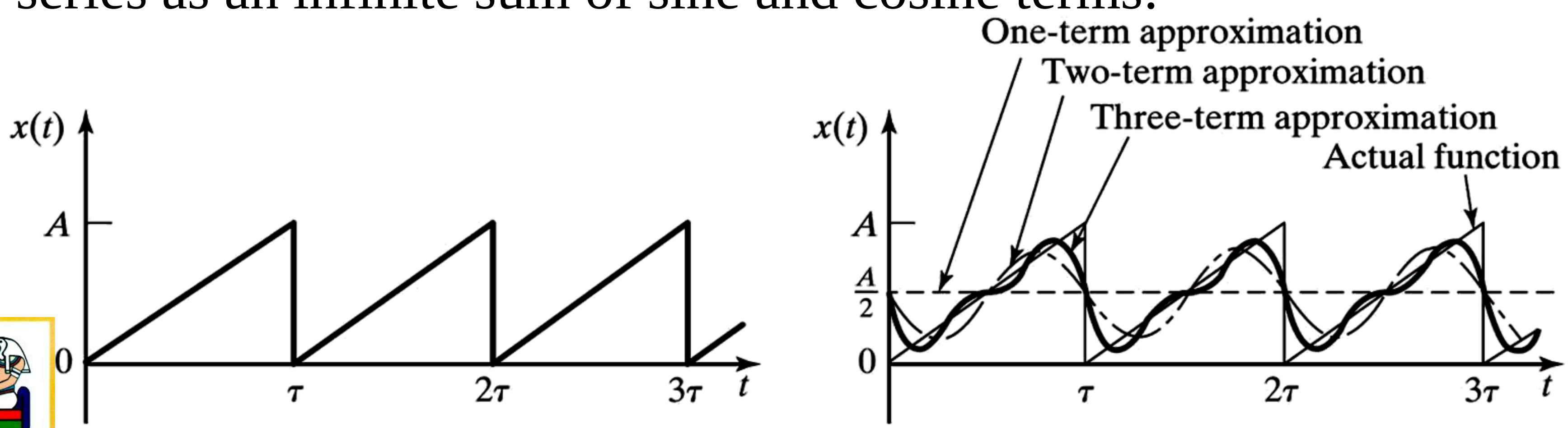
$$x_2 = A_2 \sin(\omega t + \phi)$$



Periodic Motion – Fourier Series

- The motion of many vibratory systems is not harmonic. In many cases the vibrations are periodic.

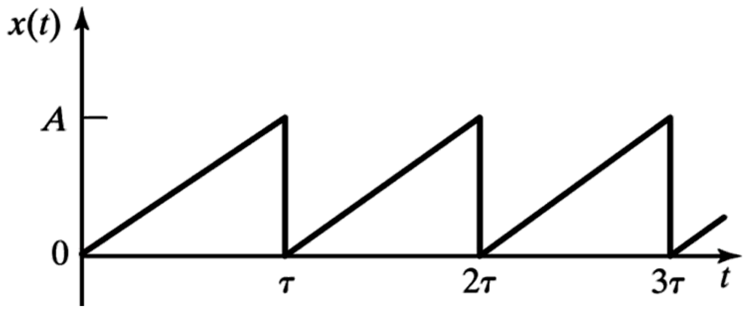
- Any periodic function of time can be represented by Fourier series as an infinite sum of sine and cosine terms:

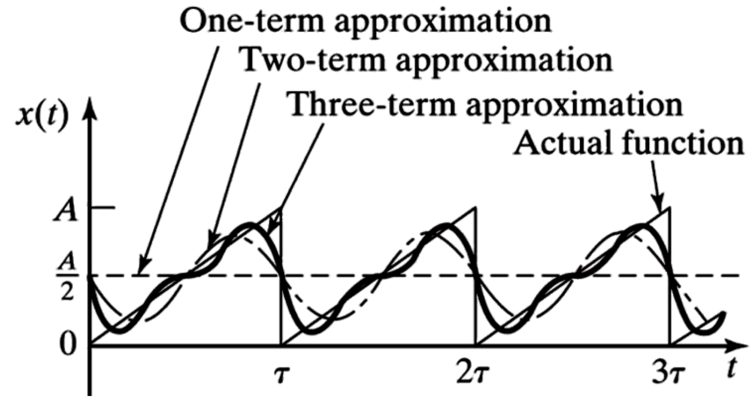


(a) (b)

Periodic Motion – Fourier Series

- Any periodic function of time can be represented by Fourier series as an infinite sum of sine and cosine terms:





$$x(t) = \frac{a_0}{2} + a_1 \cos \omega t + a_2 \cos 2\omega t + \cdots + b_1 \sin \omega t + b_2 \sin 2\omega t + \cdots$$

$$= \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$

$$a_0 = \frac{\omega}{\pi} \int_0^{2\pi/\omega} x(t)\, dt = \frac{2}{\tau} \int_0^{\tau} x(t)\, dt$$

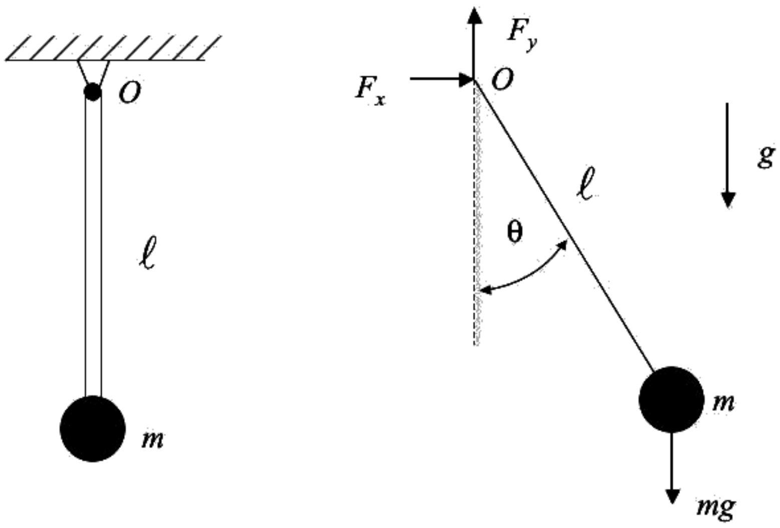
$$a_n = \frac{\omega}{\pi} \int_0^{2\pi/\omega} x(t) \cos n\omega t\, dt = \frac{2}{\tau} \int_0^{\tau} x(t) \cos n\omega t\, dt$$

$$b_n = \frac{\omega}{\pi} \int_0^{2\pi/\omega} x(t) \sin n\omega t\, dt = \frac{2}{\tau} \int_0^{\tau} x(t) \sin n\omega t\, dt$$

Introduction to Vibration

Example: The Pendulum

- Sketch in general position
- Write all forces on free body diagram
- Use Newton's 2nd Law to find the equations of motion



Newton's 2nd Law $\sum \mathbf{M}_0 = J_0 \alpha, \quad J_0 = m\ell^2$

$$J_0 \alpha(t) = -mgl\sin\theta(t) \Rightarrow m\ell^2 \ddot{\theta}(t) + mg\ell\sin\theta(t) = 0$$